

Name: \_\_\_\_\_ UIN: \_\_\_\_\_ Section: \_\_\_\_\_ Score: \_\_\_\_\_

A system has three generators with the following cost curves.

$$C_1(P_1) = 8600 + 13 P_1 + 0.002 P_1^2$$

$$C_2(P_2) = 3400 + 10 P_2 + 0.005 P_2^2$$

$$C_3(P_3) = 1700 + 4 P_3 + 0.012 P_3^2$$

$C_1, C_2,$  and  $C_3$  are the generator costs, in \$/hr

$P_1, P_2,$  and  $P_3$  are the generator real power outputs, in MW

Each generator must be dispatched within the following limits

$$25 \text{ MW} \leq P_1 \leq 450 \text{ MW}$$

$$15 \text{ MW} \leq P_2 \leq 500 \text{ MW}$$

$$35 \text{ MW} \leq P_3 \leq 425 \text{ MW}$$

If the total system load is 700 MW, what should be  $P_1, P_2,$  and  $P_3$  to minimize the total system cost  $C_1 + C_2 + C_3$ ? Also give the system marginal cost  $\lambda$ . (Remember that for generators not at a limit, the incremental cost (dC/dP) should be equal to  $\lambda$ .) You may use either the direct method or lambda iteration method. If you use the lambda iteration method, start with  $\lambda_H = 18$  and  $\lambda_L = 10$  and complete up to four iterations.

### Direct Method

$$\lambda = 13 + 0.004P_1 = 10 + 0.01P_2 = 4 + 0.024P_3$$

$$P_1 + P_2 + P_3 = 700$$

Solving these equations results in  $P_1 = 15, P_2 = 306, P_3 = 378$ . Since this results in a limit violation for  $P_1$ , set  $P_1 = 25$  and resolve

$$\lambda = 10 + 0.01P_2 = 4 + 0.024P_3$$

$$25 + P_2 + P_3 = 700$$

Which results in  $P_1 = 25 \text{ MW}, P_2 = 300 \text{ MW}, P_3 = 375 \text{ MW}, \lambda = 13 \text{ \$/MWh}$

### Lambda iteration method

Solve for powers as a function of lambda, subject to limits

$$P_1 = (\lambda - 13)/0.004$$

$$P_2 = (\lambda - 10)/0.01$$

$$P_3 = (\lambda - 4)/0.024$$

$\lambda_H$	$\lambda_L$	$\lambda_M$	$P_1$	$P_2$	$P_3$	$P_{total}$
18	10	14	250	400	416.67	1066.67
14	10	12	25	200	333.33	558.33
14	12	13	25	300	375	700

1. At the optimal solution, what is the value of the system marginal cost  $\lambda$ ?

Answer: 13.00 \$/MWh

2. What is the optimal dispatch of generator three ( $P_3$ ) ?

Answer: 375 MW