ECEN 460, Spring 2026
Power System Operation and Control

Class 16: Power System Economic Modeling

Prof. Adam Birchfield
Dept. of Electrical and Computer Engineering
Texas A&M University
abirchfield@tamu.edu

TEXAS A&M UNIVERSITY ®

Announcements

- Homework #7: 6th edition book problems 6.62, 6.63, 6.65 or 7th edition book problems 7.1, 7.2, 7.4
  - Due Tuesday, March 24th

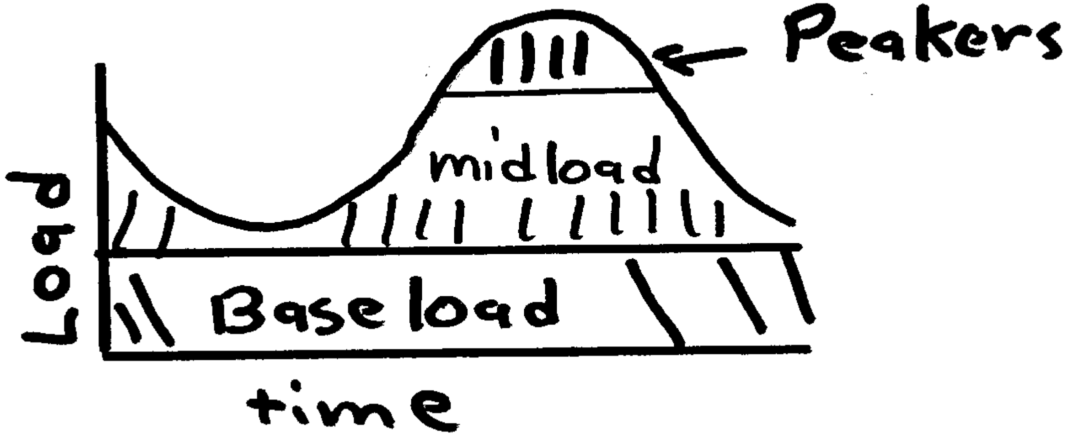
Introduction to Power System Economics

- Today we're going to have introduce the economic side of power system operations
- The key challenge is the economic dispatch (ED) problem, which asks how we can
  - Serve all of the load
  - Using the cheapest dispatch of generators
  - Subject to other reliability and efficiency constraints
- The Optimal Power Flow problem (OPF), which we'll discuss later, combines ED with the Power Flow

Source: http://images.pennnet.com/articles/pe/cap/cap_gephoto.jpg

Generator Costs and Types

- Generator costs depend on the fuel type, unit design, and system conditions
- Traditionally there are three broad categories
  - Baseload: large coal and nuclear, operated 24/7 at maximum
  - Cycling or Midload: coal and gas plants that cycle on and off daily
  - Peakers: smaller gas, petroleum, hydro units that are mainly used during periods of high demand



- Non-dispatchable sources such as wind and solar can be quite variable, but have a low incremental cost. Usually they are operated at the maximum available power.
- For hydro the fuel (water) is free but there may be many constraints on operation: total water availability, reservoir level coordination, downstream flow for wildlife

Steam Plant (Coal)

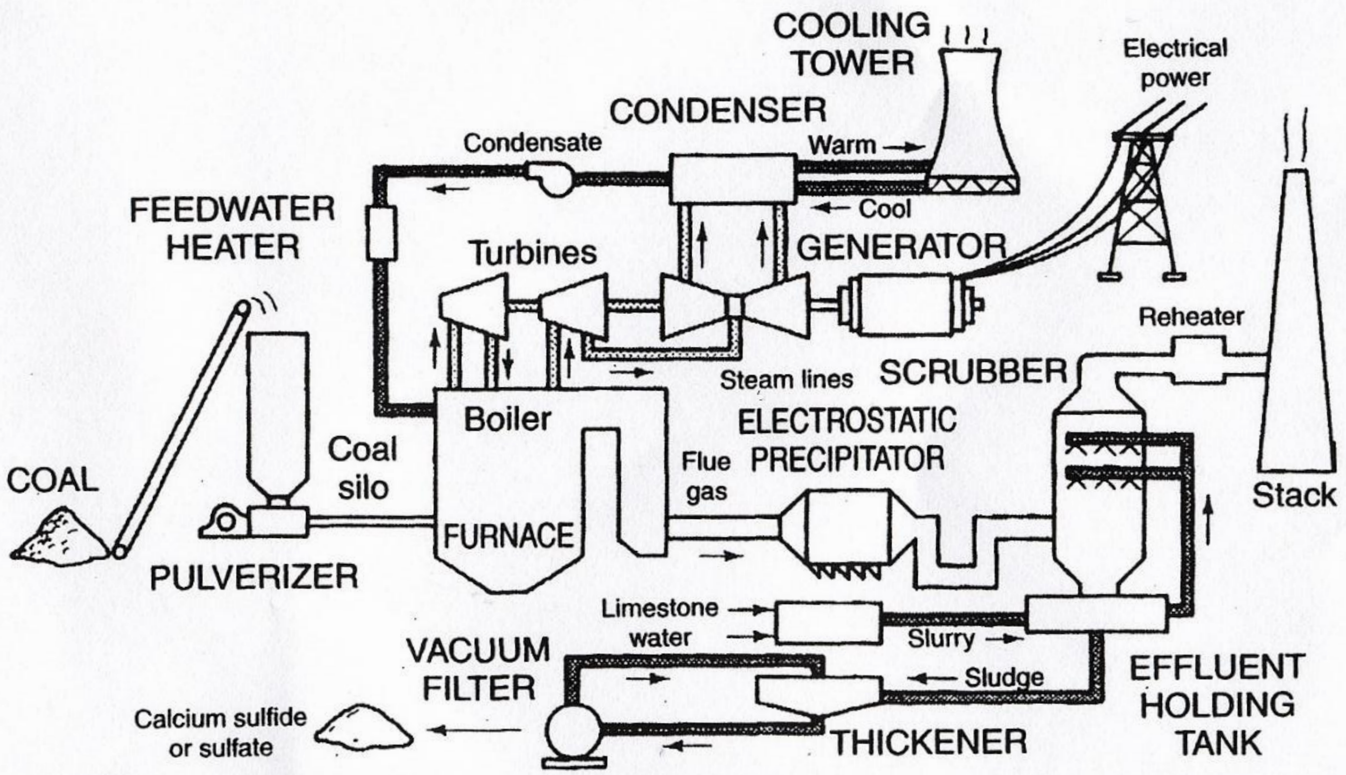
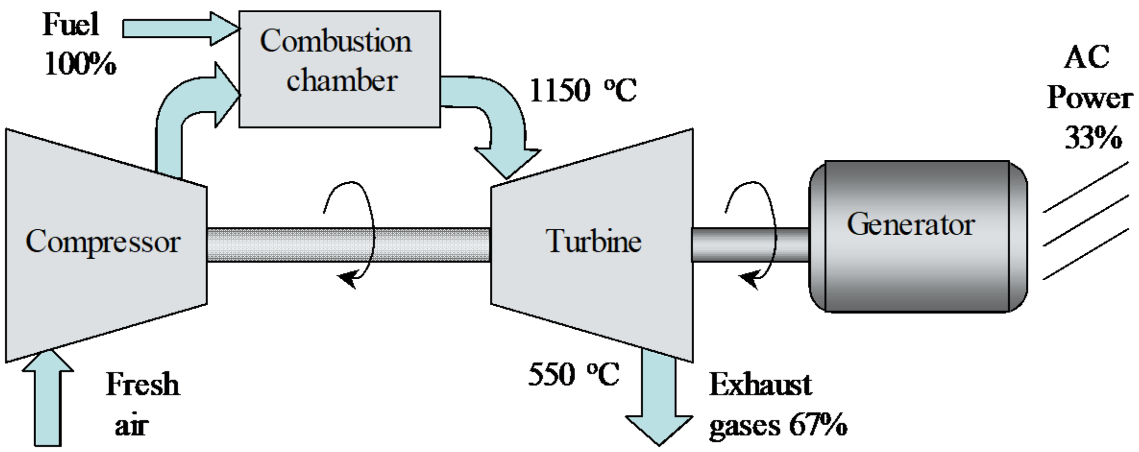


Figure 3.19 Typical modern coal-fired power plant using an electrostatic precipitator for particulate control and a limestone-based SO_2 scrubber. A cooling tower is shown for thermal pollution control. From Masters (1998).

Source: Masters, Renewable and Efficient Electric Power Systems, 2004

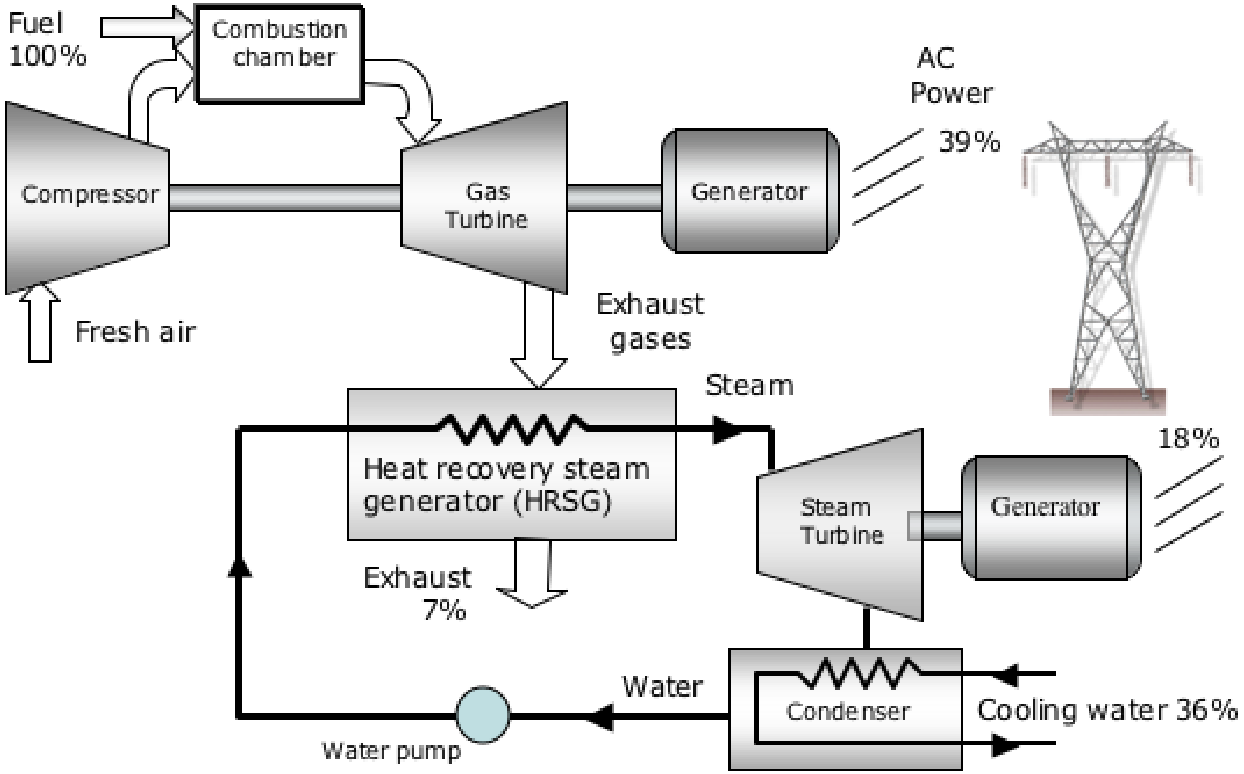
Natural Gas Simple Cycle and Combined Cycle

Simple Cycle



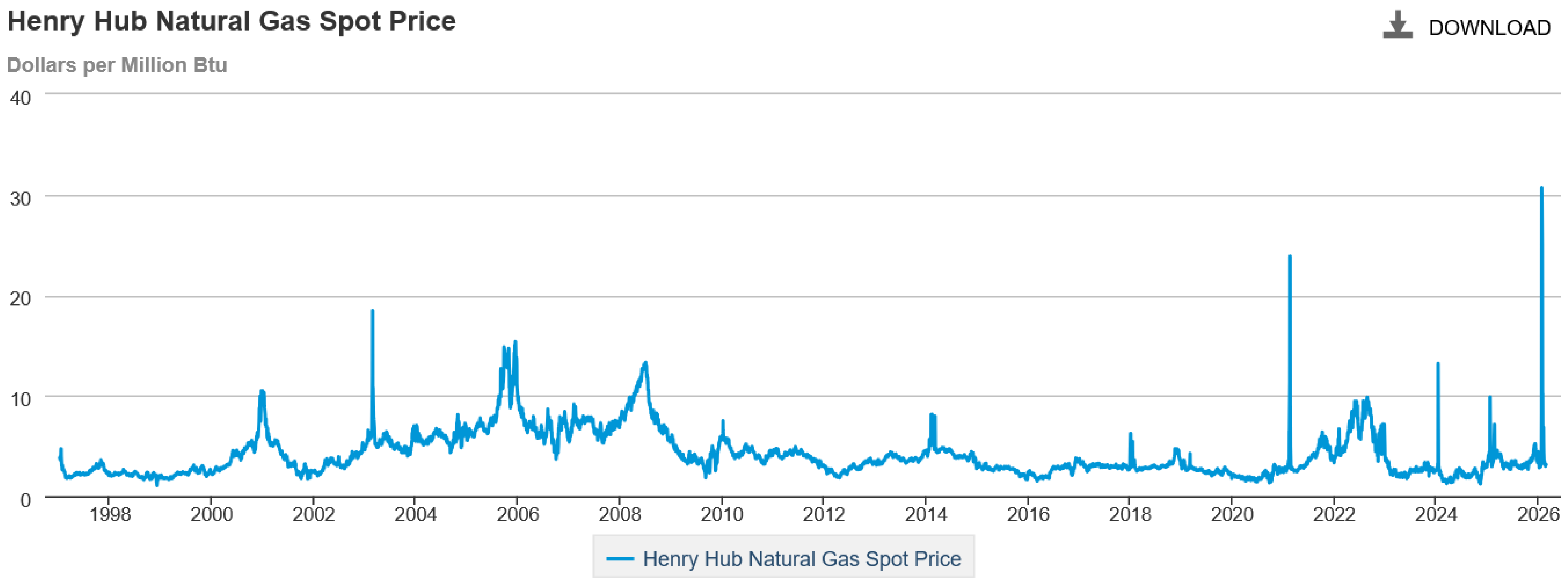
Typical efficiency is around 30 to 35%

Combined Cycle



Efficiencies of up to 60% can be achieved, with even higher values when the steam is used for heating. Fuel is usually natural gas

Variation in Natural Gas Prices



Our recent cold weather set a new record. Marginal cost for natural gas fired electricity price in $/MWh is about 7-10 times gas price.

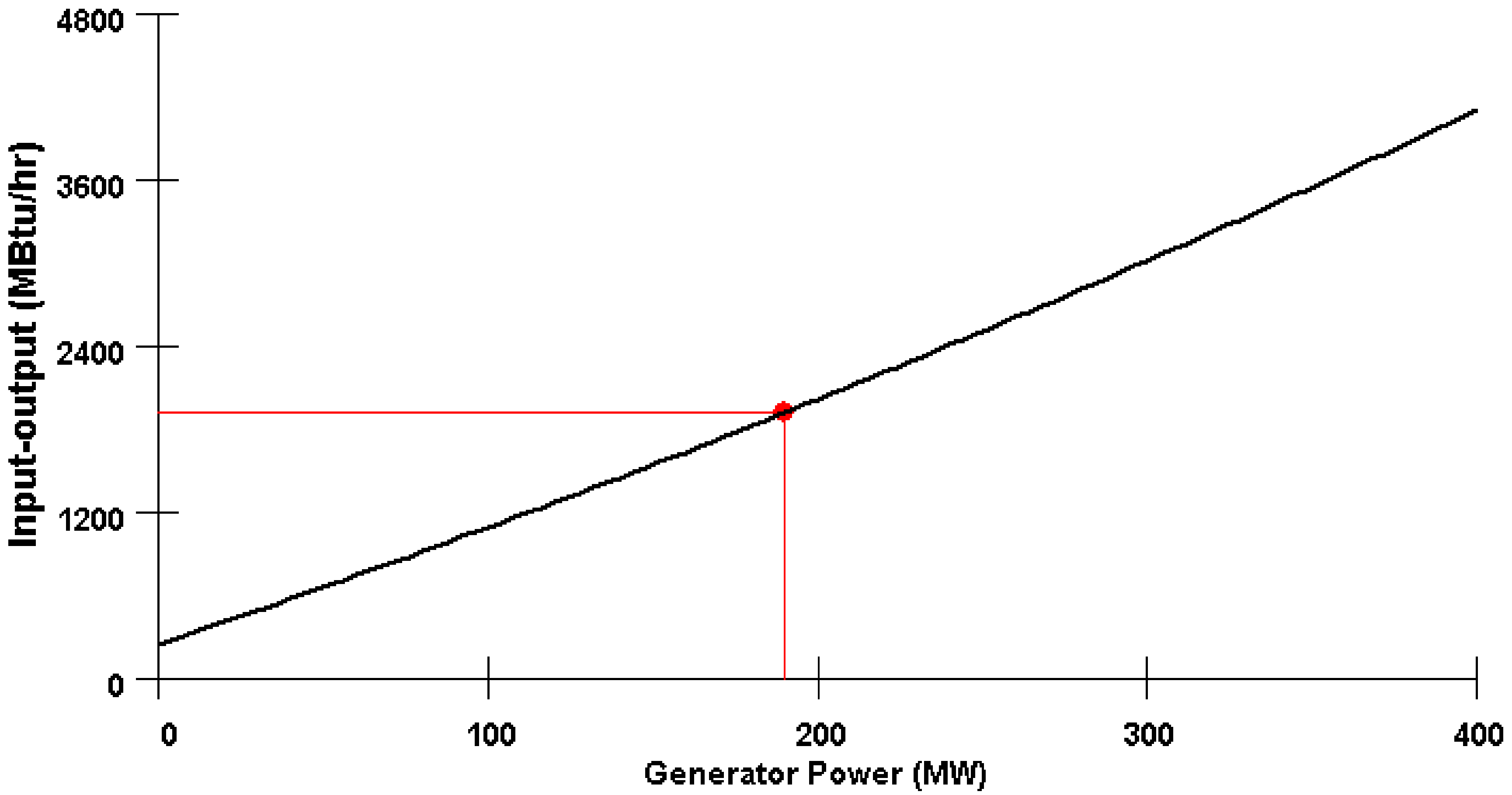
Source: EIA

Generator Cost Curves

- Generator costs are typically represented by up to four different curves
  - input/output (I/O) curve
  - fuel-cost curve
  - heat-rate curve
  - incremental cost curve
- For reference
  - 1 Btu (British thermal unit) = 1054 J
  - 1 MBtu = 1×10^6 Btu
  - 1 MBtu = 0.293 MWh
  - 3.41 Mbtu = 1 MWh

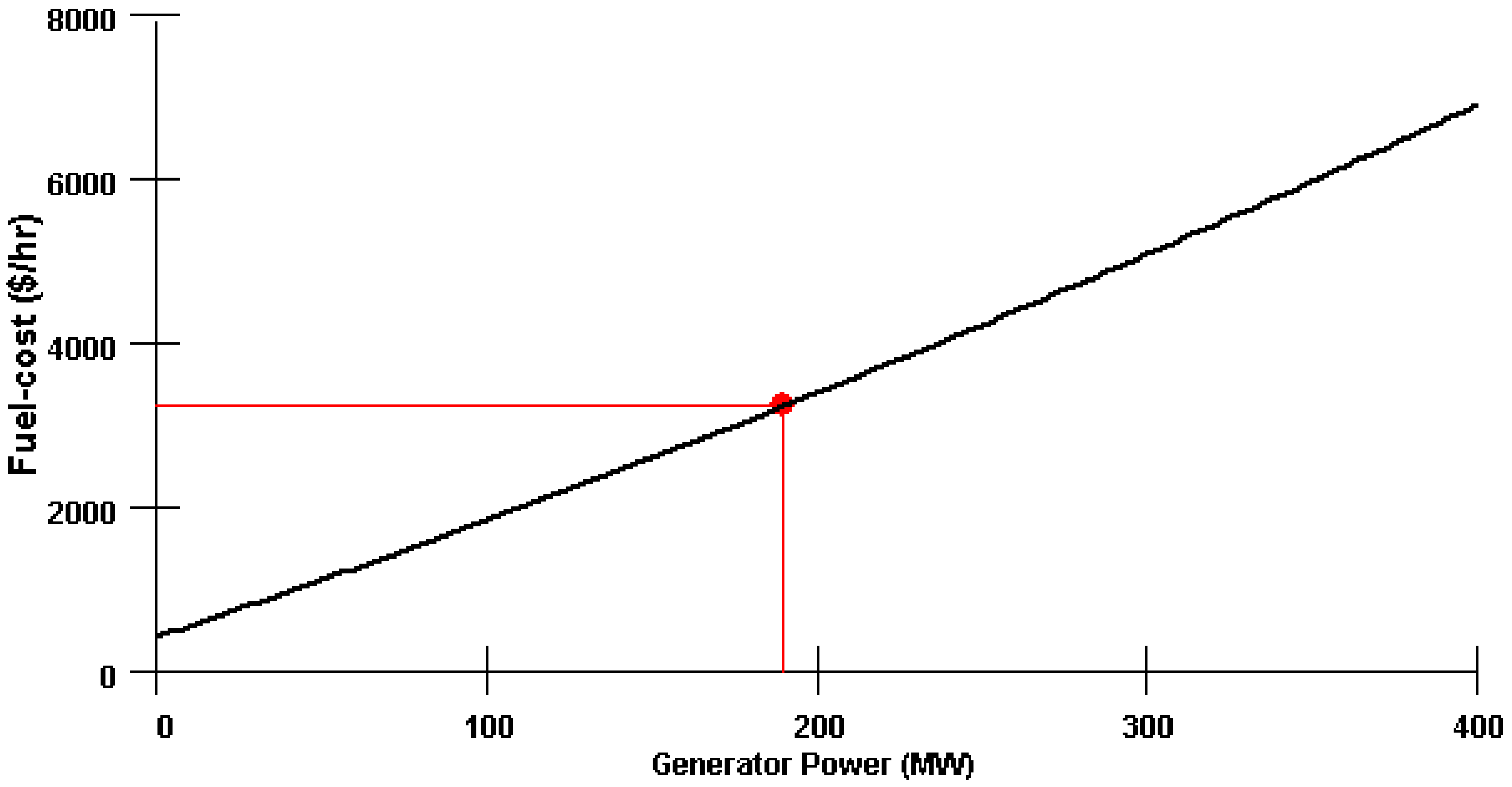
I/O Curve

- The IO curve plots fuel input (in MBtu/hr) versus net MW output.



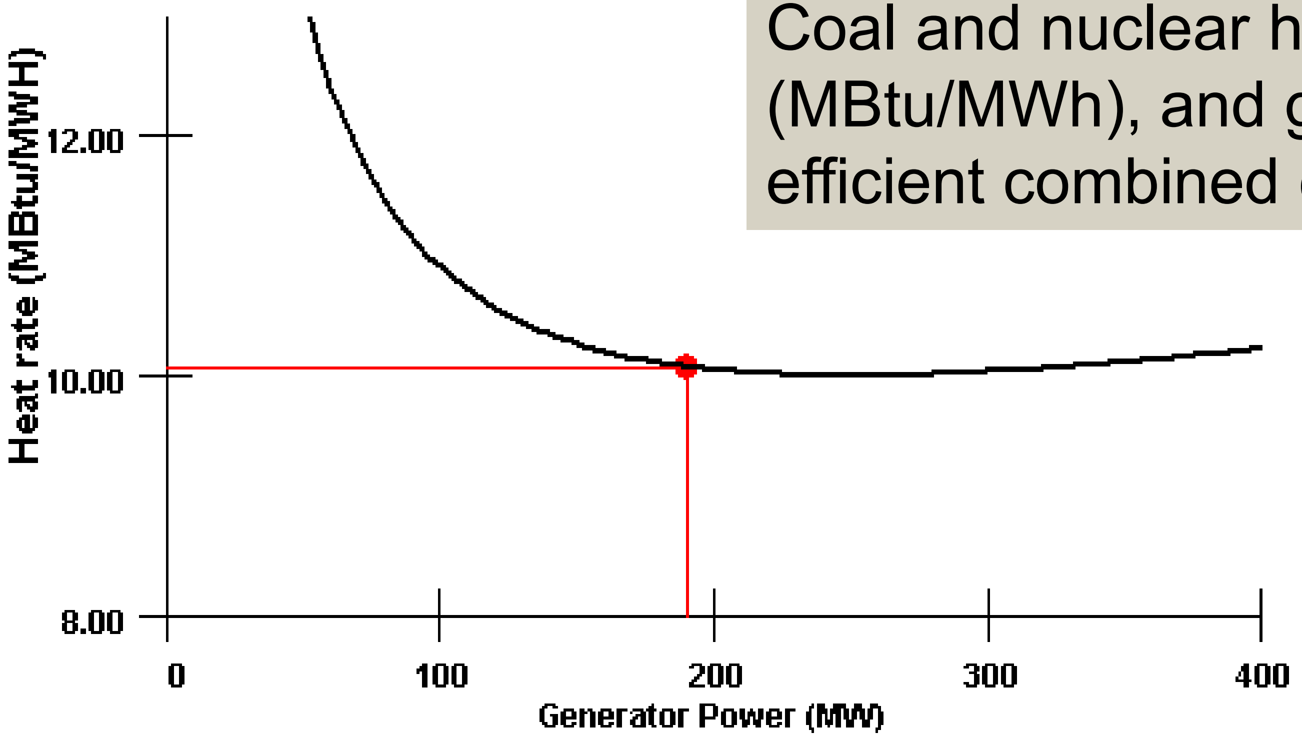
Fuel-cost Curve

- The fuel-cost curve is the I/O curve scaled by fuel cost. Fuel costs vary (with the latest values in Chapter 4 of EIA's Electricity Monthly)



Heat-rate Curve

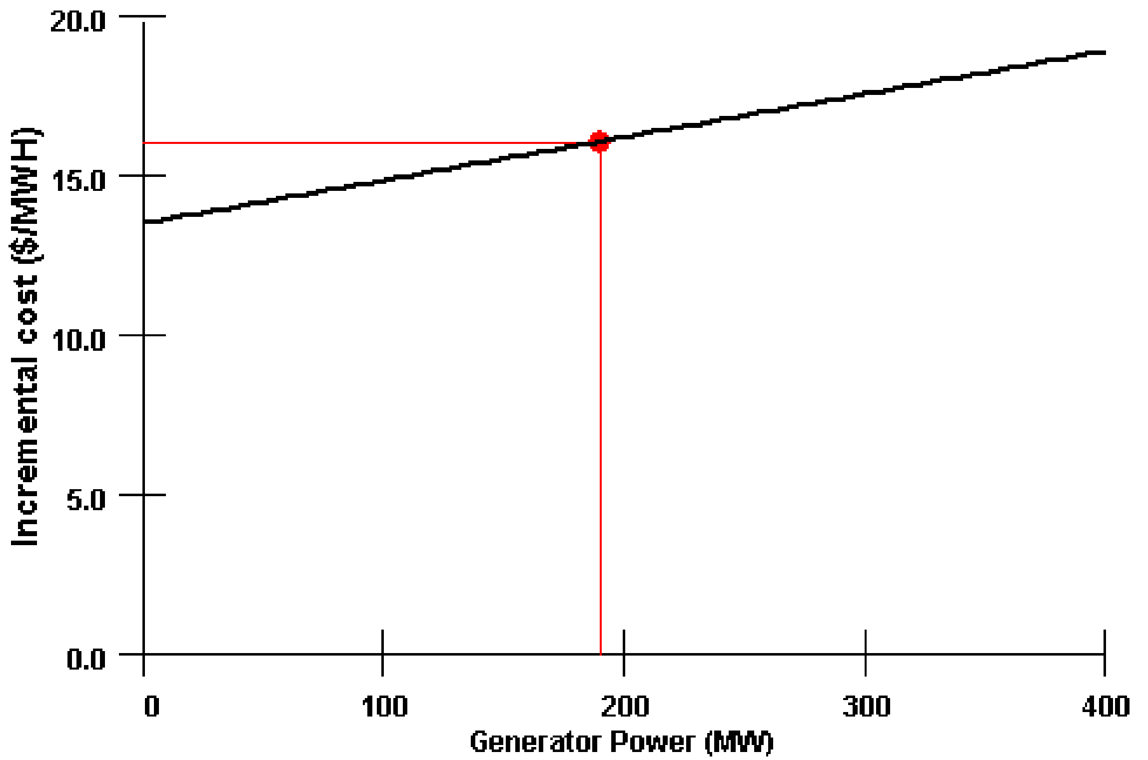
- Plots the average number of MBtu/hr of fuel input needed per MW of output.
- Heat-rate curve is the I/O curve scaled by MW; often heat rates are given in Btu/kWh, which is the Mbtu/MWh value multiplied by 1000



Coal and nuclear heat rates are around 10 (MBtu/MWh), and get down to 6.2 for highly efficient combined cycle plants

Incremental (Marginal) cost Curve

- Plots the incremental $/MWh as a function of MW.
- Found by differentiating the cost curve



Mathematical Formulation of Costs

- Generator cost curves are usually not smooth. However the curves can usually be adequately approximated using piece-wise smooth, functions.
- Two representations predominate
  - quadratic or cubic functions
  - piecewise linear functions
- In 460 we'll generally assume a quadratic representation

$$C_i(P_{Gi}) = \alpha_i + \beta P_{Gi} + \gamma P_{Gi}^2 \text{ \$/hr (fuel}-\text{cost)}$$

$$IC_i(P_{Gi}) = \frac{dC_i(P_{Gi})}{dP_{Gi}} = \beta + 2\gamma P_{Gi} \text{ \$/MWh}$$

Incremental Cost Example

- For a two generator system assume

$$C_1(P_{G1}) = 1000 + 20P_{G1} + 0.01P_{G1}^2 \text{ \$/hr}$$
$$C_2(P_{G2}) = 400 + 15P_{G2} + 0.03P_{G2}^2 \text{ \$/hr}$$

-

Then

$$IC_1(P_{G1}) = \frac{dC_1(P_{G1})}{dP_{G1}} = 20 + 0.02P_{G1} \text{ \$/MWh}$$
$$IC_2(P_{G2}) = \frac{dC_2(P_{G2})}{dP_{G2}} = 15 + 0.06P_{G2} \text{ \$/MWh}$$

If $P_{G1} = 250$ MW
and $P_{G2} = 150$ MW

Then

$C_1(250)$
$= 1000 + 20 \times 250 + 0.01 \times 250^2$
= \$ 6625/hr

$C_2(150)$
$= 400 + 15 \times 150 + 0.03 \times 150^2$
= \$6025/hr

Then

$IC_1(250)$
$= 20 + 0.02 \times 250 =$ \$ 25/MWh

$IC_2(150)$
$= 15 + 0.06 \times 150 =$ \$ 24/MWh

Economic Dispatch Problem

- The goal of economic dispatch is to determine the generation dispatch that minimizes the instantaneous operating cost, subject to the constraint that total generation = total load + losses

$$\text{Minimize} \quad C_T = \sum_{i=1}^{m} C_i(P_{Gi})$$

Such that:

$$\sum_{i=1}^{m} P_{Gi} = P_D + P_{Losses}$$

C_T = System total cost

$C_i(P_{Gi})$ = Cost for generator i as a function of the generator's real power P

P_D = Total system power demand

Solution to the Economic Dispatch Problem

- Economic dispatch is formulated as a constrained minimization
  - The cost function is often total generation cost in an area
  - Single equality constraint is the real power balance equation
- Solved by setting up the Lagrangian (with P_D the load and P_L the losses, which are a function the generation)

$$\mathrm{L}(\mathbf{P}_{\mathrm{G}},\lambda) = \sum_{i=1}^{m} C_i(P_{Gi}) + \lambda\left(P_D + P_L(\mathbf{P}_G) - \sum_{i=1}^{m} P_{Gi}\right)$$

- A necessary condition for a minimum is that the gradient is zero. Without losses this occurs when all generators are dispatched at the same marginal cost (except when they hit a limit)

Solution to the Economic Dispatch Problem, cont.

$$\mathrm{L}(\mathbf{P}_{\mathrm{G}},\lambda) \;=\sum_{i=1}^{m} C_i(P_{Gi}) + \lambda(P_D + P_L(P_G) - \sum_{i=1}^{m} P_{Gi})$$

$$\frac{\partial \mathrm{L}(\mathbf{P}_{\mathrm{G}},\lambda)}{\partial P_{Gi}} = \frac{dC_i(P_{Gi})}{dP_{Gi}} - \lambda(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}) = 0$$

$$P_D + P_L(P_G) - \sum_{i=1}^{m} P_{Gi} = 0$$

If losses are neglected then there is a single marginal cost (lambda); if losses are included then each bus could have a different marginal cost

Economic Dispatch Penalty Factors

Solving each equation for λ we get

$$\frac{dC_i(P_{Gi})}{dP_{Gi}} - \lambda(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}} = 0$$

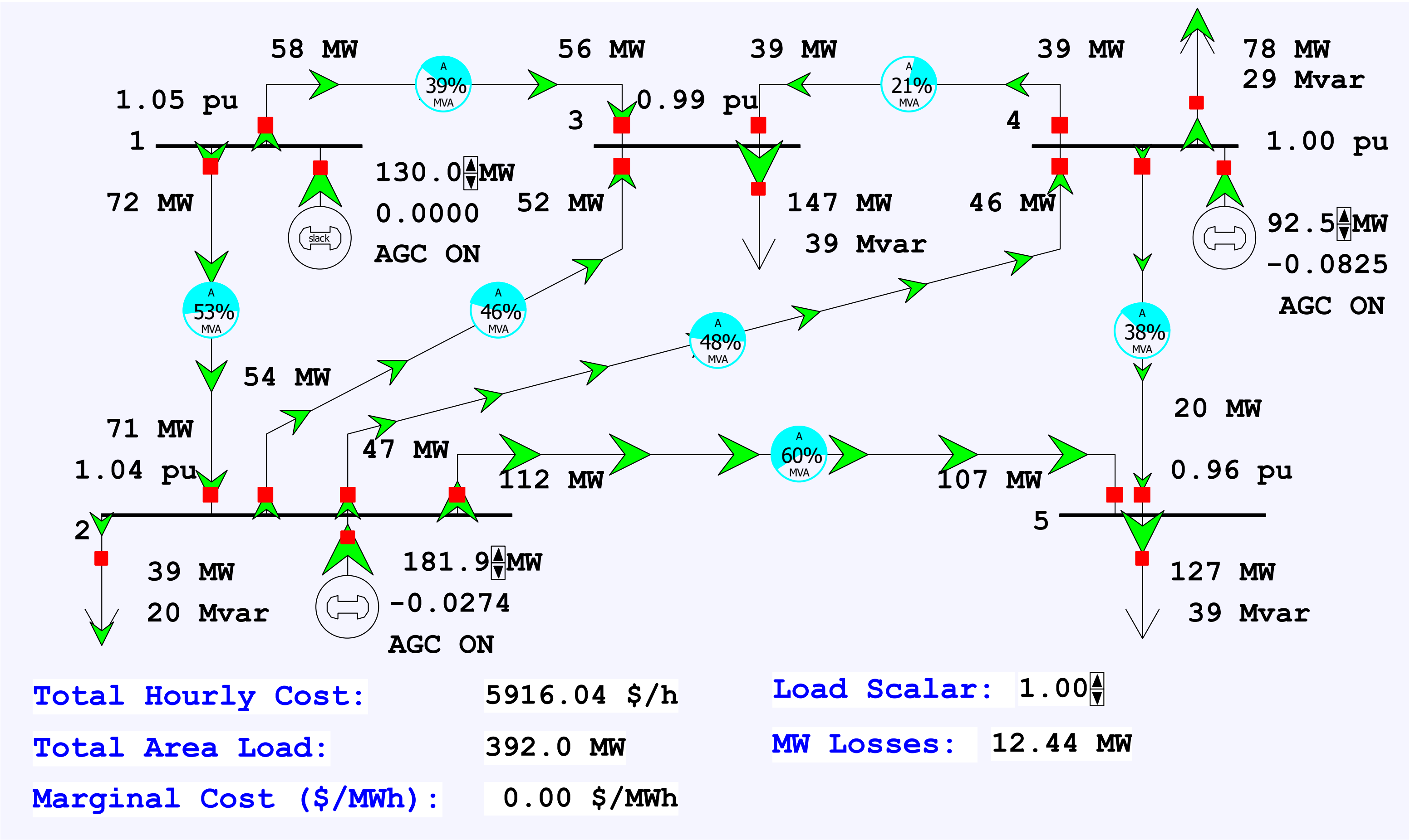
$$\lambda = \frac{1}{\left(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}\right)} \frac{dC_i(P_{Gi})}{dP_{Gi}}$$

Define the penalty factor L_i for the i^{th} generator

$$L_i = \frac{1}{\left(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}\right)}$$

The penalty factor at the slack bus is always unity!

Economic Dispatch Example



Case is Example7_3 from the book; use **Power Flow Solution Options, Advanced Options** to set penalty factor options

Unconstrained Minimization

- Assume we have a function f(**x**) and wish to determine its minimum. This is an unconstrained minimization. A necessary (but not sufficient) condition for a minimum is the gradient of the function must be zero.
- The gradient generalizes the first derivative for multi-variable problems:

$$\nabla \mathbf{f}(\mathbf{x}) \triangleq \left[\frac{\partial \mathbf{f}(\mathbf{x})}{\partial x_1}, \frac{\partial \mathbf{f}(\mathbf{x})}{\partial x_2}, \ldots, \frac{\partial \mathbf{f}(\mathbf{x})}{\partial x_n}\right] \qquad \nabla \mathbf{f}(\mathbf{x}) = \mathbf{0}$$

- This is not sufficient since the gradient is also zero at the maximum and at any saddle points

Minimization with Equality Constraint

- When the minimization is constrained with one or more equality constraints we can solve the problem using the method of Lagrange Multipliers
- Key idea is to modify a constrained minimization problem to be an unconstrained problem

That is, for the general problem

minimize $\mathbf{f}(\mathbf{x})$ s.t. $\mathbf{g}(\mathbf{x}) = \mathbf{0}$

We define the Lagrangian $\mathrm{L}(\mathbf{x}, \boldsymbol{\lambda})$

$= \mathbf{f}(\mathbf{x}) + \boldsymbol{\lambda}^T \mathbf{g}(\mathbf{x})$

Then a necessary condition for a minimum is the

$\nabla \mathrm{L}_{\mathbf{x}}(\mathbf{x}, \boldsymbol{\lambda}) = 0$and $\nabla \mathrm{L}_{\boldsymbol{\lambda}}(\mathbf{x}, \boldsymbol{\lambda}) = 0$

Economic Dispatch Lagrangian

For the economic dispatch we have a minimization constrained with a single equality constraint

$$\mathrm{L}(\mathbf{P}_\mathrm{G}, \lambda) = \sum_{i=1}^{m} C_i(P_{Gi}) + \lambda(P_D - \sum_{i=1}^{m} P_{Gi}) \quad \text{(no losses)}$$

The necessary conditions for a minimum are

$$\frac{\partial \mathrm{L}(\mathbf{P}_\mathrm{G}, \lambda)}{\partial P_{Gi}} = \frac{dC_i(P_{Gi})}{dP_{Gi}} - \lambda = 0 \quad (\text{for i } = 1 \text{ to m})$$

$$P_D - \sum_{i=1}^{m} P_{Gi} = 0$$

Economic Dispatch Example

What is economic dispatch for a two generator system $P_D = P_{G1} + P_{G2} = 500$ MW and

$$C_1(P_{G1}) = 1000 + 20P_{G1} + 0.01P_{G1}^2 \ \$/hr$$

$$C_2(P_{G2}) = 400 + 15P_{G2} + 0.03P_{G2}^2 \ \$/hr$$

Using the Largrange multiplier method we know

$$\frac{dC_1(P_{G1})}{dP_{G1}} - \lambda = 20 + 0.02P_{G1} - \lambda = 0$$

$$\frac{dC_2(P_{G2})}{dP_{G2}} - \lambda = 15 + 0.06P_{G2} - \lambda = 0$$

$$500 - P_{G1} - P_{G2} = 0$$

Economic Dispatch Example, cont'd

We therefore need to solve three linear equations

$$20 + 0.02P_{G1} - \lambda = 0$$

$$15 + 0.06P_{G2} - \lambda = 0$$

$$500 - P_{G1} - P_{G2} = 0$$

$$\begin{bmatrix} 0.02 & 0 & -1 \\ 0 & 0.06 & -1 \\ -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} P_{G1} \\ P_{G2} \\ \lambda \end{bmatrix} = \begin{bmatrix} -20 \\ -15 \\ -500 \end{bmatrix}$$

$$\begin{bmatrix} P_{G1} \\ P_{G2} \\ \lambda \end{bmatrix} = \begin{bmatrix} 312.5 \text{ MW} \\ 187.5 \text{ MW} \\ 26.2 \text{ \$/MWh} \end{bmatrix}$$

Lambda-Iteration Solution Method

- The direct solution only works well if the incremental cost curves are linear and no generators are at their limits
- A more general method is known as the lambda-iteration
  - the method requires that there be a unique mapping between a value of lambda and each generator's MW output
  - the method then starts with values of lambda below and above the optimal value, and then iteratively brackets the optimal value

Lambda-Iteration Algorithm

Pick λ^{L} and λ^{H} such that

$$\sum_{i=1}^{m} P_{Gi}(\lambda^{L}) - P_D < 0 \quad \sum_{i=1}^{m} P_{Gi}(\lambda^{H}) - P_D > 0$$

While $\left|\lambda^{H} - \lambda^{L}\right| > \varepsilon$ Do

$\lambda^{M} = (\lambda^{H} + \lambda^{L})/2$

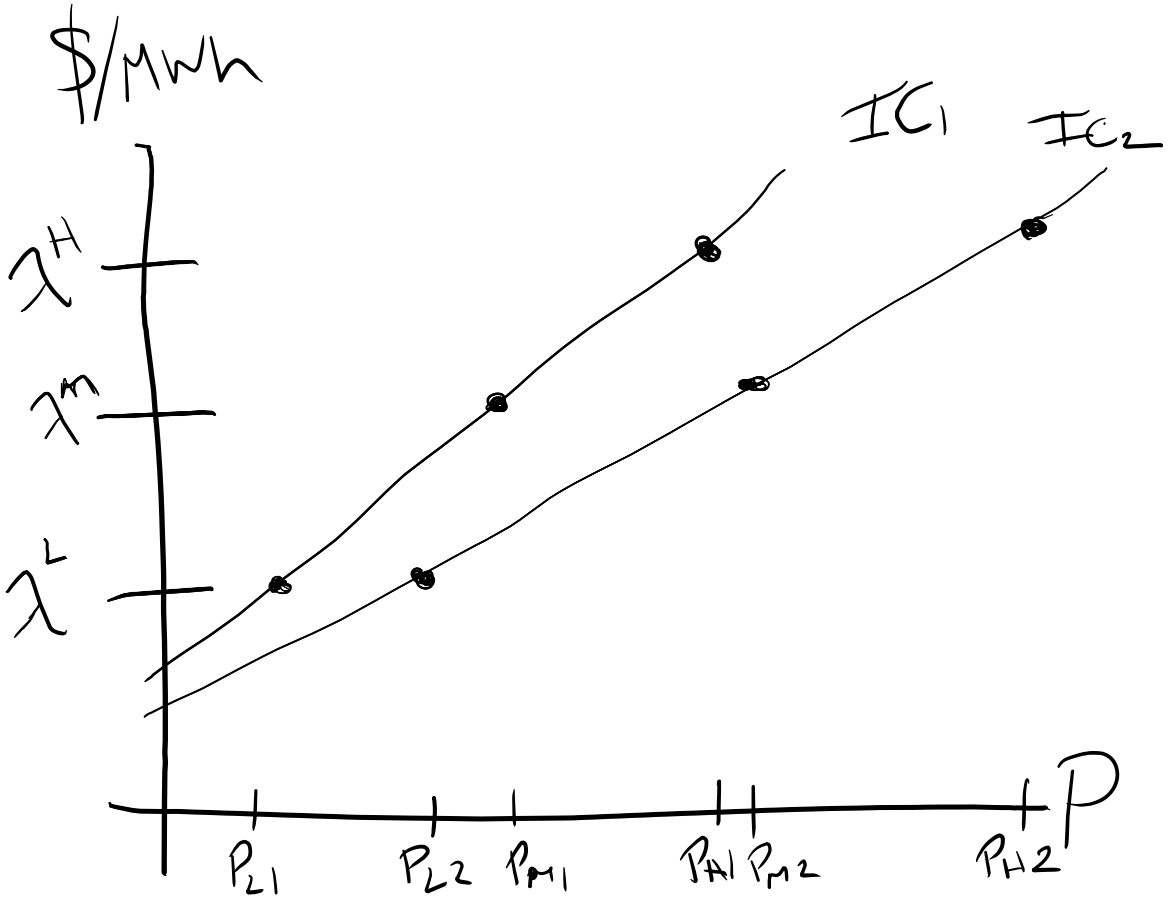
If $\sum_{i=1}^{m} P_{Gi}(\lambda^{M}) - P_D > 0$ Then $\lambda^{H} = \lambda^{M}$

Else $\lambda^{L} = \lambda^{M}$

End While

Lambda-Iteration: Graphical View

In the graph shown, for each value of lambda there is a unique P_{Gi} for each generator. This relationship is the $P_{Gi}(\lambda)$ function.



Lambda-Iteration Example

Consider a three generator system with

$$IC_1(P_{G1}) = 15 + 0.02P_{G1} = \lambda\$/\text{MWh}$$
$$IC_2(P_{G2}) = 20 + 0.01P_{G2} = \lambda\$/\text{MWh}$$
$$IC_3(P_{G3}) = 18 + 0.025P_{G3} = \lambda\$/\text{MWh}$$

and with constraint $P_{G1} + P_{G2} + P_{G3} = 1000\text{MW}$

Rewriting as a function of λ, $\text{P}_{\text{Gi}}(\lambda)$, we have

$$\text{P}_{\text{G1}}(\lambda) = \frac{\lambda - 15}{0.02} \quad \text{P}_{\text{G2}}(\lambda) = \frac{\lambda - 20}{0.01}$$
$$\text{P}_{\text{G3}}(\lambda) = \frac{\lambda - 18}{0.025}$$

Lambda-Iteration Example, cont'd

Pick λ^L so $\sum_{\mathrm{i=1}}^{\mathrm{m}} \mathrm{P_{Gi}}(\lambda^L) - 1000 < 0$ and

$$\sum_{\mathrm{i=1}}^{\mathrm{m}} \mathrm{P_{Gi}}(\lambda^H) - 1000 > 0$$

Try $\lambda^L = 20$ then $\sum_{i=1}^{m} P_{Gi}(20) - 1000 =$

$$\frac{\lambda - 15}{0.02} + \frac{\lambda - 20}{0.01} + \frac{\lambda - 18}{0.025} - 1000 = -670 \text{ MW}$$

Try $\lambda^{\mathrm{H}} = 30$ then $\sum_{i=1}^{m} P_{Gi}(30) - 1000 = 1230 \text{ MW}$

Lambda-Iteration Example, cont'd

Pick convergence tolerance $\varepsilon = 0.05$ \$/MWh

Then iterate since $|\lambda^H - \lambda^L| > 0.05$

$\lambda^M = (\lambda^H + \lambda^L)/2 = 25$

Then since $\sum_{i=1}^{m} P_{Gi}(25) - 1000 = 280$ we set λ^H

$= 25$

Since $|25 - 20| > 0.05$

$\lambda^M = (25 + 20)/2 = 22.5$

$\sum_{i=1}^{m} P_{Gi}(22.5) - 1000 = -195$ we set $\lambda^L = 22.5$

Lambda-Iteration Example, cont'd

Continue iterating until $\left|\lambda^{\mathrm{H}} - \lambda^{L}\right| < 0.05$

The solution value of λ, λ^*, is 23.53 \$/MWh

Once λ^* is known we can calculate the P_{Gi}

$$\mathrm{P}_{\mathrm{G1}}(23.5) = \frac{23.53 - 15}{0.02} = 426 \text{ MW}$$

$$\mathrm{P}_{\mathrm{G2}}(23.5) = \frac{23.53 - 20}{0.01} = 353 \text{ MW}$$

$$\mathrm{P}_{\mathrm{G3}}(23.5) = \frac{23.53 - 18}{0.025} = 221 \text{ MW}$$

Generator MW Limits

- Generators have limits on the minimum and maximum amount of power they can produce
- Often times the minimum limit is not zero. This represents a limit on the generator's operation with the desired fuel type
- Because of varying system economics usually many generators in a system are operated at their maximum MW limits.
- In the Lambda-iteration method the limits are taken into account when calculating $P_{gi}(\lambda)$:
  - If $P_{Gi}(\lambda) > P_{Gi,\max}$ then $P_{Gi(\lambda)} = P_{Gi,\max}$
  - If $P_{Gi}(\lambda) < P_{Gi,\min}$ then $P_{Gi(\lambda)} = P_{Gi,\min}$

Area Supply Curve

- The area supply curve shows how the marginal cost for an area (or the entire case) varies assuming an economic dispatch
  - The curves for an area or super area are available in their case information displays by right-clicking and selecting **Plot** then one of its options
  - This is a quick way to get an overall feel for a case