

ECEN 460, Spring 2026

Power System Operation and Control

Class 8: Transmission Lines, Part 2

Prof. Adam Birchfield

Dept. of Electrical and Computer Engineering

Texas A&M University

abirchfield@tamu.edu



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Homework 4 Due Next Week



- No homework on generators. Make sure you understand the lecture notes and labs 2 and 3.
- Homework 3 on transformers: book problems 3.4, 3.5, 3.23, due Feb. 3rd.
- Homework 4 on transmission lines: book regular problems 4.10, 4.11, 4.20, and 4.41, 5.14 (a,b), 5.38, and 5.41 (a,b), due Feb. 10th.

Transmission Lines



- In this class, we give an overview of line and transformer modeling
 - More detailed coverage of some models is in ECEN 459
 - Our focus is on how to use the models to study power systems
- Primary methods for long distance electric power transfer
 - Overhead ac
 - Underground ac
 - HVDC – overhead or underground



Conductor Table Example



TABLE A.4 Characteristics of aluminum cable, steel, reinforced (Aluminum Company of America)—ACSR

Code Word	Circular Mils Aluminum	Aluminum			Steel		Outside Diameter (inches)	Copper Equivalent* Circular Mils or A.W.G.	Ultimate Strength (pounds)	Weight (pounds per mile)	Geometric Mean Radius at 60 Hz (feet)	Approx. Current Carrying Capacity† (amps)	r_a Resistance (Ohms per Conductor per Mile)							
				Strand Diameter (inches)	Strand Diameter (inches)	25°C (77°F) Small Currents							50°C (122°F) Current Approx. 75% Capacity‡							
						dc							25 Hz	50 Hz	60 Hz	dc	25 Hz	50 Hz	60 Hz	
Joree	2 515 000	76	...	0.1819	19	0.0849	1.880	61 700	10 777	0.0621	1 380	0.0587	0.0588	0.0590	0.0591	0.0646	0.0656	0.0675	0.0684	
Thrasher	2 312 000	76	...	0.1744	19	0.0814	1.802	57 300	10 237	0.0595	1 340	0.0618	0.0619	0.0621	0.0622	0.0680	0.0690	0.0710	0.0720	
Kiwi	2 167 000	72	4	0.1735	7	0.1157	1.735	49 800	9 699	0.0570	1 300	0.0652	0.0653	0.0655	0.0656	0.0718	0.0729	0.0749	0.0760	
Bluebird	2 156 000	84	4	0.1602	19	0.0961	1.762	60 300	9 160	0.0588	1 250	0.0691	0.0692	0.0694	0.0695	0.0761	0.0771	0.0792	0.0803	
Chukar	1 781 000	84	4	0.1456	19	0.0874	1.602	51 000	8 621	0.0534	1 200	0.0734	0.0735	0.0737	0.0738	0.0808	0.0819	0.0840	0.0851	
Falcon	1 590 000	54	3	0.1716	19	0.1030	1.545	1 000 000	8 082	0.0520	1 160	0.0783	0.0784	0.0786	0.0788	0.0862	0.0872	0.0894	0.0906	
Parrot	1 510 500	54	3	0.1673	19	0.1004	1.506	950 000	7 544	0.0507	1 110	0.0839	0.0840	0.0842	0.0844	0.0924	0.0935	0.0957	0.0969	
Plover	1 431 000	54	3	0.1628	19	0.0977	1.465	900 000	7 019	0.0493	1 060	0.0903	0.0905	0.0907	0.0909	0.0994	0.1005	0.1025	0.1035	
Martin	1 351 000	54	3	0.1582	19	0.0949	1.424	850 000	6 479	0.0479	1 010	0.0979	0.0980	0.0981	0.0982	0.1078	0.1088	0.1118	0.1128	
Pheasant	1 272 000	54	3	0.1535	19	0.0921	1.382	800 000	6 112	0.0465	970	0.104	0.104	0.104	0.104	0.1145	0.1155	0.1175	0.1185	
Grackle	1 192 500	54	3	0.1486	19	0.0892	1.338	750 000	5 940	0.0450	950	0.107	0.107	0.107	0.108	0.1178	0.1188	0.1218	0.1228	
Finch	1 113 000	54	3	0.1436	19	0.0862	1.293	700 000	5 399	0.0435	900	0.117	0.117	0.117	0.117	0.1288	0.1288	0.1288	0.1288	
Goldfinch	1 033 500	54	3	0.1384	7	0.1384	1.243	650 000	4 859	0.0420	830	0.131	0.131	0.131	0.132	0.1442	0.1452	0.1472	0.1482	
Cardinal	954 000	54	3	0.1329	7	0.1329	1.196	600 000	4 513	0.0403	800	0.147	0.147	0.147	0.147	0.1618	0.1618	0.1618	0.1618	
Canary	900 000	54	3	0.1291	7	0.1291	1.162	566 000	4 109	0.0391	750	0.154	0.155	0.155	0.155	0.1695	0.1715	0.1755	0.1775	
Crane	874 500	54	3	0.1273	7	0.1273	1.146	550 000	3 859	0.0386	730	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	
Condor	795 000	54	3	0.1214	7	0.1214	1.093	500 000	3 419	0.0368	700	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	
Drake	795 000	26	2	0.1749	7	0.1360	1.108	500 000	3 150	0.0375	670	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	
Mallard	795 000	30	2	0.1628	19	0.0977	1.140	500 000	2 720	0.0393	650	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	
Crow	715 500	54	3	0.1151	7	0.1151	1.036	450 000	2 450	0.0349	600	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	
Starling	715 500	26	2	0.1659	7	0.1290	1.051	450 000	2 100	0.0355	580	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	
Redwing	715 500	30	2	0.1544	19	0.0926	1.081	450 000	1 800	0.0372	560	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	
Flamingo	666 600	54	3	0.1111	7	0.1111	1.000	419 000	1 600	0.0337	500	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	
Rook	636 000	54	3	0.1085	7	0.1085	0.977	400 000	1 400	0.0329	480	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	
Grosbeak	636 000	26	2	0.1564	7	0.1216	0.990	400 000	1 200	0.0335	460	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	
Egret	636 000	30	2	0.1456	19	0.0874	1.019	400 000	1 000	0.0351	440	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	
Peacock	605 000	54	3	0.1059	7	0.1059	0.953	380 500	1 000	0.0321	420	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	
Squab	605 000	26	2	0.1525	7	0.1186	0.966	380 500	900	0.0327	400	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	
Dove	556 500	26	2	0.1463	7	0.1138	0.927	350 000	800	0.0313	380	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	
Eagle	556 500	30	2	0.1362	7	0.1362	0.953	350 000	700	0.0328	360	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	
Hawk	477 000	26	2	0.1355	7	0.1054	0.858	300 000	600	0.0290	340	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	

Additional Transmission Topics



- **Multi-circuit lines:** Multiple lines often share a common transmission right-of-way. This does cause mutual inductance and capacitance, but that is often ignored in system analysis.
- **Cables:** There are about 3000 miles of underground ac cables in U.S. Cables are primarily used in urban areas. In a cable the conductors are tightly spaced, ($< 1\text{ft}$) with oil impregnated paper commonly used to provide insulation
 - inductance is lower
 - capacitance is higher, greatly limiting cable length
- **Ground wires:** Transmission lines are usually protected from lightning strikes with a ground wire. This topmost wire (or wires) helps to attenuate the transient voltages/currents that arise during a lightning strike. The ground wire is typically grounded at each pole.

Additional Transmission Topics

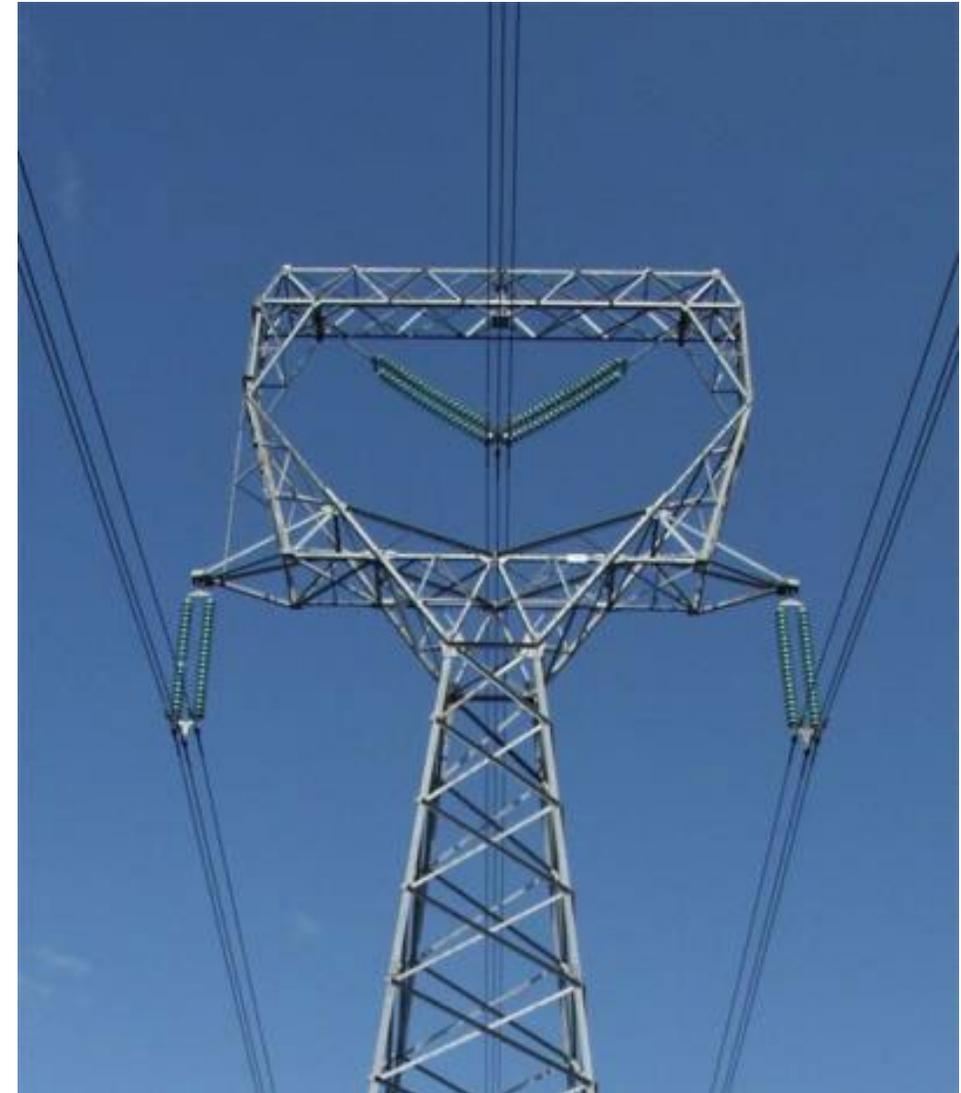


- **Corona discharge:** Due to high electric fields around lines, the air molecules become ionized. This causes a crackling sound and may cause the line to glow!
- **Shunt conductance:** Usually ignored. A small current may flow through contaminants on insulators.
- **DC Transmission:** Because of the large fixed cost necessary to convert ac to dc and then back to ac, dc transmission is only practical for several specialized applications
 - long distance overhead power transfer (> 400 miles)
 - long cable power transfer such as underwater
 - providing an asynchronous means of joining different power systems (such as ERCOT to Eastern or Western grids)
- **Line insulation:** Driven by voltage, including the 60-Hz power voltage as well as surge voltages caused by lightning and switching. Key concept is avoiding flashover of the insulation. The basic lightning impulse insulation level (BIL) is often 4-8 times the nominal rated voltage.

Transmission Line Models



- Previous slides have covered how to calculate the distributed inductance, capacitance and resistance of transmission lines.
- In this section we will use these distributed parameters to develop the transmission line models used in power system analysis.



Transmission Line Equivalent Circuit



- Our current model of a transmission line is shown below

Units on z and y are per unit length!

For operation at frequency ω , let $z = r + j\omega L$
and $y = g + j\omega C$ (with g usually equal 0)

Propagation Constant, Characteristic and Surge Impedance



Define the propagation constant γ as

$$\gamma = \sqrt{yz} = \alpha + j\beta$$

where

α = the attenuation constant

β = the phase constant

Define $Z_c = \sqrt{\frac{z}{y}}$ = characteristic impedance

Lossless Transmission Lines



For a lossless line the characteristic impedance, Z_c , is known as the surge impedance.

$$Z_c = \sqrt{\frac{j\omega l}{j\omega c}} = \sqrt{\frac{l}{c}} \Omega \text{ (a real value)}$$

If a lossless line is terminated at bus R in impedance

$$Z_c = \frac{V_R}{I_R}$$

Then $I_R Z_c = V_R$ so we get...

Lossless Transmission Lines



$$V(x) = V_R \cosh \gamma x + I_R Z_c \sinh \gamma x$$

$$I(x) = I_R \cosh \gamma x + \frac{V_R}{Z_c} \sinh \gamma x$$

$$\frac{V(x)}{I(x)} = Z_c$$

This equation is derived in Chapter 5, but this derivation is not part of ECEN 460

Define $\frac{|V(x)|^2}{Z_c}$ as the surge impedance load (SIL).

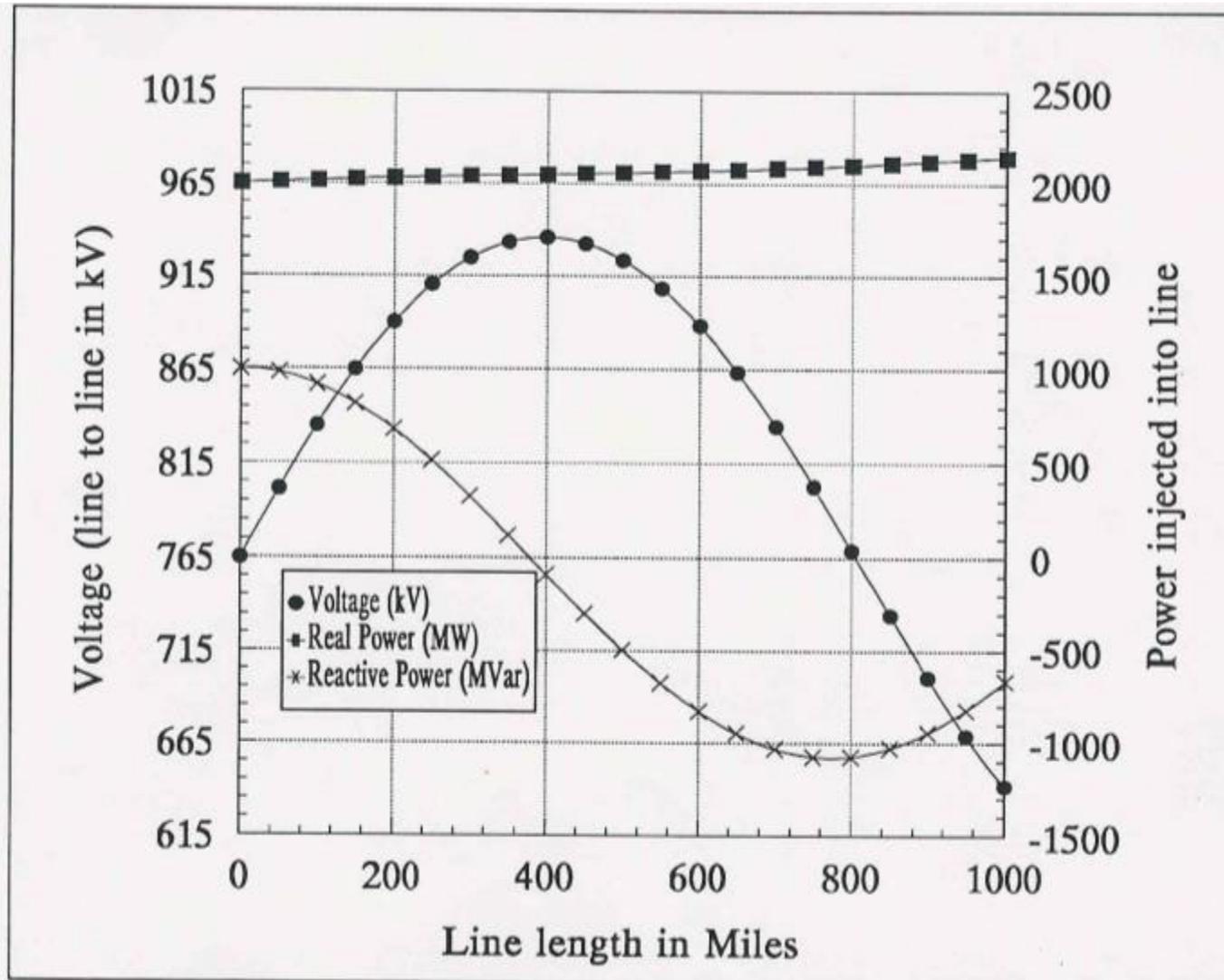
Since the line is lossless this implies

$$|V(x)| = |V_R|$$

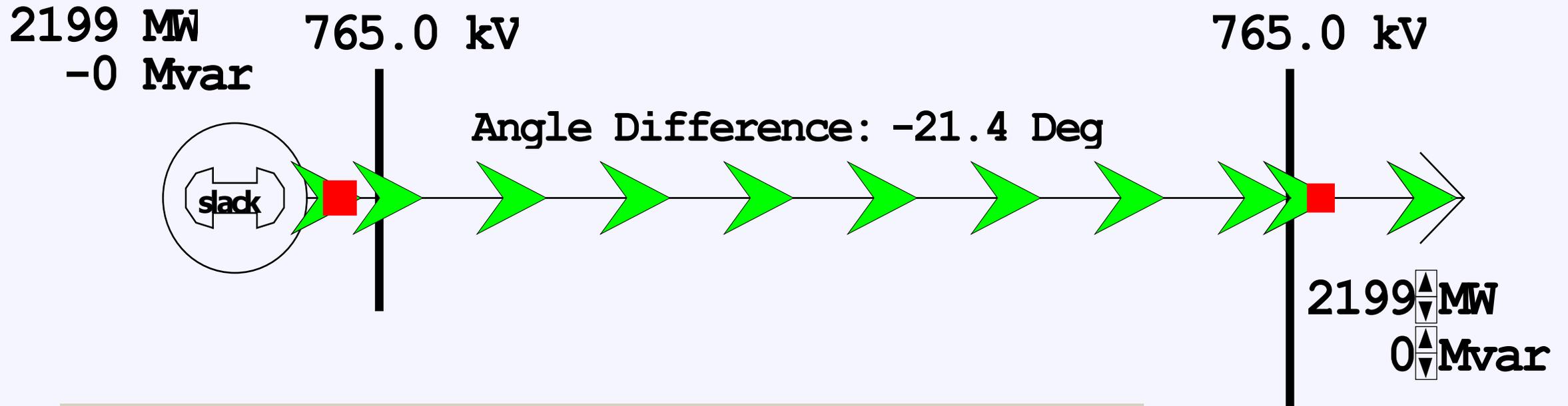
$$|I(x)| = |I_R|$$

If $P > \text{SIL}$ then line consumes vars; otherwise line generates vars.

Transmission Line Example, cont'd



Example 5.4



The 2199 MW represents the surge impedance loading; for values above this the line consumes reactive power, for values below this it generates reactive power.

Equivalent Circuit Model



The common representation is the π equivalent circuit

Next, we'll present simplified ways to determine the parameters Z' and Y' .

Simplified Parameters



These values can be derived to be

$$\begin{aligned}
 Z' &= Z_c \sinh \gamma l = \sqrt{\frac{z l z}{y l z}} \sinh \gamma l \\
 &= Z \frac{\sinh \gamma l}{\gamma l} \text{ with } Z \triangleq z l \text{ (recalling } \gamma = \sqrt{z y})
 \end{aligned}$$

$$\begin{aligned}
 \frac{Y'}{2} &= \frac{1}{Z_c} \tanh \frac{\gamma l}{2} = \sqrt{\frac{y l y}{z l y}} \tanh \frac{\gamma l}{2} \\
 &= \frac{Y}{2} \frac{\tanh \frac{\gamma l}{2}}{\frac{\gamma l}{2}} \text{ with } Y \triangleq y l
 \end{aligned}$$

Simplified Parameters, cont.



For most lines make the following approximations:

$$Z' = Z \quad (\text{assumes } \frac{\sinh \gamma l}{\gamma l} \approx 1)$$

$$\frac{Y'}{2} = \frac{Y}{2} \quad (\text{assumes } \frac{\tanh(\gamma l/2)}{\gamma l/2} \approx 1)$$

Length	$\frac{\sinh \gamma l}{\gamma l}$	$\frac{\tanh(\gamma l/2)}{\gamma l/2}$
50 miles	0.998 \angle 0.02°	1.001 \angle - 0.01°
100 miles	0.993 \angle 0.09°	1.004 \angle - 0.04°
200 miles	0.972 \angle 0.35°	1.014 \angle - 0.18°

Three Line Models



Long Line Model (longer than 250 miles)

$$\text{use } Z' = Z \frac{\sinh \gamma l}{\gamma l}, \quad \frac{Y'}{2} = \frac{Y}{2} \frac{\tanh \frac{\gamma l}{2}}{\frac{\gamma l}{2}}$$

Medium Line Model (less than 250 miles)

$$\text{use } Z \text{ and } \frac{Y}{2}$$

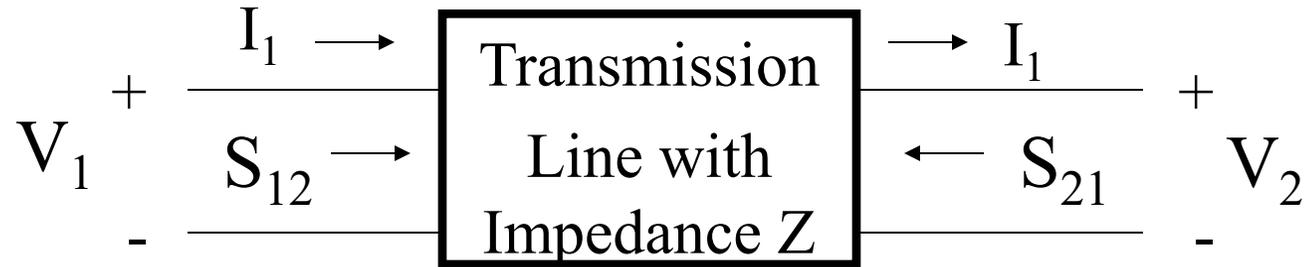
Short Line Model (not used much but useful for insight)

use Z (i.e., assume Y is zero; it small for short or low voltage lines)

Power Transfer in Short Lines



- Often we'd like to know the maximum power that could be transferred through a short transmission line



$$S_{12} = V_1 I_1^* = V_1 \left(\frac{V_1 - V_2}{Z} \right)^*$$

$$\text{with } V_1 = |V_1| \angle \theta_1, \quad V_2 = |V_2| \angle \theta_2 \quad Z = |Z| \angle \theta_Z$$

$$S_{12} = \frac{|V_1|^2}{|Z|} \angle \theta_Z - \frac{|V_1| |V_2|}{|Z|} \angle \theta_Z + \theta_{12}$$

Power Transfer in Lossless Lines



If we assume a line is lossless with impedance jX and are just interested in the real power transfer then:

$$P_{12} + jQ_{12} = \frac{|V_1|^2}{|Z|} \angle 90^\circ - \frac{|V_1||V_2|}{|Z|} \angle (90^\circ + \theta_{12})$$

Since $-\cos(90^\circ + \theta_{12}) = \sin \theta_{12}$, we get

$$P_{12} = \frac{|V_1||V_2|}{X} \sin \theta_{12}$$

Hence the maximum power transfer is

$$P_{12}^{Max} = \frac{|V_1||V_2|}{X}$$

Limits Affecting Max. Power Transfer



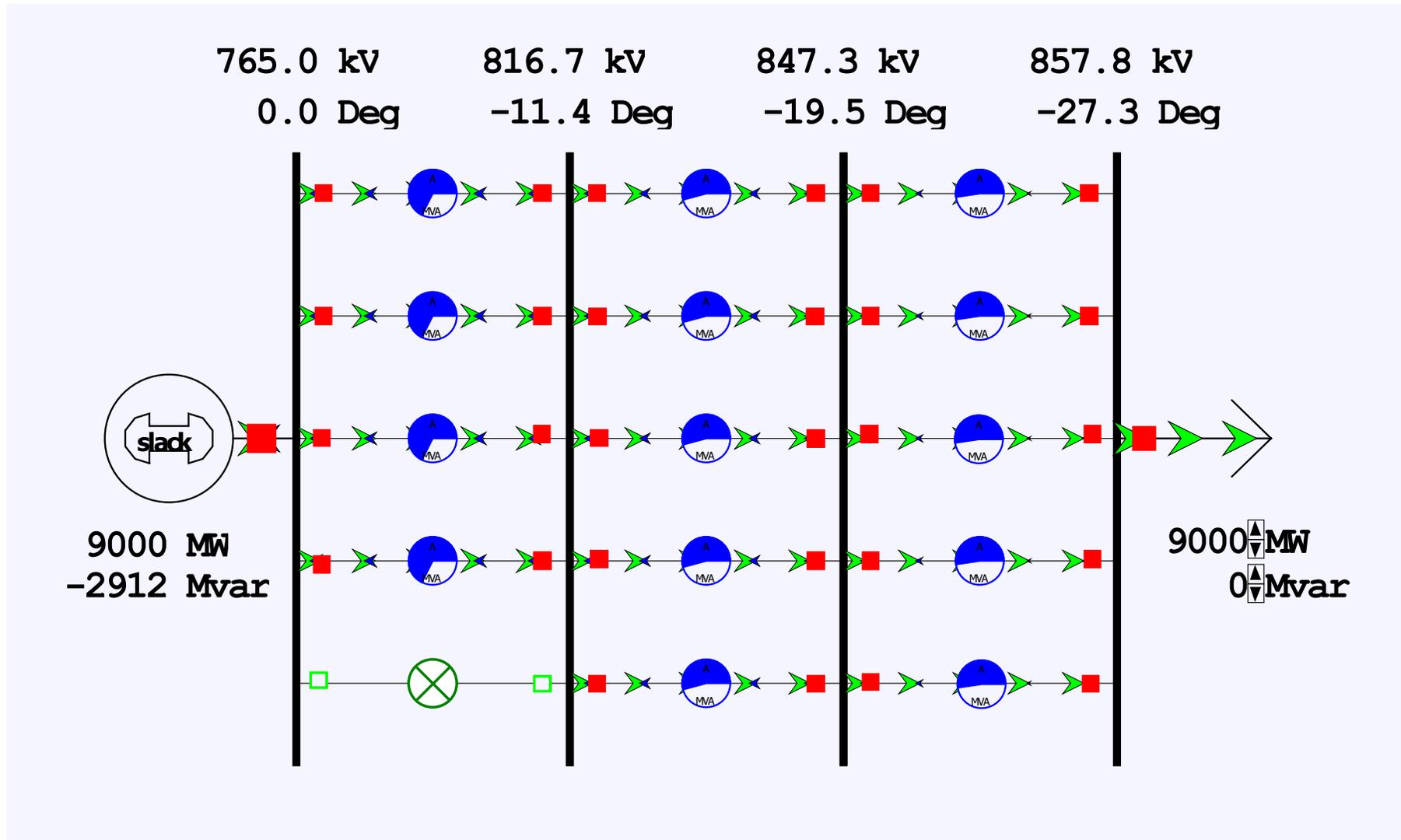
- Thermal limits
 - limit is due to heating of conductor and hence depends heavily on ambient conditions.
 - For many lines, sagging is the limiting constraint.
 - Newer conductors limit can limit sag. For example 3M has lines with a core consisting of ceramic Nextel fibers. These lines can operate at 200 degrees C.
 - Higher currents and operating temperatures mean higher losses!
 - Trees grow, and many will eventually hit lines if they are planted under the line.

Other Limits Affecting Power Transfer



- Angle limits
 - while the maximum power transfer occurs when line angle difference is 90 degrees, actual limit is substantially less due to multiple lines in the system
- Voltage stability limits
 - as power transfers increases, reactive losses increase as I^2X . As reactive power increases the voltage falls, resulting in a potentially cascading voltage collapse.

Example 5.8



Transmission Line Series Compensation



- One way to increase the transmission capacity of a transmission line that is limited by its reactance is to add series compensation
 - Capacitors are placed in series with the transmission line (covered in Example 5.10)



Image shows BPA series capacitors in a 500 kV line

Transmission Line Series Compensation



- Amount of series compensation is expressed as a percentage of the total line reactance (e.g., 50%)
- The series capacitance is usually setup so that it can be bypassed sometimes
 - There can be excessive reactive power generation on the system during light loads, like at night
- There can be a concern with sub-synchronous interactions (SSI)

$$f_n = \frac{1}{\sqrt{LC}} = f_0 \sqrt{\frac{X_C}{X_L}}$$

Example 5.10

