ECEN 460, Spring 2026
Power System Operation and Control

Class 7: Transmission Lines, Part 1

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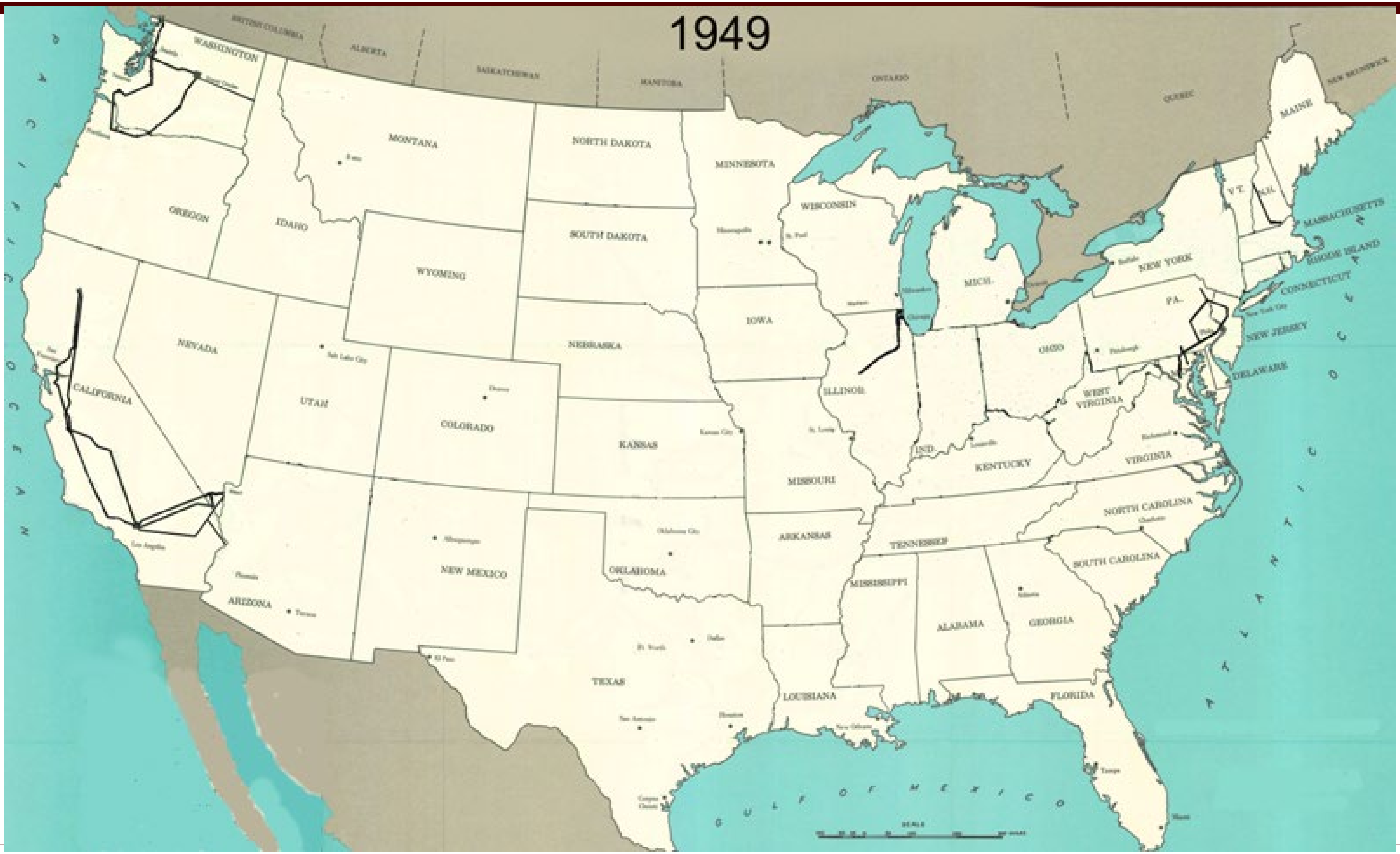
Homework 4 Due Next Week

- No homework on generators. Make sure you understand the lecture notes and labs 2 and 3.
- Homework 3 on transformers: book problems 3.4, 3.5, 3.23, due Feb. 3rd.
- Homework 4 on transmission lines: book regular problems 4.10, 4.11, 4.20, and 4.41, 5.14 (a,b), 5.38, and 5.41 (a,b), due Feb. 10th.

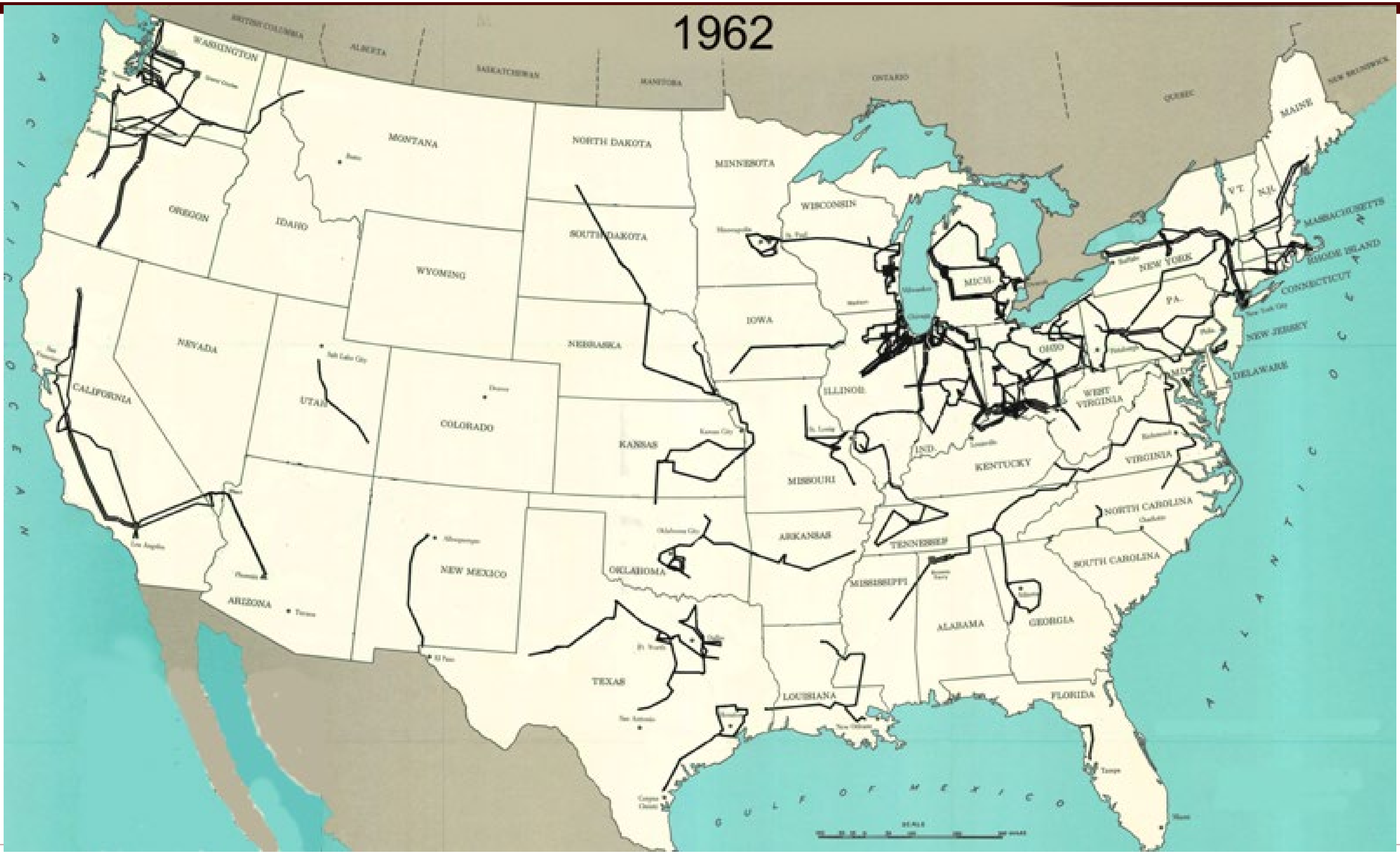
Transmission Lines

- In this class, we give an overview of line and transformer modeling
  - More detailed coverage of some models is in ECEN 459
  - Our focus is on how to use the models to study power systems
- Primary methods for long distance electric power transfer
  - Overhead ac
  - Underground ac
  - HVDC – overhead or underground

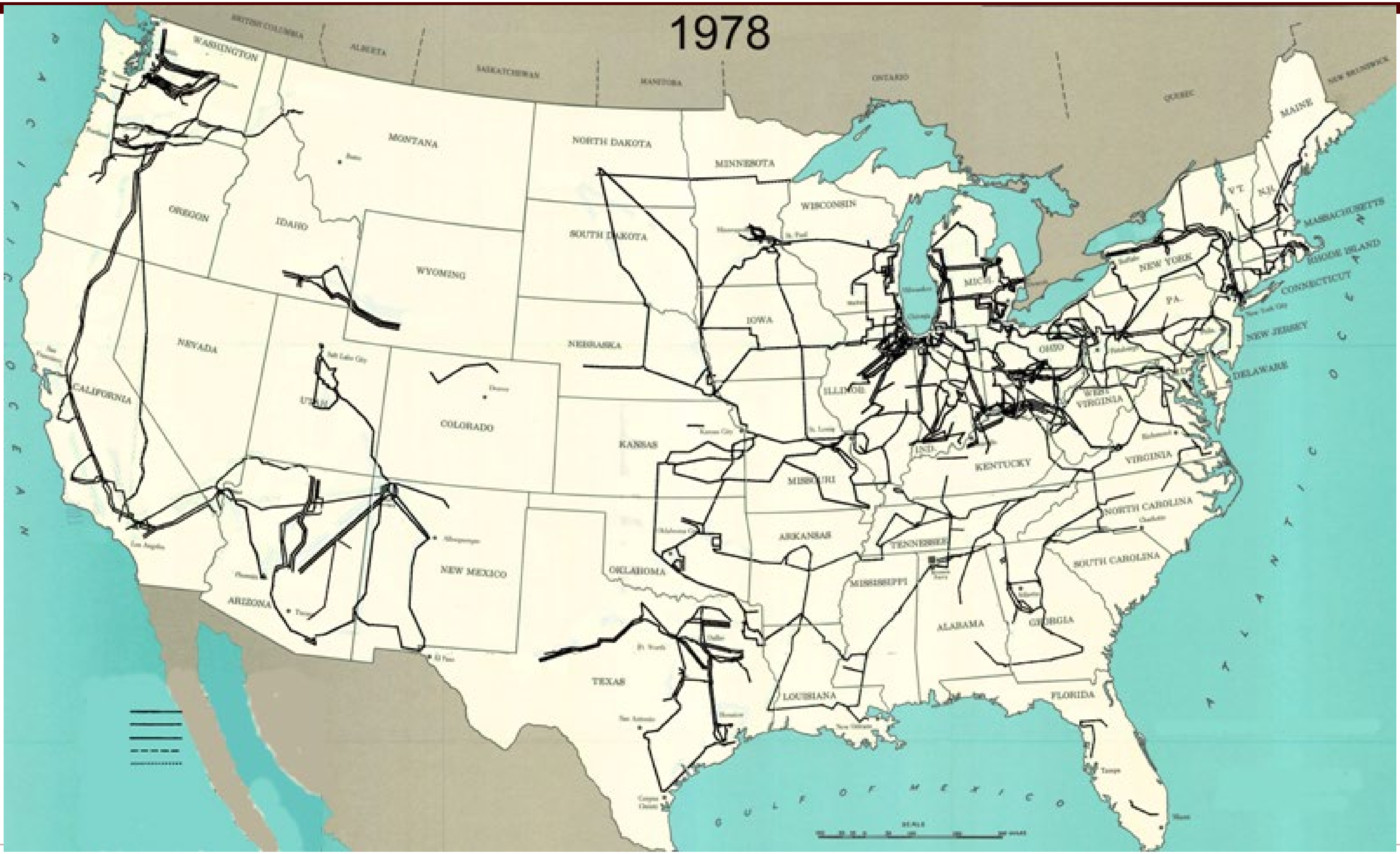
345 kV+ Transmission Growth at a Glance



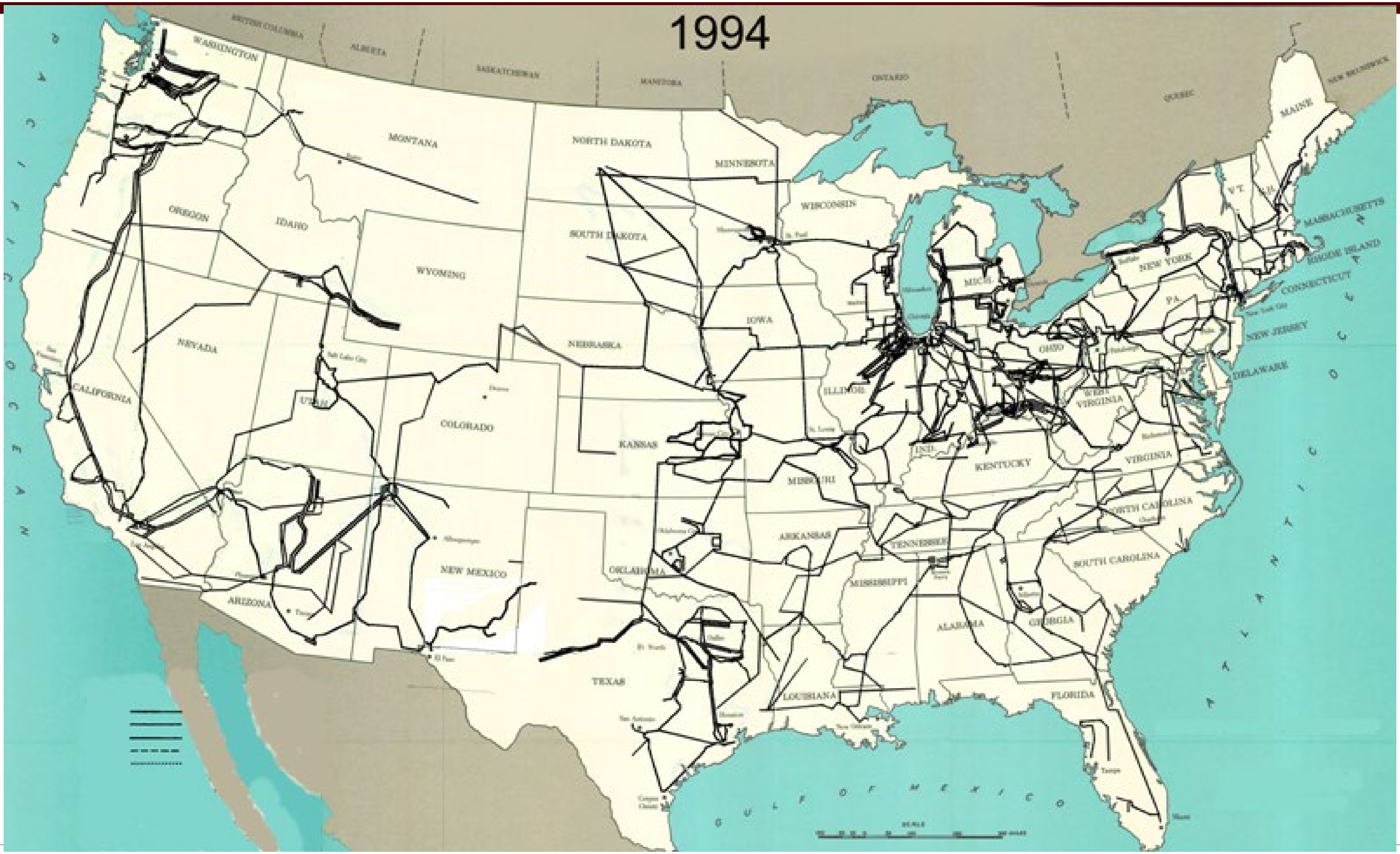
345 kV+ Transmission Growth at a Glance



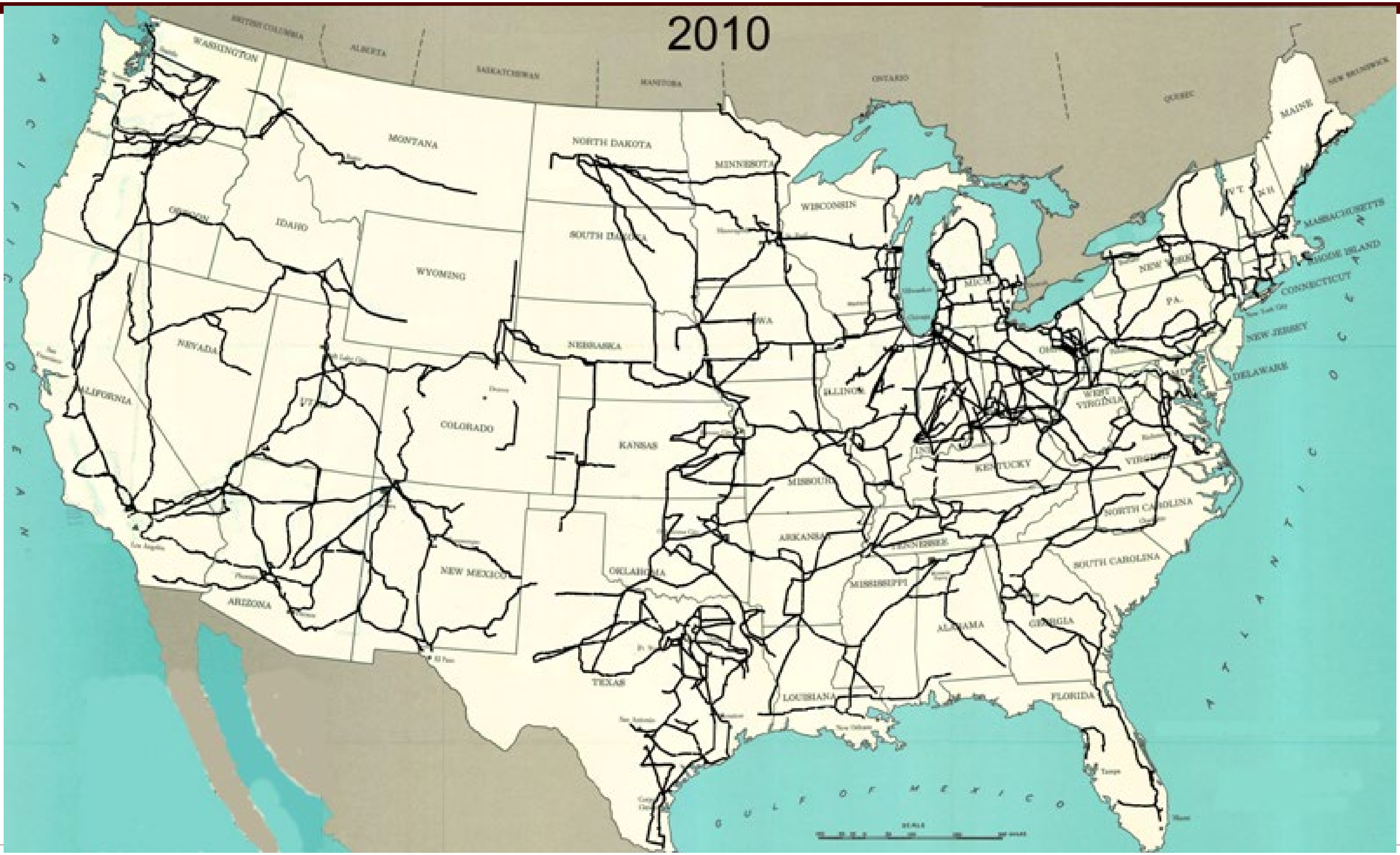
345 kV+ Transmission Growth at a Glance



345 kV+ Transmission Growth at a Glance



345 kV+ Transmission Growth at a Glance



HVDC Lines in North America

http://www.grainbeltexpresscleanline.com/site/page/history_of_hvdc_transmission

Line Conductors

- Typical transmission lines use multi-strand conductors
- ACSR (aluminum conductor steel reinforced) conductors are most common. A typical Al. to St. ratio is about 4 to 1.
- AAC (all aluminum conductors) are lighter but have less strength; used in urban areas with shorter spans
- Copper is heavier, but has better conductance; used in cables where weight is not an issue

Line Conductors, cont'd

- Total conductor area is given in circular mils. One circular mil is the area of a circle with a diameter of 0.001 = $\pi \times 0.0005^2$ square inches
- **Example:** what is the the area of a solid, 1" diameter circular wire?
  Answer: 1000 kcmil (kilo circular mils)
- Because conductors are stranded, the equivalent radius must be provided by the manufacturer. In tables this value is known as the GMR and is usually expressed in feet.

Transmission Line Parameters

- We'll discuss how to calculate transmission line parameters, which will then form part of the input to our power system model, such as in PowerWorld:
  - Resistance
  - Inductance
  - Capacitance
  - Ampacity

Line Resistance

- Line resistance per unit length is given by

  $R = \frac{\rho}{A}$ where ρ is the resistivity
- Resistivity of Copper = $1.68 \times 10^{-8} \Omega\text{−m}$
- Resistivity of Aluminum = $2.65 \times 10^{-8} \Omega\text{−m}$

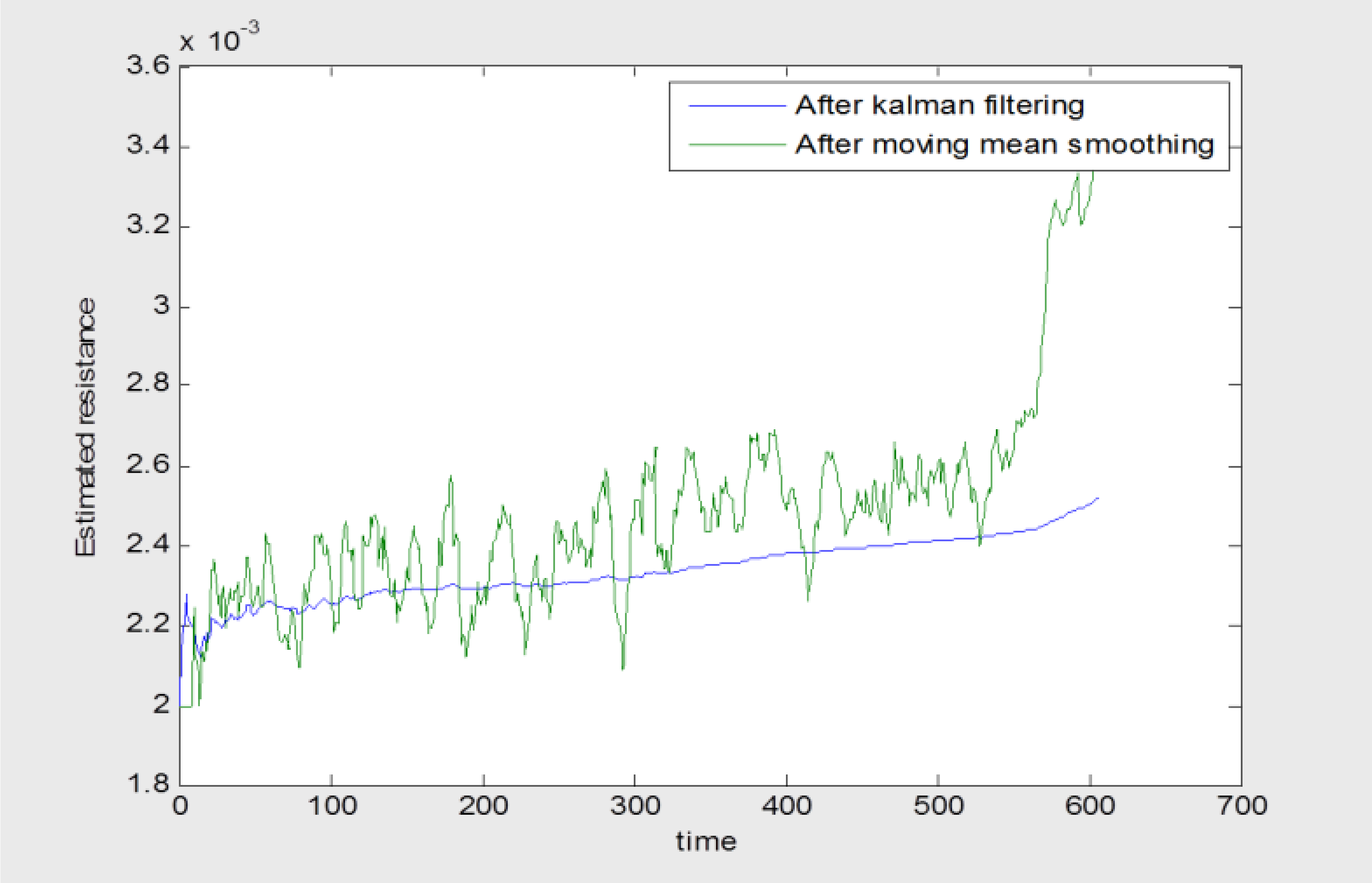
Example: What is the resistance in Ω / mile of a 1” diameter solid aluminum wire (at dc)?

$$R = \frac{2.65 \times 10^{-8} \Omega\text{−m}}{\pi \times 0.0127\text{m}^2} 1609 \frac{\text{m}}{\text{mile}} = \frac{0.084\Omega}{\text{mile}}$$

Line Resistance, cont'd

- Because ac current tends to flow towards the surface of a conductor, the resistance of a line at 60 Hz is slightly higher than at dc.
- Resistivity and hence line resistance increase linearly as conductor temperature increases (changes is about 0.4% per degree C)
  - In some locations conductor temperatures can vary by up to 100° C!
- Because ACSR conductors are stranded, actual resistance, inductance and capacitance needs to be determined from tables.

Variation in Line Resistance Example

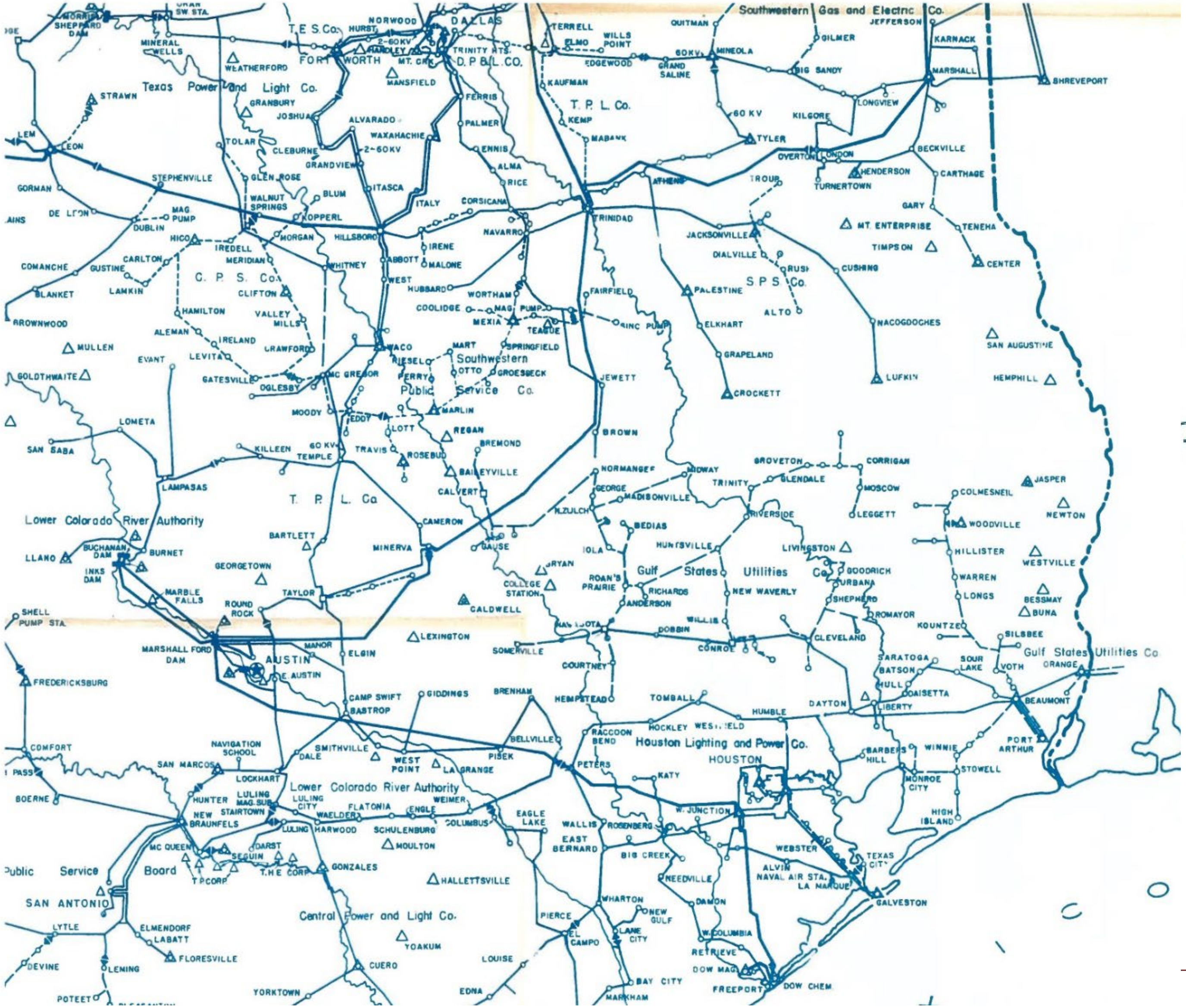


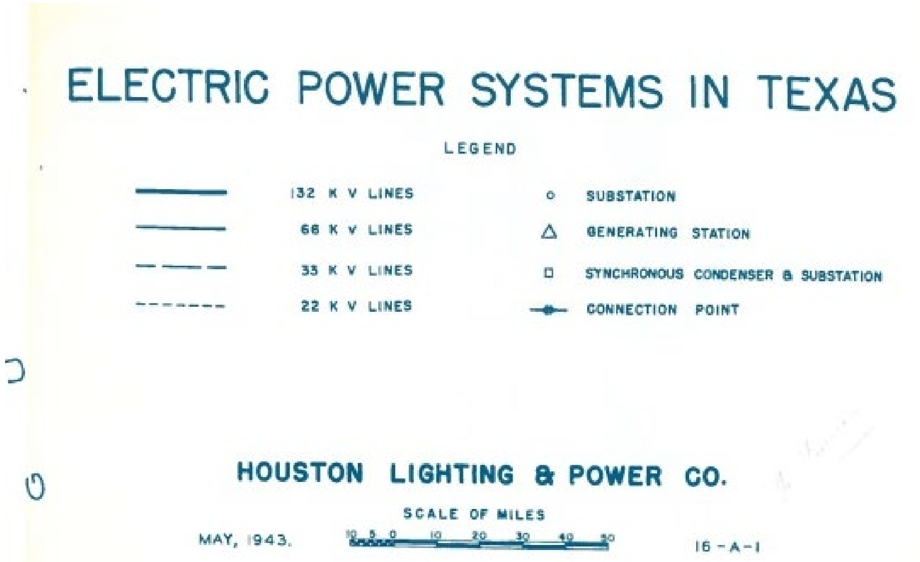
Time is in minutes. Input was 30 second ICCP data. Conductor resistance increased by about 50%, indicating a 50/0.4 = 125 degree C rise in temperature.

High Voltage Transmission Line Worker

Source: www.youtube.com/watch?v=LIjC7DjoVe8

Texas Grid, 1943





Magnetics Review

Ampere's circuital law:

$$F = \oint_C \mathbf{H} \cdot \mathrm{d}\boldsymbol{l} = \boldsymbol{I}_{\mathrm{enc}}$$

F = mmf = magnetomotive force (amp-turns)

H = magnetic field intensity (amp-turns/meter)

dl = Vector differential path length (meters)

Line integral about closed path C

Ienc = Algebraic sum of current linked by C

Ampere's law is most useful in cases of symmetry, such as with an infinitely long line

Line integrals are a generalization of traditional integration

Integrate along X-axis

Integrate along a general path, which may be closed.

Magnetic Flux Density

Magnetic fields are usually measured in terms of flux density

$\mathbf{B} =$ flux density (Tesla $[\mathrm{T}]$ or Gauss $[\mathrm{G}]$)
(1T $=$ 10,000G)
For a linear a linear magnetic material
$\mathbf{B} = \mu\mathbf{H}$ where μ is the called the permeability

$\mu = \mu_0\mu_r$

$\mu_0 =$ permeability of freespace $= 4\pi \times 10^{-7}\frac{H}{m}$

$\mu_r =$ relative permeability $\approx$ 1 for air

Magnetic Flux

Total flux passing through a surface A is

$$\phi = \int_A \mathbf{B} \cdot d\boldsymbol{a}$$

da = vector with direction normal to surface

If flux density **B** is uniform and perpendicular to an area A then

$$\phi = BA$$

Magnetic Fields from Single Wire

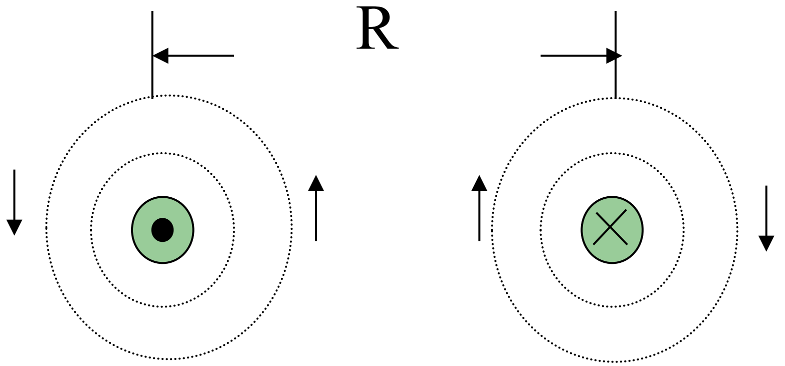
Assume we have an infinitely long wire with current of 1000A. How much magnetic flux passes through a 1 meter square, located between 4 and 5 meters from the wire?

Direction of H is given by the “Right-hand” Rule

Easiest way to solve the problem is to take advantage of symmetry. For an integration path we’ll choose a circle with a radius of x.

Two Conductor Line Inductance

Key problem with the previous derivation is we assumed no return path for the current. Now consider the case of two wires, each carrying the same current I, but in opposite directions; assume the wires are separated by distance R.

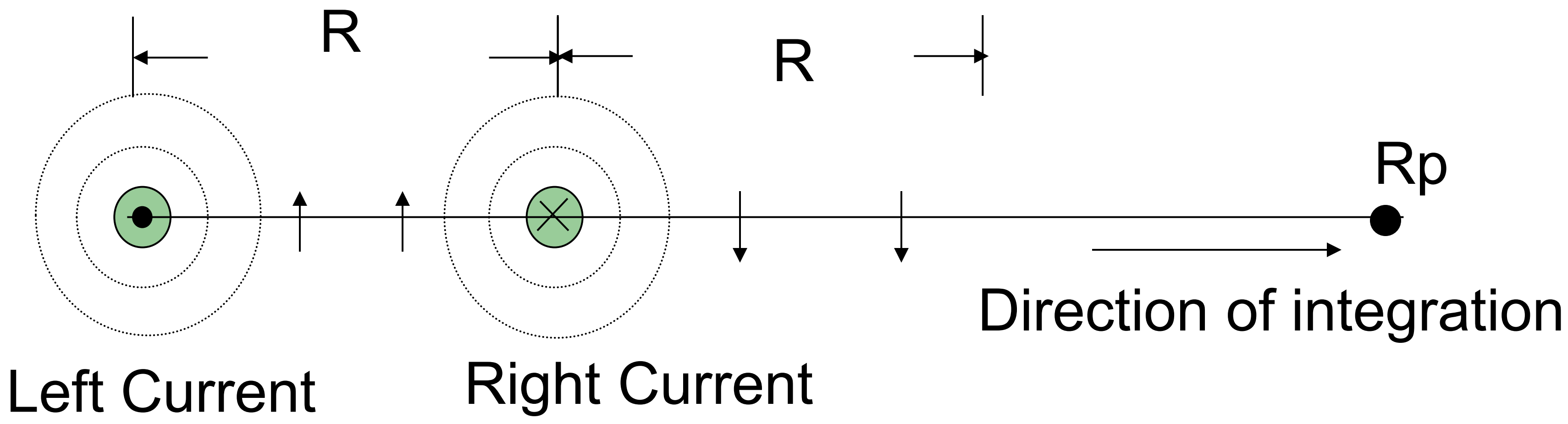


Creates counter-clockwise field

Creates a clockwise field

To determine the inductance of each conductor we integrate as before. However now we get some field cancellation

Two Conductor Case, cont'd



For distance Rp, greater than 2R, from left line

$$\lambda_{\text{left}} = \frac{\mu_0}{2\pi} I \ln \frac{Rp}{r'} - \frac{\mu_0}{2\pi} I \ln \left(\frac{Rp - R}{R} \right)$$

Key Point: As we integrate for the left line, at distance 2R from the left line the net flux linked due to the Right line is zero!

Use superposition to get total flux linkage.

Two Conductor Inductance

Simplifying (with equal and opposite currents)

$$\lambda_{\text{left}} = \frac{\mu_0}{2\pi} I \left(\ln \frac{Rp}{r'} - \ln \left(\frac{Rp - R}{R} \right) \right)$$

$$= \frac{\mu_0}{2\pi} I (\ln R\,p - \ln r\,' - \ln(\,Rp - R) + \ln R)$$

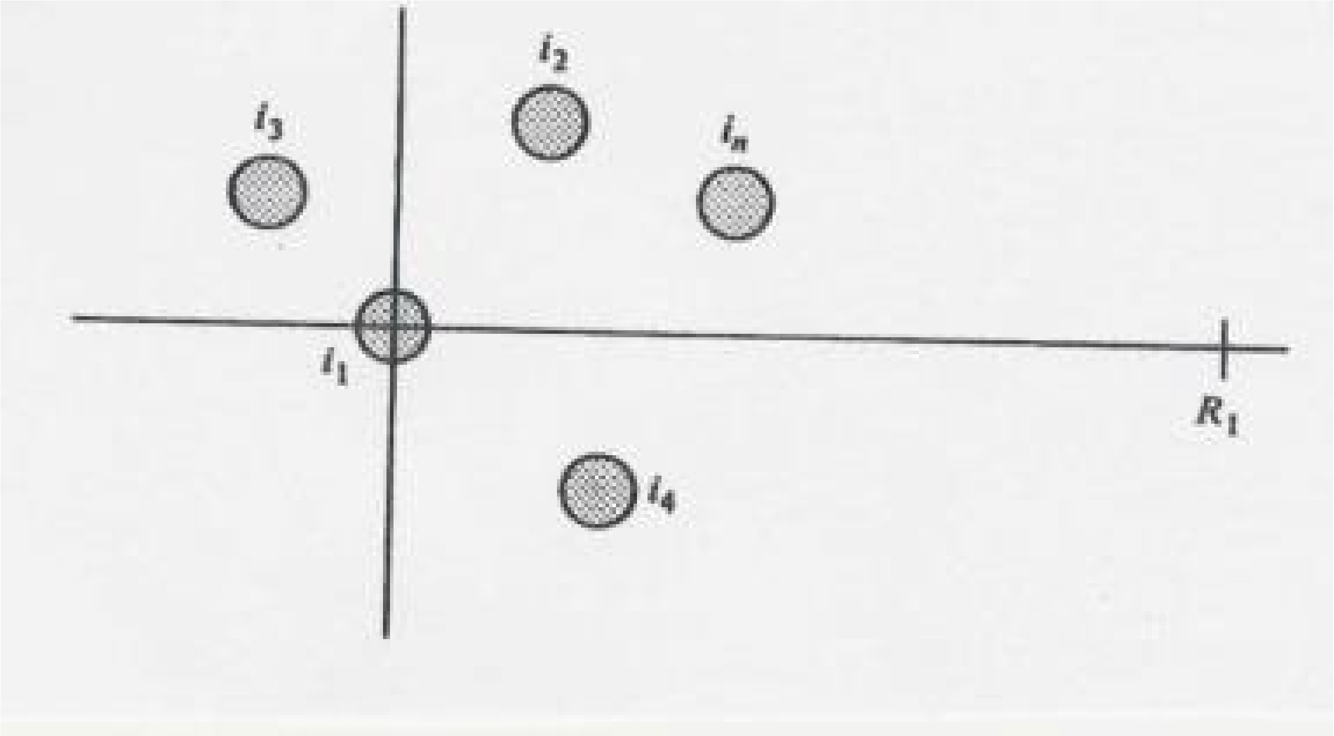
$$= \frac{\mu_0}{2\pi} I \left(\ln \frac{R}{r'} + \ln \frac{Rp}{Rp - R} \right)$$

$$= \frac{\mu_0}{2\pi} I \left(\ln \frac{R}{r'} \right) \quad \text{as Rp} \rightarrow \infty$$

$$L_{left} = \frac{\mu_0}{2\pi} \left(\ln \frac{R}{r'} \right) \quad \text{H/m}$$

Many-Conductor Case

Now assume we now have k conductors, each with current i_k, arranged in some specified geometry. We'd like to find flux linkages of each conductor.



Each conductor's flux linkage, l_k, depends upon its own current and the current in all the other conductors.

To derive l_1 we'll be integrating from conductor 1 (at origin) to the right along the x-axis.

Many-Conductor Case, cont'd

Therefore, if $\sum_{j=1}^{n} i_j = 0$,

which is true in a balanced three phase system, then the second term is zero and

$$\lambda_1 = \frac{\mu_0}{2\pi}\left[i_1 \ln\frac{1}{r_1'} + i_2 \ln\frac{1}{d_{12}} + \cdots + i_n \ln\frac{1}{d_{1n}}\right]$$

$$\lambda_1 = L_{11} i_1 + L_{12} i_2 \cdots + L_{1n} i_n$$

System has self and mutual inductance. However, the mutual inductance can be canceled for balanced 3ϕ systems with symmetry.

Symmetric Line Spacing – 69 kV

Line Inductance Example

Calculate the reactance for a balanced 3ϕ, 60Hz transmission line with a conductor geometry of an equilateral triangle with D = 5m, r = 1.24cm (Rook conductor) and a length of 5 miles.

Since system is assumed balanced

$$\mathrm{i_a} = -i_b - i_c$$

$$\lambda_a = \frac{\mu_0}{2\pi}\left[i_a \ln(\frac{1}{r'}) + i_b \ln(\frac{1}{D}) + i_c \ln(\frac{1}{D})\right]$$

Line Inductance Example, cont'd

Substituting

$$\mathrm{i_a} = -i_b - i_c$$

Hence

$$\lambda_\mathrm{a} = \frac{\mu_0}{2\pi}\left[i_a \ln\left(\frac{1}{r'}\right) - i_a \ln\left(\frac{1}{D}\right)\right]$$

$$= \frac{\mu_0}{2\pi} i_a \ln\left(\frac{D}{r'}\right)$$

$$L_a = \frac{\mu_0}{2\pi} \ln\left(\frac{D}{r'}\right) = \frac{4\pi \times 10^{-7}}{2\pi} \ln\left(\frac{5}{9.67 \times 10^{-3}}\right)$$

$= 1.25 \times 10^{-6}$ H/m

Transmission Tower Configurations

- The problem with the line analysis we've done so far is we have assumed a symmetrical tower configuration. Such a tower figuration is seldom practical.

Therefore, in general $D_{ab} \neq D_{ac} \neq D_{bc}$

Unless something was done this would result in unbalanced phases

Typical transmission tower configurations

Transposition

- To keep system balanced, over the length of a transmission line the conductors are rotated so each phase occupies each position on tower for an equal distance. This is known as transposition.
- In practice, not all lines are transposed, but it is a very common assumption for simplicity of calculation

Aerial or side view of conductor positions over the length of the transmission line.

Inductance of Transposed Line

Define the geometric mean distance (GMD)

$$\mathrm{D_m} \triangleq (d_{12}d_{13}d_{23})^{\frac{1}{3}}$$

Then for a balanced 3ϕ system $(I_a = -I_b - I_c)$

$$\lambda_\mathrm{a} = \frac{\mu_0}{2\pi}\left[I_a \ln\frac{1}{r'} - I_a \ln\frac{1}{D_m}\right] = \frac{\mu_0}{2\pi} I_a \ln\frac{D_m}{r'}$$

Hence

$$L_a = \frac{\mu_0}{2\pi}\ln\frac{D_m}{r'} = 2\times 10^{-7}\ln\frac{D_m}{r'} \quad \text{H/m}$$

Conductor Bundling

To increase the capacity of high voltage transmission lines it is very common to use a number of conductors per phase. This is known as conductor bundling. Typical values are two conductors for 345 kV lines, three for 500 kV and four for 765 kV.

Bundled Conductor Pictures

The AEP Wyoming-Jackson Ferry 765 kV line uses 6-bundle conductors. Conductors in a bundle are at the same voltage!

Photo Source: BPA and American Electric Power

Inductance of Lines with Bundled Conductors

- The per phase inductance is

$$\mathrm{L_a} = \frac{\mu_0}{2\pi} \ln\left(\frac{D}{R_b}\right)$$

where

$R_b \triangleq$ geometric mean radius (GMR) of bundle

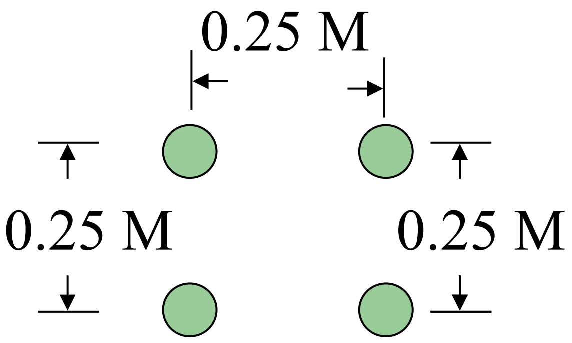
$= (r' d_{12} \dots d_{1b})^{\frac{1}{b}}$ in general

The detailed derivation is given in ECEN 459

When calculating the per phase resistance of bundled lines, the total resistance is R per conductor divided by b, where b is the number of conductors in the bundle

Bundle Inductance Example

Consider the previous example of the three phases symmetrically spaced 5 meters apart using wire with a radius of r = 1.24 cm. Except now assume each phase has 4 conductors in a square bundle, spaced 0.25 meters apart. What is the new inductance per meter?



$r = 1.24 \times 10^{-2}$ m $r' = 9.67 \times 10^{-3}$ m

$$R_b = \left(9.67 \times 10^{-3} \times 0.25 \times 0.25 \times \sqrt{2} \times 0.25\right)^{\frac{1}{4}}$$

$$= 0.12 \text{ m} \quad \text{(ten times bigger!)}$$

$$L_a = \frac{\mu_0}{2\pi} \ln \frac{5}{0.12} = 7.46 \times 10^{-7} \text{ H/m}$$

Inductance Example

- Calculate the per phase inductance and reactance of a balanced 3ϕ, 60 Hz, line with horizontal phase spacing of 10m using three conductor bundling with a spacing between conductors in the bundle of 0.3m. Assume the line is uniformly transposed and the conductors have a 1cm radius.

Answer: D_m = 12.6 m, R_b= 0.0889 m
Inductance = 9.9 x 10^{-7} H/m, Reactance = 0.6 Ω/Mile

Line Capacitance

- High voltage transmission lines and cables can have significant capacitance

For the case of uniformly transposed lines we use the same D_m and a similar GMR as with inducance,

$$C = \frac{2\pi\varepsilon}{\ln \frac{D_m}{R_b^c}}$$

where $D_m = [d_{ab} d_{ac} d_{bc}]^{\frac{1}{3}}$

$R_b^c = (r d_{12} \cdots d_{1n})^{\frac{1}{n}}$ (note r NOT r′)

ε in air $= \varepsilon_o = 8.854 \times 10^{-12}$ F/m

Line Capacitance Example

- Calculate the per phase capacitance and susceptance of a balanced 3ϕ, 60 Hz, transmission line with horizontal phase spacing of 10m using three conductor bundling with a spacing between conductors in the bundle of 0.3m. Assume the line is uniformly transposed and the conductors have a a 1cm radius.

Line Capacitance Example, cont'd

$$R_b^c = (0.01 \times 0.3 \times 0.3)^{\frac{1}{3}} = 0.0963 \text{ m}$$

$$\mathrm{D_m} = (10 \times 10 \times 20)^{\frac{1}{3}} = 12.6 \text{ m}$$

$$C = \frac{2\pi \times 8.854 \times 10^{-12}}{\ln \frac{12.6}{0.0963}} = 1.141 \times 10^{-11} \mathrm{F/m}$$

$$\mathrm{X_c} = \frac{1}{\omega C} = \frac{1}{2\pi 60 \times 1.141 \times 10^{-11} \mathrm{F/m}}$$

$$= 2.33 \times 10^{8}\ \Omega\text{−m (not } \Omega/\text{m)}$$

Conductor Table Example

TABLE A.4 Characteristics of aluminum cable, steel, reinforced (Aluminum Company of America)—ACSR

Code Word	Circular Mils Aluminum	Aluminum			Steel		Outside Diameter (inches)	Copper Equivalent* Circular Mils or A.W.G.	Ultimate Strength (pounds)	Weight (pounds per mile)	Geometric Mean Radius at 60 Hz (feet)	Approx. Current Carrying Capacity† (amps)	r_a Resistance (Ohms per Conductor per Mile)							
													25°C (77°F) Small Currents				50°C (122°F) Current Approx. 75% Capacity‡			
				Strand Diameter (inches)		Strand Diameter (inches)							dc	25 Hz	50 Hz	60 Hz	dc	25 Hz	50 Hz	60 Hz
Joree	2 515 000	76	...	0.1819	19	0.0849	1.880		61 700		0.0621									0.0450
Thrasher	2 312 000	76	...	0.1744	19	0.0814	1.802		57 300		0.0595									0.0482
Kiwi	2 167 000	72	4	0.1735	7	0.1157	1.735		49 800		0.0570									0.0511
Bluebird	2 156 000	84	4	0.1602	19	0.0961	1.762		60 300		0.0588									0.0505
Chukar	1 781 000	84	4	0.1456	19	0.0874	1.602		51 000		0.0534									0.0598
Falcon	1 590 000	54	3	0.1716	19	0.1030	1.545	1 000 000	56 000	10 777	0.0520	1 380	0.0587	0.0588	0.0590	0.0591	0.0646	0.0656	0.0675	0.0684
Parrot	1 510 500	54	3	0.1673	19	0.1004	1.506	950 000	53 200	10 237	0.0507	1 340	0.0618	0.0619	0.0621	0.0622	0.0680	0.0690	0.0710	0.0720
Plover	1 431 000	54	3	0.1628	19	0.0977	1.465	900 000	50 400	9 699	0.0493	1 300	0.0652	0.0653	0.0655	0.0656	0.0718	0.0729	0.0749	0.0760
Martin	1 351 000	54	3	0.1582	19	0.0949	1.424	850 000	47 600	9 160	0.0479	1 250	0.0691	0.0692	0.0694	0.0695	0.0761	0.0771	0.0792	0.0803
Pheasant	1 272 000	54	3	0.1535	19	0.0921	1.382	800 000	44 800	8 621	0.0465	1 200	0.0734	0.0735	0.0737	0.0738	0.0808	0.0819	0.0840	0.0851
Grackle	1 192 500	54	3	0.1486	19	0.0892	1.338	750 000	43 100	8 082	0.0450	1 160	0.0783	0.0784	0.0786	0.0788	0.0862	0.0872	0.0894	0.0906
Finch	1 113 000	54	3	0.1436	19	0.0862	1.293	700 000	40 200	7 544	0.0435	1 110	0.0839	0.0840	0.0842	0.0844	0.0924	0.0935	0.0957	0.0969
[illegible]	1 033 500	54	3	0.1384	7	0.1384	[illegible]	650 000	37 100	7 019	[illegible]	[illegible]	0.0903	0.0905	0.0907	0.0909	0.0994	0.1005	0.1025	[illegible]
Cardinal	954 000	54	3	0.1329	7	0.1329	1.196	600 000	34 200	6 479	0.0403	1 010	0.0979	0.0980	0.0981	0.0982	0.1078	0.1088	0.1118	0.1128
Canary	900 000	54	3	0.1291	7	0.1291	1.162	566 000	32 300	6 112	0.0391	970	0.104	0.104	0.104	0.104	0.1145	0.1155	0.1175	0.1185
Crane	874 500	54	3	0.1273	7	0.1273	1.146	550 000	31 400	5 940	0.0386	950	0.107	0.107	0.107	0.108	0.1178	0.1188	0.1218	0.1228
Condor	795 000	54	3	0.1214	7	0.1214	1.093	500 000	28 500	5 399	0.0368	900	0.117	0.118	0.118	0.119	0.1288	0.1308	0.1358	0.1378
Drake	795 000	26	2	0.1749	7	0.1360	1.108	500 000	31 200	5 770	0.0375	900	0.117	0.117	0.117	0.117	0.1288	0.1288	0.1288	0.1288
Mallard	795 000	30	2	0.1628	19	0.0977	1.140	500 000	38 400	6 517	0.0393	910	0.117	0.117	0.117	0.117	0.1288	0.1288	0.1288	0.1288
Crow	715 500	54	3	0.1151	7	0.1151	1.036	450 000	26 300	4 859	0.0349	830	0.131	0.131	0.131	0.132	0.1442	0.1452	0.1472	0.1482
Starling	715 500	26	2	0.1659	7	0.1290	1.051	450 000	28 100	5 193	0.0355	840	0.131	0.131	0.131	0.131	0.1442	0.1442	0.1442	0.1442
Redwing	715 500	30	2	0.1544	19	0.0926	1.081	450 000	34 600	5 865	0.0372	840	0.131	0.131	0.131	0.131	0.1442	0.1442	0.1442	0.1442
Flamingo	666 600	54	3	0.1111	7	0.1111	1.000	419 000	24 500	4 527	0.0337	800	0.140	0.140	0.141	0.141	0.1541	0.1571	0.1591	0.1601
Rook	636 000	54	3	0.1085	7	0.1085	0.977	400 000	23 600	4 319	0.0329	770	0.147	0.147	0.148	0.148	0.1618	0.1638	0.1678	0.1688
Grosbeak	636 000	26	2	0.1564	7	0.1216	0.990	400 000	25 000	4 616	0.0335	780	0.147	0.147	0.147	0.147	0.1618	0.1618	0.1618	0.1618
Egret	636 000	30	2	0.1456	19	0.0874	1.019	400 000	31 500	5 213	0.0351	780	0.147	0.147	0.147	0.147	0.1618	0.1618	0.1618	0.1618
Peacock	605 000	54	3	0.1059	7	0.1059	0.953	380 500	22 500	4 109	0.0321	750	0.154	0.155	0.155	0.155	0.1695	0.1715	0.1755	0.1775
Squab	605 000	26	2	0.1525	7	0.1186	0.966	380 500	24 100	4 391	0.0327	760	0.154	0.154	0.154	0.154	0.1700	0.1720	0.1720	0.1720
Dove	556 500	26	2	0.1463	7	0.1138	0.927	350 000	22 400	4 039	0.0313	730	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859
Eagle	556 500	30	2	0.1362	7	0.1362	0.953	350 000	27 200	4 588	0.0328	730	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859
Hawk	477 000	26	2	0.1355	7	0.1054	0.858	300 000	19 430	3 462	0.0290	670	0.196	0.196	0.196	0.196	0.216			