ECEN 460, Spring 2026
Power System Operation and Control

Class 14: More Power Flow Examples

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Announcements

- Homework #6: book problems 6.9, 6.12, 6.18, 6.25, 6.38, due Thursday, Mar 5, 2026

The Power Flow Problem

- What we know:
  - Generator real power
  - Generator voltage magnitude
  - Load real and reactive power
- What we don't know:
  - Bus voltage angles
  - Non-generator bus voltage magnitude
  - Generator reactive power
  - Current injections of generators or loads
- Note that the problem is formulated with *power* values, not current, so we cannot use the Y-bus equations directly
- Power flow problem is ***non-linear***

Power Flow Problem Variables

- These two equations must be satisfied for all buses in the system
- Total of 2n equations
- Parameters g and b are known from the Y-bus
- Each bus has four variables:
  - Bus voltage magnitude V_i
  - Bus voltage angle θ_i
  - Bus real power injection P_i
  - Bus reactive power injection Q_i

$$\mathrm{P_i} = \sum_{k=1}^{n} |V_i||V_k| \left(g_{ik}\cos\theta_{ik} + b_{ik}\sin\theta_{ik}\right) = P_{Gi} - P_{Di}$$

$$\mathrm{Q_i} = \sum_{k=1}^{n} |V_i||V_k| \left(g_{ik}\sin\theta_{ik} - b_{ik}\cos\theta_{ik}\right) = Q_{Gi} - Q_{Di}$$

- So in general, we need to pre-specify 2 additional variables at each bus before we can solve

Power Flow Bus Types

Each bus has one of three basic types, and for each one there are two known variables and two unknown variables:

- Load (PQ) bus
  - Assume P/Q are fixed at most buses in the system
  - Known: P and Q, Unknown: V and θ
- Slack bus
  - Only one in the whole system
  - Usually assigned to a large generator bus
  - Known: V and θ, Unknown: P and Q
  - Provides angle reference ($\theta = 0$) and picks up the slack P
- Generator (PV) bus
  - Other generators (5-10% of buses in a large system)
  - Assume generator is controlling power output and voltage
  - Known: V and P, Unknown: θ and Q

Newton-Raphson Power Flow Procedure

Same procedure we have been using for N-R solutions so far

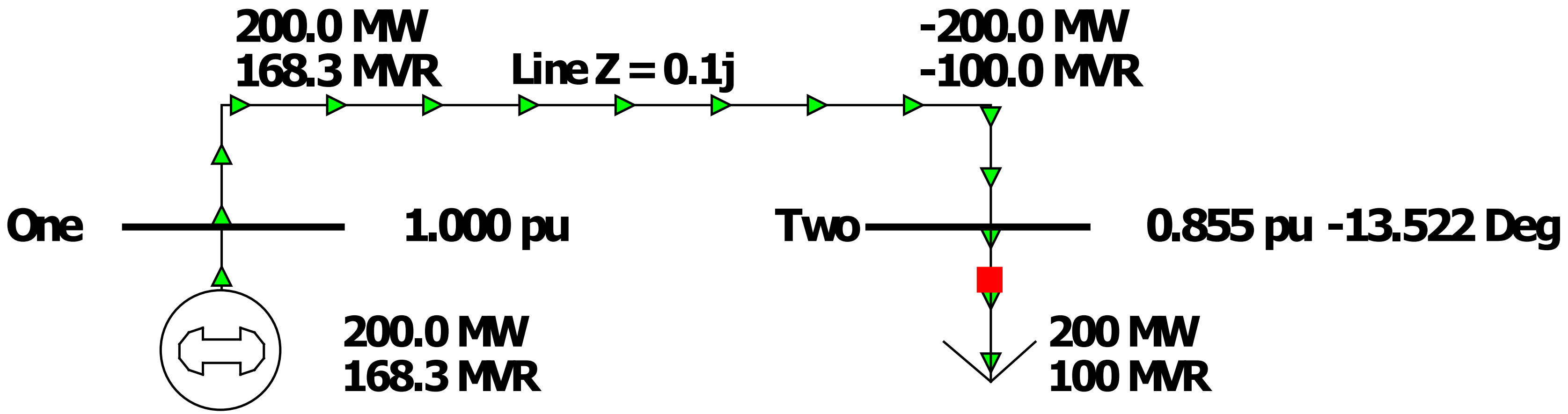
1. Build the Y-bus matrix
2. Write the list the variables and power balance equations
3. Make the power flow Jacobian
4. Make an initial guess of $\mathbf{x}$
5. While $\left\|\mathbf{f}\left(\mathbf{x}^{(v)}\right)\right\| > \epsilon$

$$\mathbf{x}^{(v+1)} = \mathbf{x}^{(v)} - \mathbf{J}(\mathbf{x}^{(v)})^{-1}\mathbf{f}(\mathbf{x}^{(v)})$$

$$v = v + 1$$

Two Bus Solved Values

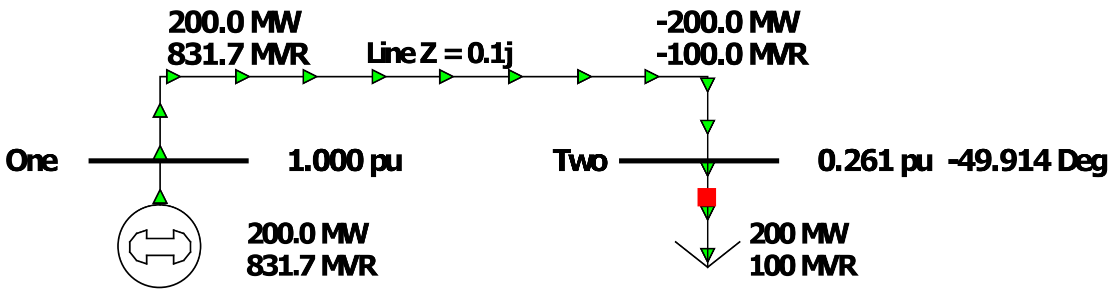
- Once the voltage angle and magnitude at bus 2 are known we can calculate all the other system values, such as the line flows and the generator reactive power output



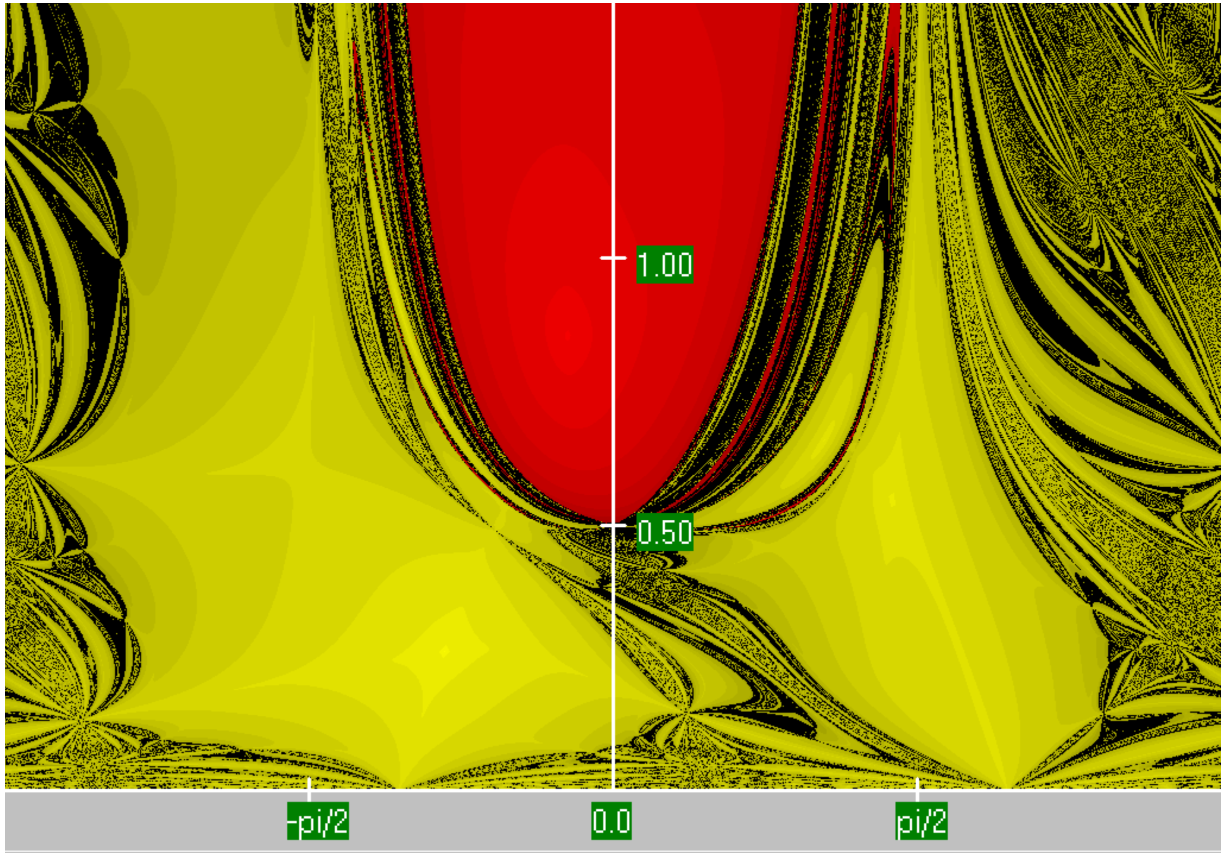
Low Voltage Solution!

- This case actually has two solutions! The second "low voltage" is found by using a low initial guess.

  Set $v = 0$, guess $\mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 0.25 \end{bmatrix}$



This shows the region of convergence for different initial guesses of bus 2 angle (x-axis) and magnitude (y-axis)



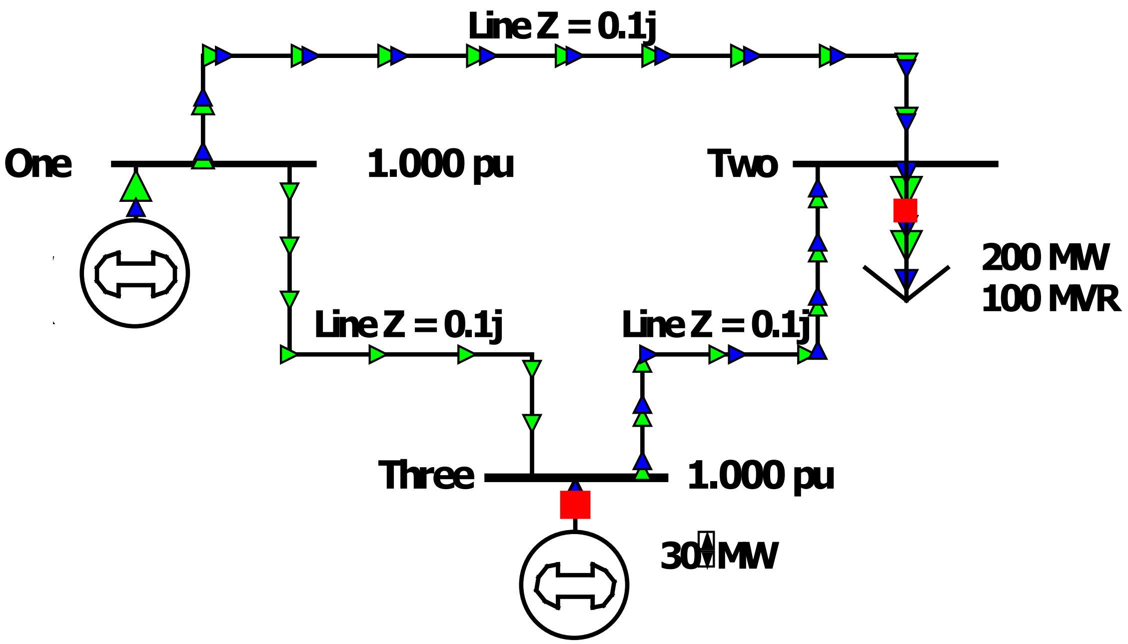
Red region converges to the high voltage solution, while the yellow region converges to the low voltage solution

Generator Reactive Power Limits

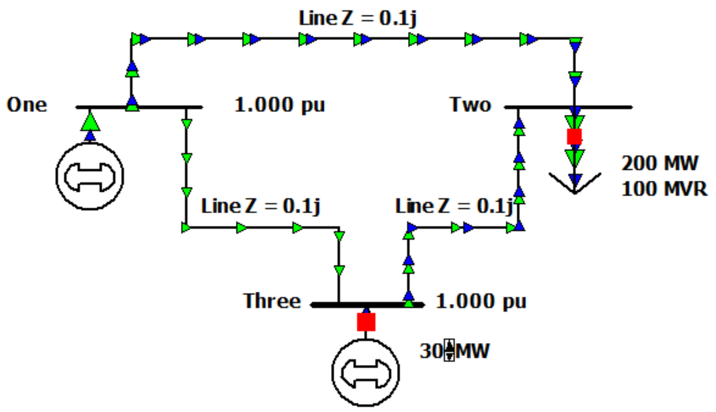
- The reactive power output of generators varies to maintain the terminal voltage; on a real generator this is done by the exciter
- To maintain higher voltages requires more reactive power
- Generators have reactive power limits, which are dependent upon the generator's MW output
- These limits must be considered during the power flow solution
  - During power flow, once a solution is obtained, check to make generator reactive power output is within its limits
  - If the reactive power is outside of the limits, fix Q at the max or min value, and resolve treating the generator as a PQ bus
    - this is know as "type-switching"
    - also need to check if a PQ generator can again regulate
- Rule of thumb: to raise system voltage we need to supply more vars

Power Flow Example #2

- Bus 1 contains a large generating unit operating at V=1.0 pu
- Bus 2 has a load consuming 200 MW and 100 MVR
- Bus 3 has a generator that is set to produce 30 MW and control its terminal voltage to 1.0 pu
- Want to solve for all bus voltages and line flows



Power Flow Example #2



$$P_i = \sum_{k=1}^{n} |V_i||V_k| \left(g_{ik}\cos\theta_{ik} + b_{ik}\sin\theta_{ik}\right) = P_{Gi} - P_{Di}$$

$$Q_i = \sum_{k=1}^{n} |V_i||V_k| \left(g_{ik}\sin\theta_{ik} - b_{ik}\cos\theta_{ik}\right) = Q_{Gi} - Q_{Di}$$

Power Flow Example #2

$$\mathrm{P_i} = \sum_{k=1}^{n} |V_i||V_k| \left(g_{ik} \cos\theta_{ik} + b_{ik} \sin\theta_{ik}\right) = P_{Gi} - P_{Di}$$

$$\mathrm{Q_i} = \sum_{k=1}^{n} |V_i||V_k| \left(g_{ik} \sin\theta_{ik} - b_{ik} \cos\theta_{ik}\right) = Q_{Gi} - Q_{Di}$$

Power Flow Example #2

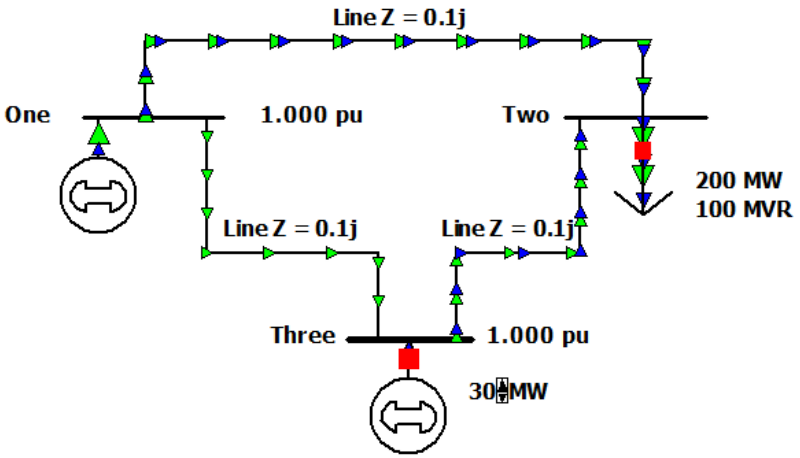
Power Flow Example #2

Power Flow Example #2

Power Flow Example #2, Solution

- First, figure out bus types, knowns, and unknowns
- Build Y-bus
- Identify variables and equations (unknown θ, V and known P, Q)

Bus	Type	Known	Unknown
1	Slack	θ, V	P, Q
2	PQ	P, Q	θ, V
3	PV	P, V	θ, Q



$$\text{Y-bus}=\begin{bmatrix} -20j & 10j & 10j \\ 10j & -20j & 10j \\ 10j & 10j & -20j \end{bmatrix}$$

$$x = \begin{bmatrix} \theta_2 \\ \theta_3 \\ V_2 \end{bmatrix};\ f(x) = \begin{bmatrix} P_2(x) \\ P_3(x) \\ Q_2(x) \end{bmatrix}$$

Power Flow Example #2, Solution

Bus	Type	Known	Unknown
1	Slack	θ, V	P, Q
2	PQ	P, Q	θ, V
3	PV	P, V	θ, Q

$$\text{Y-bus}=\begin{bmatrix} -20j & 10j & 10j \\ 10j & -20j & 10j \\ 10j & 10j & -20j \end{bmatrix};\ x=\begin{bmatrix}\theta_2\\ \theta_3\\ V_2\end{bmatrix};\ f(x)=\begin{bmatrix}P_2(x)\\ P_3(x)\\ Q_2(x)\end{bmatrix}$$

$$\mathrm{P_i} = \sum_{k=1}^{n} |V_i||V_k|\,(g_{ik}\cos\theta_{ik} + b_{ik}\sin\theta_{ik}) = P_{Gi} - P_{Di}$$

$$\mathrm{Q_i} = \sum_{k=1}^{n} |V_i||V_k|\,(g_{ik}\sin\theta_{ik} - b_{ik}\cos\theta_{ik}) = Q_{Gi} - Q_{Di}$$

Write out power balance equations to get f(x):

$$P_2(x) = V_2V_1b_{12}\sin(\theta_2-\theta_1) + V_2^2(b_{22}\sin 0) + V_2V_3b_{23}\sin(\theta_2-\theta_3) = 10V_2\sin\theta_2 + 10V_2\sin(\theta_2-\theta_3)$$

$$P_3(x) = V_3V_1b_{12}\sin(\theta_3-\theta_1) + V_3V_2b_{23}\sin(\theta_3-\theta_2) + V_3^2(b_{33}\sin 0) = 10\sin\theta_3 + 10V_2\sin(\theta_3-\theta_2)$$

$$Q_2(x) = V_2V_1(-b_{21}\cos(\theta_2-\theta_1)) + V_2^2(-b_{22}\cos 0) + V_2V_3(-b_{23}\cos(\theta_3-\theta_2)) = -10\mathrm{V}_2\cos\theta_2 + 20\mathrm{V}_2^2 - 10\mathrm{V}_2\cos(\theta_3-\theta_2)$$

$$f(x) = \begin{bmatrix} 10V_2\sin\theta_2 + 10V_2\sin(\theta_2-\theta_3) + \mathrm{P_{D2}} \\ 10\sin\theta_3 + 10V_2\sin(\theta_3-\theta_2) - P_{G3} \\ -10\mathrm{V}_2\cos\theta_2 + 20\mathrm{V}_2^2 - 10\mathrm{V}_2\cos(\theta_3-\theta_2) + Q_{D2} \end{bmatrix}$$

And the Jacobian from all the partial derivatives

$$J(x) = \begin{bmatrix} 10V_2\cos\theta_2 + 10V_2\cos\theta_{32} & -10V_2\cos\theta_{23} & 10\sin\theta_2 - 10\sin\theta_{23} \\ -10V_2\cos\theta_{23} & 10\cos\theta_3 + 10V_2\cos\theta_{32} & 10V_2\sin\theta_{23} \\ 10\sin\theta_2 + 10\sin\theta_{23} & 10\sin(\theta_{32}) & -10\cos\theta_2 + 40V_2 - 10\cos\theta_{32} \end{bmatrix}$$

Power Flow Example #2, Solution

$$x = \begin{bmatrix} \theta_2 \\ \theta_3 \\ V_2 \end{bmatrix};\ f(x) = \begin{bmatrix} P_2(x) + P_{d2} \\ P_3(x) - P_{g2} \\ Q_2(x) + Q_{d3} \end{bmatrix} = \begin{bmatrix} 10V_2 \sin\theta_2 + 10V_2 \sin(\theta_2 - \theta_3) + P_{d2} \\ 10 \sin\theta_3 + 10V_2 \sin(\theta_3 - \theta_2) - P_{g2} \\ -10\mathrm{V}_2 \cos\theta_2 + 20\mathrm{V}_2^2 - 10\mathrm{V}_2 \cos(\theta_3 - \theta_2) + Q_{d3} \end{bmatrix}$$

$$J(x) = \begin{bmatrix} 10V_2 \cos\theta_2 + 10V_2 \cos\theta_{32} & -10V_2 \cos\theta_{23} & 10 \sin\theta_2 - 10 \sin\theta_{23} \\ -10V_2 \cos\theta_{23} & 10 \cos\theta_3 + 10V_2 \cos\theta_{32} & 10V_2 \sin\theta_{23} \\ 10 \sin\theta_2 + 10 \sin\theta_{23} & 10 \sin(\theta_{32}) & -10 \cos\theta_2 + 40V_2 - 10 \cos\theta_{32} \end{bmatrix}$$

Iteration 1

$$x^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; f\left(x^{(0)}\right) = \begin{bmatrix} 2 \\ -0.3 \\ 1 \end{bmatrix}; J\left(x^{(0)}\right) = \begin{bmatrix} 20 & -10 & 0 \\ -10 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix};\ x^{(1)} = x^{(0)} - \mathbf{J}(\mathbf{x}^{(0)})^{-1}\mathbf{f}\left(\mathbf{x}^{(0)}\right) = \begin{bmatrix} -0.1233 \\ -0.0467 \\ 0.95 \end{bmatrix}$$

Iteration 2

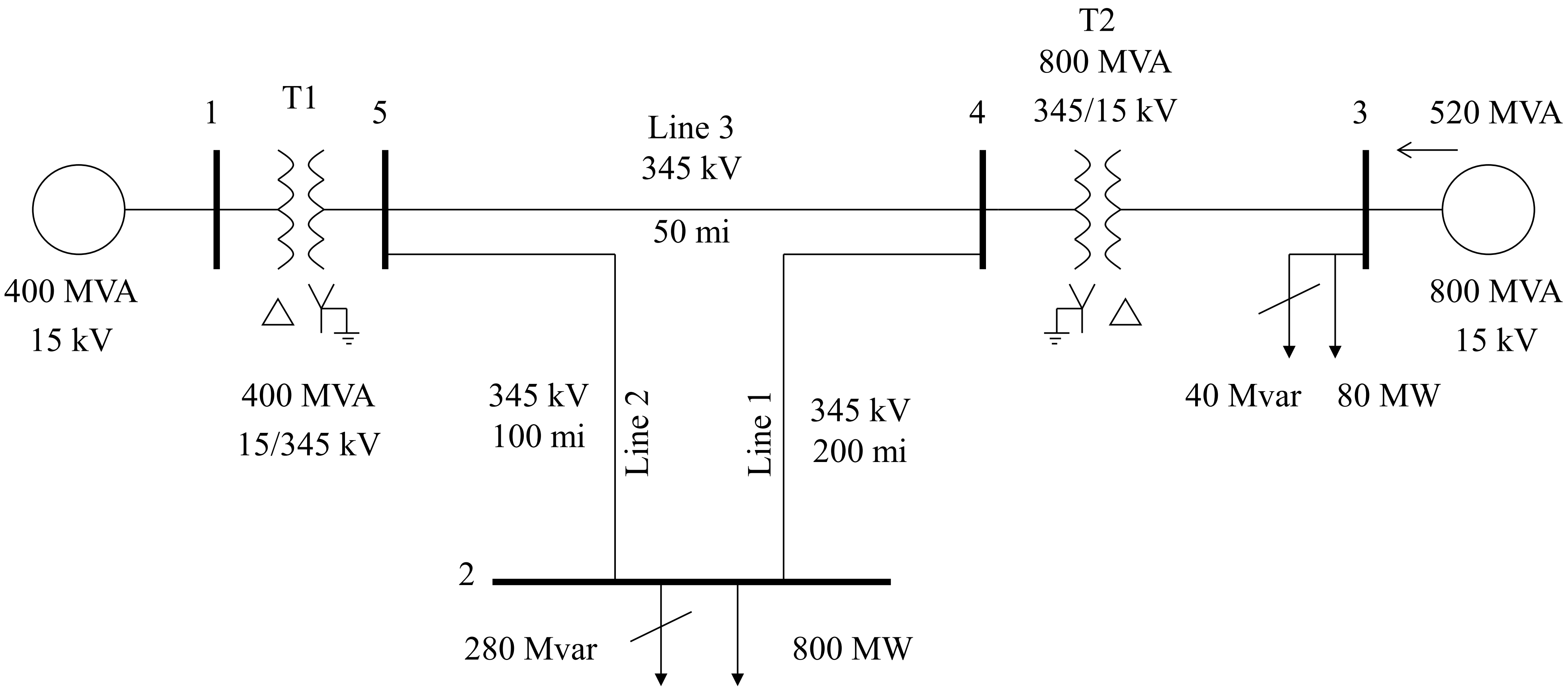
$$x^{(1)} = \begin{bmatrix} -0.1233 \\ -0.0467 \\ 0.95 \end{bmatrix}; f\left(x^{(1)}\right) = \begin{bmatrix} 0.1037 \\ -0.03887 \\ 0.1501 \end{bmatrix}; J\left(x^{(1)}\right) = \begin{bmatrix} 18.9 & -9.47 & -0.46 \\ -9.47 & 19.5 & -0.73 \\ -2.00 & 0.766 & 18.11 \end{bmatrix};\ x^{(2)} = x^{(1)} - \mathbf{J}(\mathbf{x}^{(1)})^{-1}\mathbf{f}\left(\mathbf{x}^{(1)}\right) = \begin{bmatrix} -0.1298 \\ -0.04814 \\ 0.9411 \end{bmatrix}$$

Iteration 3

$$x^{(2)} = \begin{bmatrix} -0.1298 \\ -0.04814 \\ 0.9411 \end{bmatrix}; f\left(x^{(2)}\right) = \begin{bmatrix} 0.0147 \\ -0.0138 \\ 0.00122 \end{bmatrix}; J\left(x^{(2)}\right) = \begin{bmatrix} 18.71 & -9.38 & -0.48 \\ -9.38 & 19.37 & -0.767 \\ -2.11 & 0.815 & 17.76 \end{bmatrix};\ x^{(3)} = x^{(2)} - \mathbf{J}(\mathbf{x}^{(2)})^{-1}\mathbf{f}\left(\mathbf{x}^{(2)}\right) = \begin{bmatrix} -0.1304 \\ -0.04771 \\ 0.9409 \end{bmatrix}$$

$$f\left(x^{(3)}\right) = \begin{bmatrix} 0.000259 \\ -0.000250 \\ -0.000086 \end{bmatrix}$$

The N-R Power Flow: 5-Bus Example



Single-line diagram

The N-R Power Flow: 5-Bus Example

Table 1. Bus input data

Bus	Type	V per unit	δ degrees	P_G per unit	Q_G per unit	P_L per unit	Q_L per unit	Q_{Gmax} per unit	Q_{Gmin} per unit
1	Swing	1.0	0	—	—	0	0	—	—
2	Load	—	—	0	0	8.0	2.8	—	—
3	Constant voltage	1.05	—	5.2	—	0.8	0.4	4.0	-2.8
4	Load	—	—	0	0	0	0	—	—
5	Load	—	—	0	0	0	0	—	—

Table 2. Line input data

Bus-to-Bus	R' per unit	X' per unit	G' per unit	B' per unit	Maximum MVA per unit
2-4	0.0090	0.100	0	1.72	12.0
2-5	0.0045	0.050	0	0.88	12.0
4-5	0.00225	0.025	0	0.44	12.0

The N-R Power Flow: 5-Bus Example

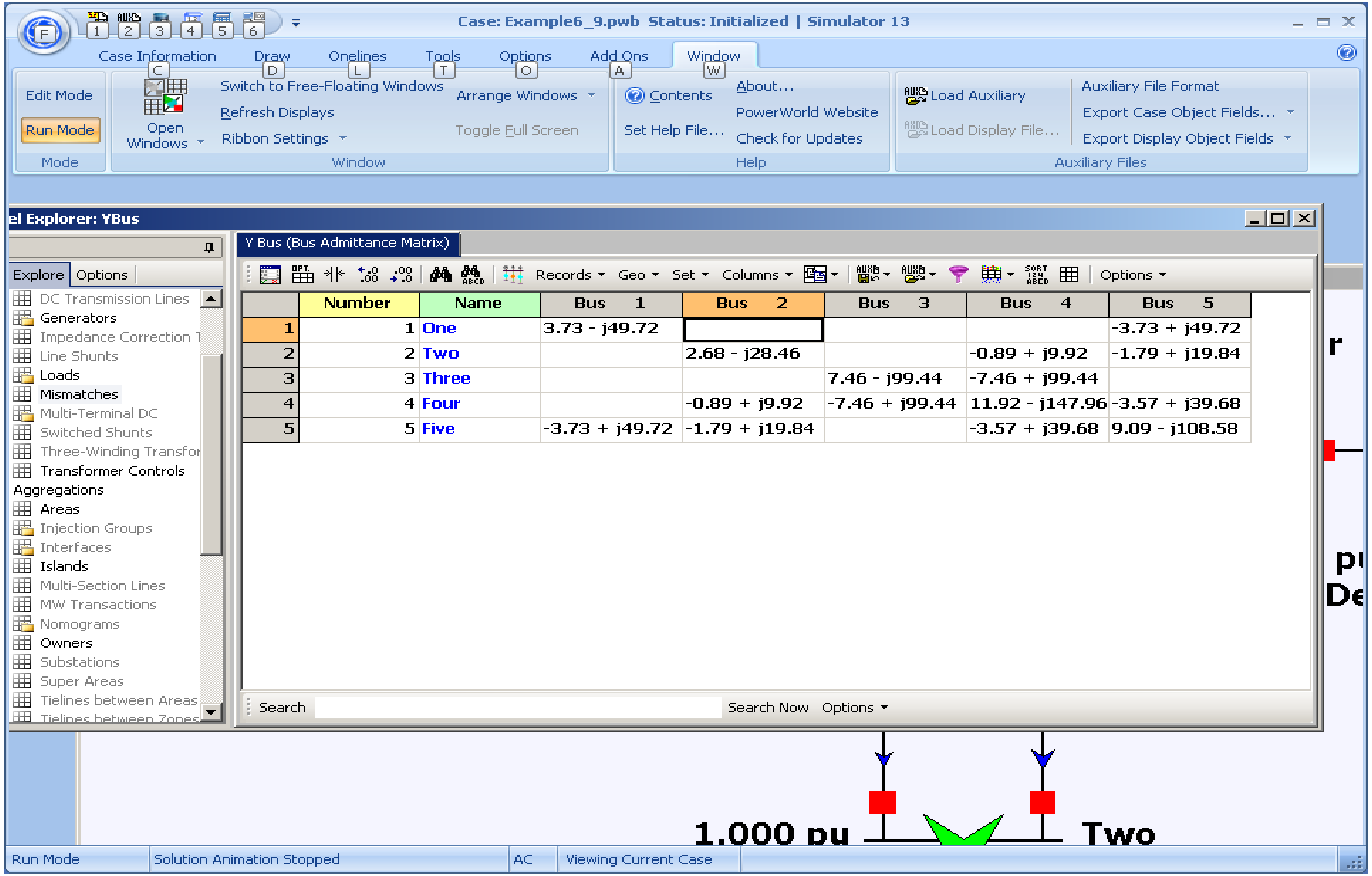
Table 3. Transformer input data

Bus-to-Bus	R per unit	X per unit	G_c per unit	B_m per unit	Maximum MVA per unit	Maximum TAP Setting per unit
1-5	0.00150	0.02	0	0	6.0	—
3-4	0.00075	0.01	0	0	10.0	—

Table 4. Input data and unknowns

Bus	Input Data	Unknowns
1	$V_1 = 1.0,\ \delta_1 = 0$	P_1, Q_1
2	$P_2 = P_{G2}-P_{L2} = -8$ $Q_2 = Q_{G2}-Q_{L2} = -2.8$	V_2, δ_2
3	$V_3 = 1.05$ $P_3 = P_{G3}-P_{L3} = 4.4$	Q_3, δ_3
4	$P_4 = 0,\ Q_4 = 0$	V_4, δ_4
5	$P_5 = 0,\ Q_5 = 0$	V_5, δ_5

Time to Close the Hood: Let the Computer Do the Math! (Ybus Shown)



	Number	Name	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5
1	1	One	3.73 - j49.72				-3.73 + j49.72
2	2	Two		2.68 - j28.46		-0.89 + j9.92	-1.79 + j19.84
3	3	Three			7.46 - j99.44	-7.46 + j99.44	
4	4	Four		-0.89 + j9.92	-7.46 + j99.44	11.92 - j147.96	-3.57 + j39.68
5	5	Five	-3.73 + j49.72	-1.79 + j19.84		-3.57 + j39.68	9.09 - j108.58

Ybus Details

Elements of Y_{bus} connected to bus 2

$$Y_{21} = Y_{23} = 0$$

$$Y_{24} = \frac{-1}{R'_{24} + jX'_{24}} = \frac{-1}{0.009 + j0.1} = -0.89276 + j9.91964 \; per \; unit$$

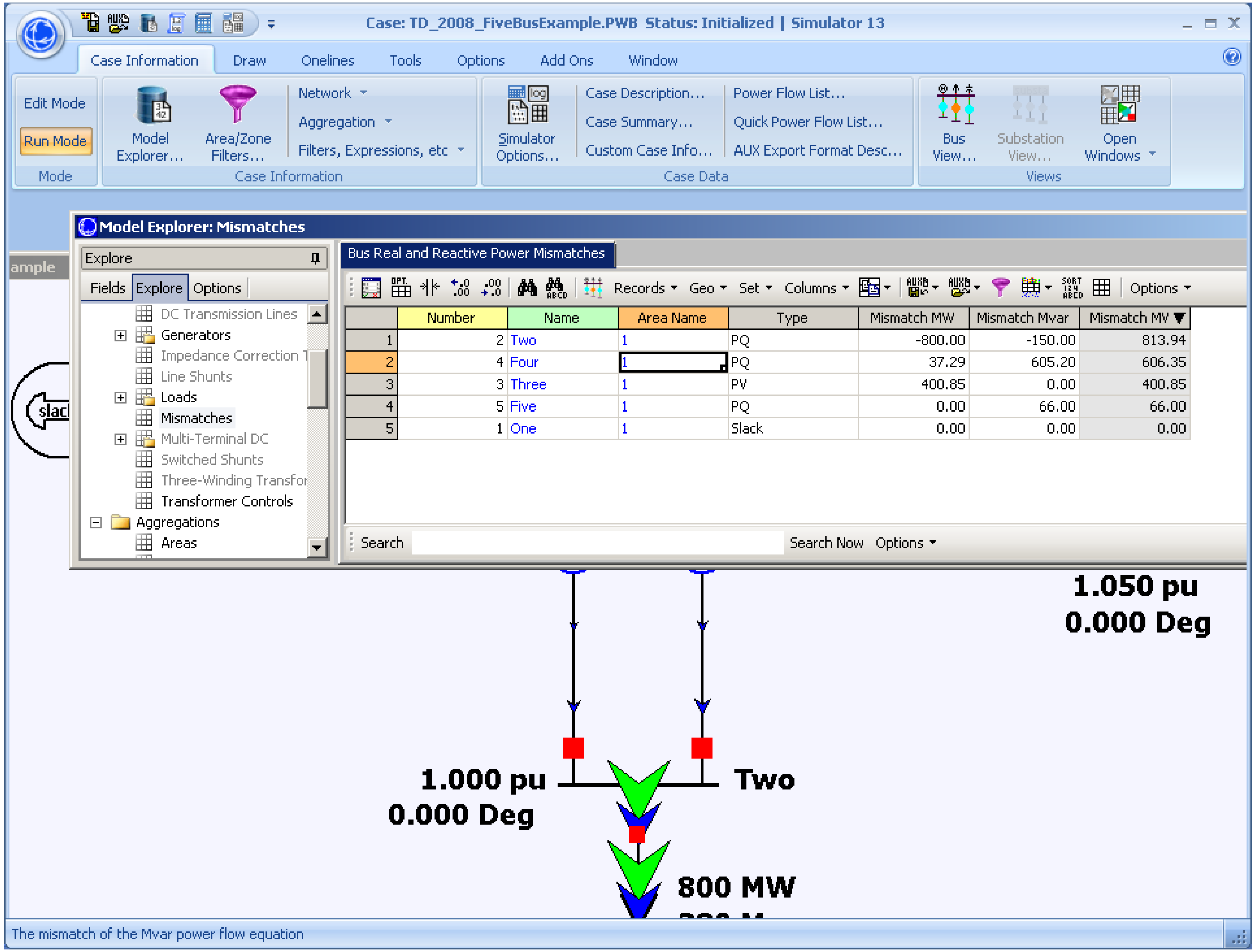
$$Y_{25} = \frac{-1}{R'_{25} + jX'_{25}} = \frac{-1}{0.0045 + j0.05} = -1.78552 + j19.83932 \; per \; unit$$

$$Y_{22} = \frac{1}{R'_{24} + jX'_{24}} + \frac{1}{R'_{25} + jX'_{25}} + j\frac{B'_{24}}{2} + j\frac{B'_{25}}{2}$$

$$= (0.89276 - j9.91964) + (1.78552 - j19.83932) + j\frac{1.72}{2} + j\frac{0.88}{2}$$

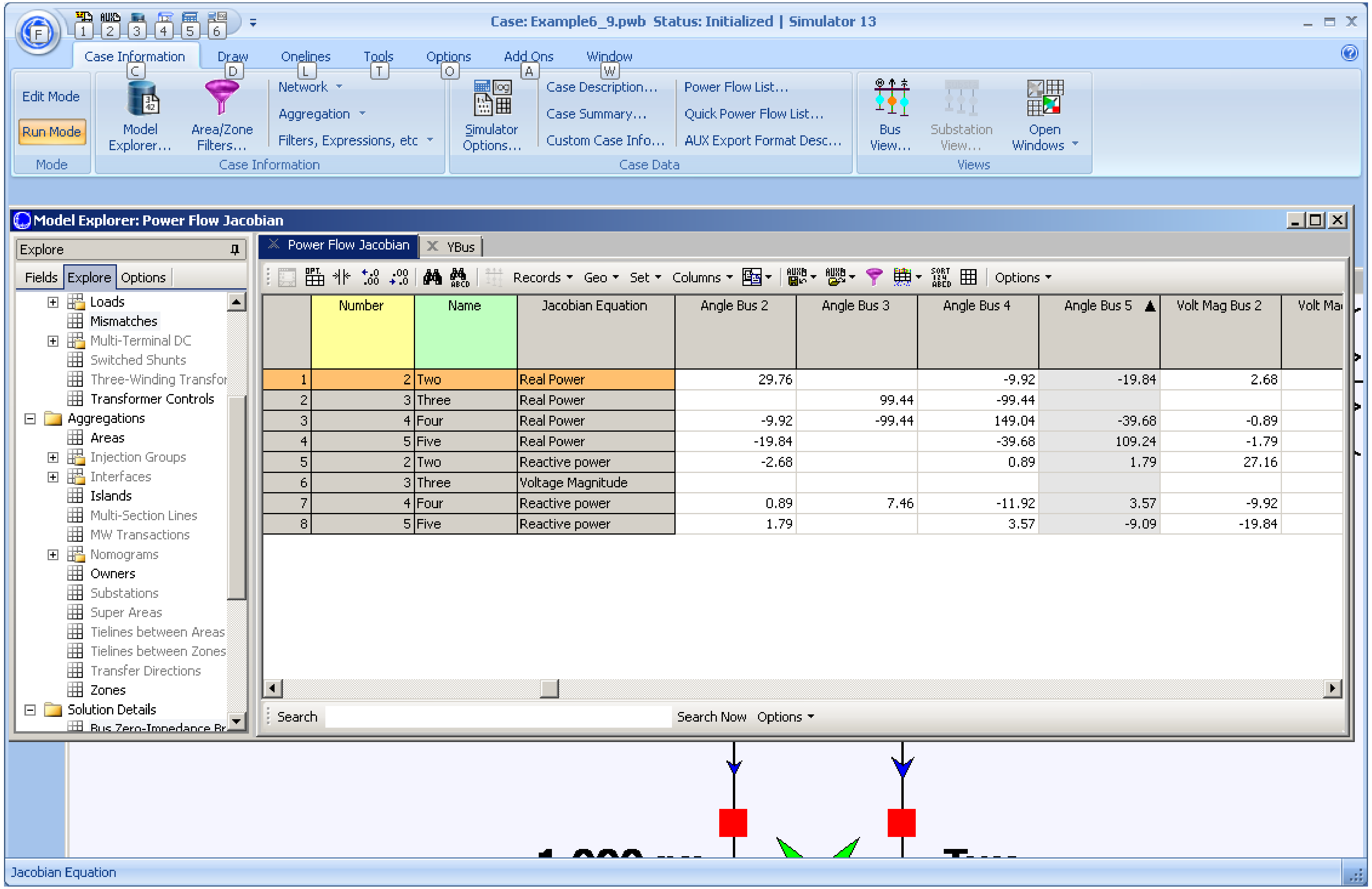
$$= 2.67828 - j28.4590 = 28.5847\angle -84.624° \; per \; unit$$

Here Are the Initial Bus Mismatches



	Number	Name	Area Name	Type	Mismatch MW	Mismatch Mvar	Mismatch MV
1	2	Two	1	PQ	-800.00	-150.00	813.94
2	4	Four	1	PQ	37.29	605.20	606.35
3	3	Three	1	PV	400.85	0.00	400.85
4	5	Five	1	PQ	0.00	66.00	66.00
5	1	One	1	Slack	0.00	0.00	0.00

And the Initial Power Flow Jacobian



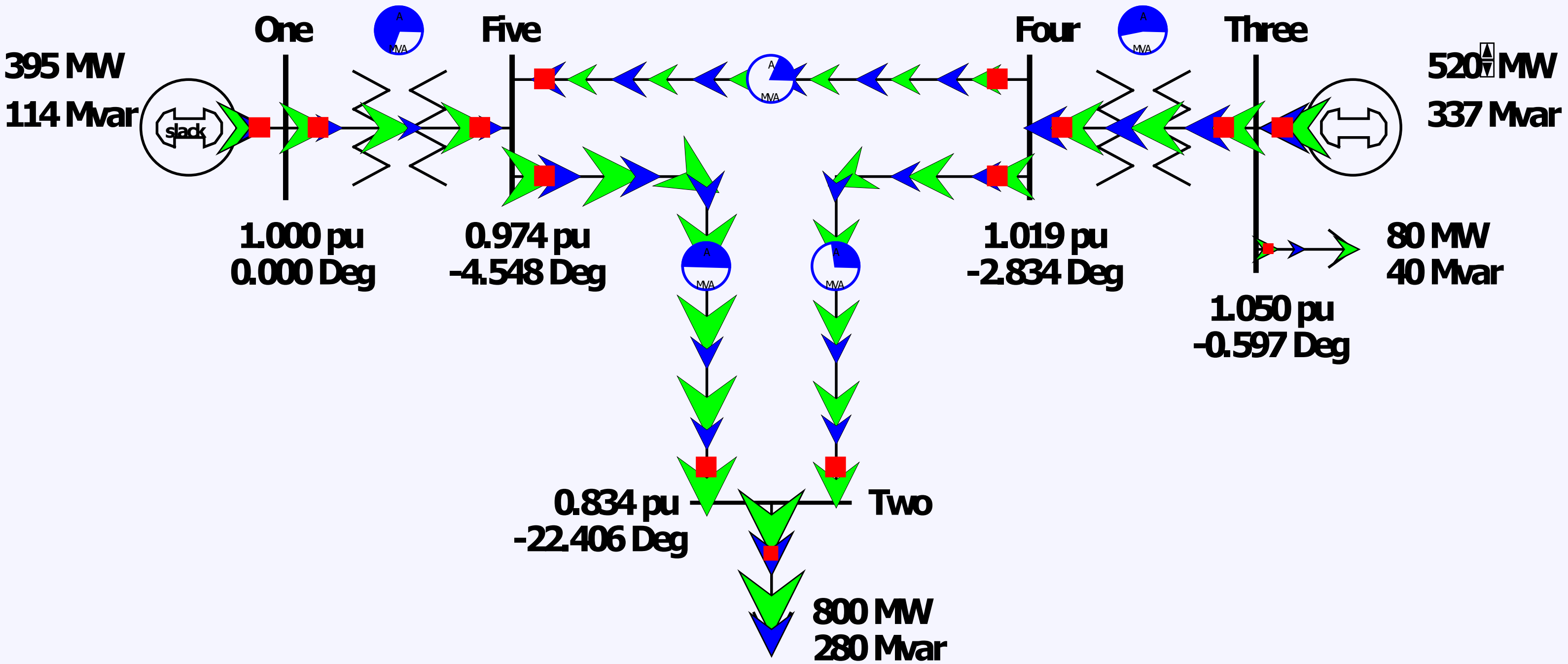
	Number	Name	Jacobian Equation	Angle Bus 2	Angle Bus 3	Angle Bus 4	Angle Bus 5	Volt Mag Bus 2
1	2	Two	Real Power	29.76		-9.92	-19.84	2.68
2	3	Three	Real Power		99.44	-99.44		
3	4	Four	Real Power	-9.92	-99.44	149.04	-39.68	-0.89
4	5	Five	Real Power	-19.84		-39.68	109.24	-1.79
5	2	Two	Reactive power	-2.68		0.89	1.79	27.16
6	3	Three	Voltage Magnitude					
7	4	Four	Reactive power	0.89	7.46	-11.92	3.57	-9.92
8	5	Five	Reactive power	1.79		3.57	-9.09	-19.84

And the Hand Calculation Details!

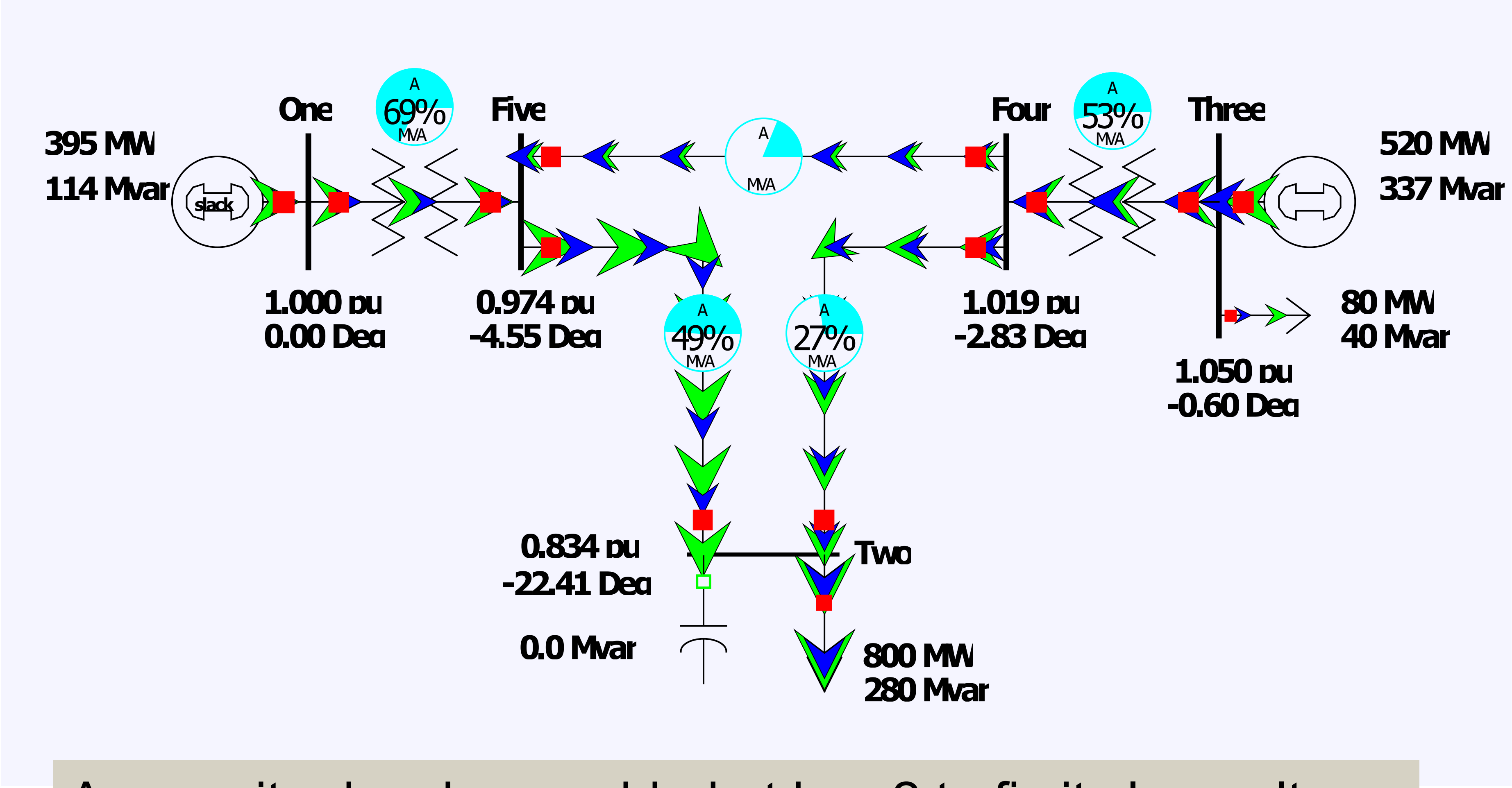
$$\Delta P_2(0) = P_2 - P_2(x) = P_2 - V_2(0)\{Y_{21}V_1\cos[\delta_2(0) - \delta_1(0) - \theta_{21}]$$
$$+Y_{22}V_2\cos[-\theta_{22}] + Y_{23}V_3\cos[\delta_2(0) - \delta_3(0) - \theta_{23}]$$
$$+Y_{24}V_4\cos[\delta_2(0) - \delta_4(0) - \theta_{24}]$$
$$+Y_{25}V_5\cos[\delta_2(0) - \delta_5(0) - \theta_{25}]\}$$
$$= -8.0 - 1.0\{28.5847(1.0)\cos(84.624°)$$
$$+9.95972(1.0)\cos(-95.143°)$$
$$+19.9159(1.0)\cos(-95.143°)\}$$
$$= -8.0 - (-2.89 \times 10^{-4}) = -7.99972 \; per \; unit$$

$$J1_{24}(0) = V_2(0)Y_{24}V_4(0)\sin[\delta_2(0) - \delta_4(0) - \theta_{24}]$$
$$= (1.0)(9.95972)(1.0)\sin[-95.143°]$$
$$= -9.91964 \; per \; unit$$

Five Bus Power System Solved

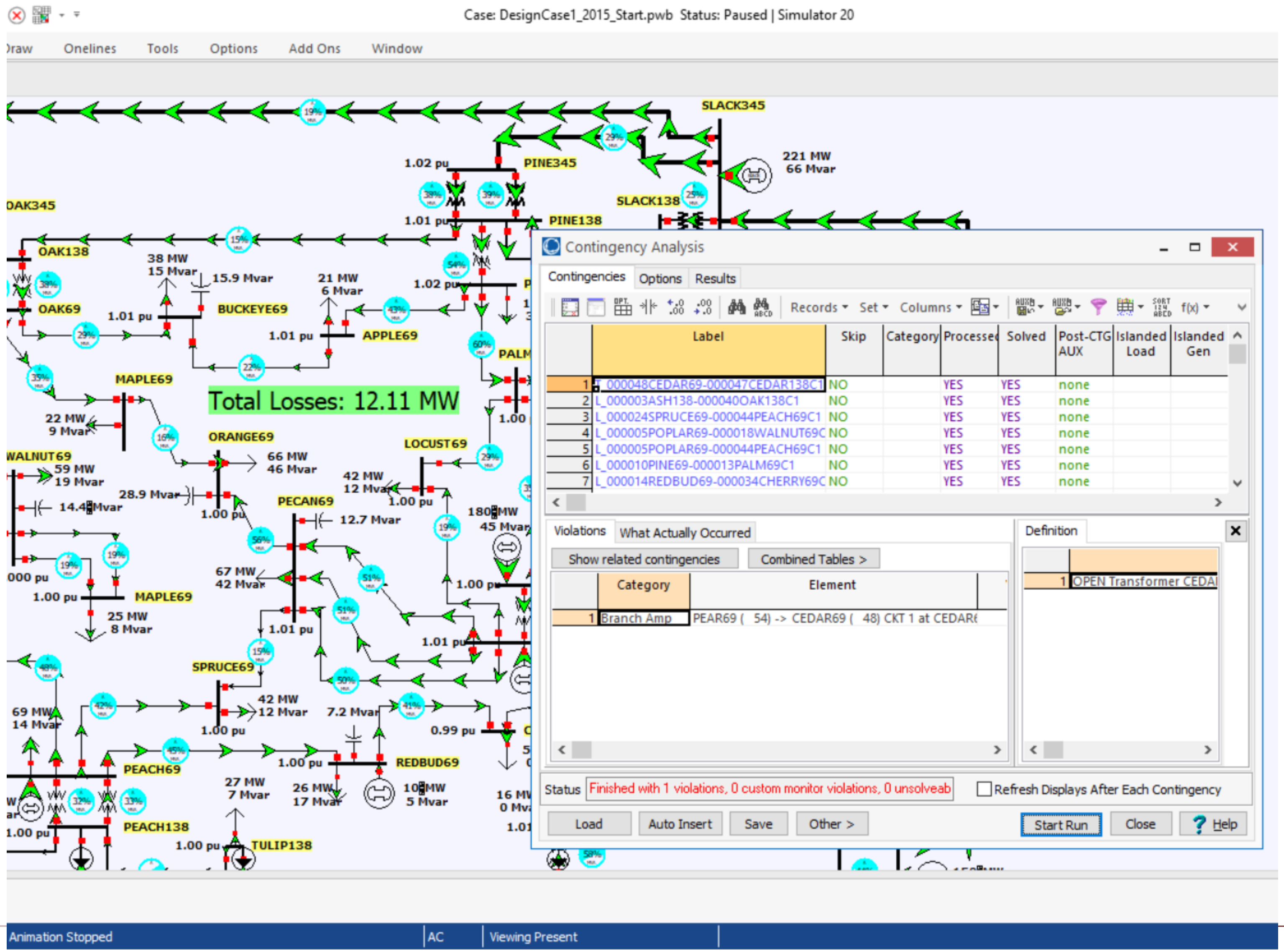


Five Bus System with Capacitor



A capacitor has been added at bus 2 to fix its low voltage

Contingency Analysis



Contingency analysis provides an automatic way of looking at all the statistically likely contingencies. In this example the contingency set is all the single line and transformer outages

Power Flow Application - PJM Control Center