ECEN 460, Spring 2026
Power System Operation and Control

Class 13: Newton-Raphson Solution Method for the Power Flow Problem

Prof. Adam Birchfield
Dept. of Electrical and Computer Engineering
Texas A&M University
abirchfield@tamu.edu

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Announcements

- Homework #5: book problems 6.30 and 6.31, due Thursday, Feb. 26, 2026
- Homework #6: book problems 6.9, 6.12, 6.18, 6.25, 6.38, due Thursday, Mar 5, 2026

Recall, Transmission Line and Transformer Models

- Transmission lines use the "pi" model

These devices reduce down to impedances

- Transformer non-ideal model is simplified with the per-unit system (and neglecting shunt elements)

Recall, Building the Y-Bus Matrix

- Here are the steps for building the Y-bus matrix
  - Start with an NxN matrix of all zeros (N is the number of buses)
  - For any shunt elements (connected to ground, such as most capacitors)
    - Add $\bar{Y}$ to the corresponding diagonal elements of the Y-bus
  - For branches between two buses,
    - Add $\bar{Y}$ to both diagonal elements of the Y-bus
    - Subtract $\bar{Y}$ from both off-diagonal elements of the Y-bus
- We add one network element at a time to the Y-bus matrix!

Remember, $\bar{Y} = 1/\bar{Z}$

$$\longrightarrow \begin{bmatrix} \bar{Y}_A + \bar{Y}_B & -\bar{Y}_A & -\bar{Y}_B & 0 \\ -\bar{Y}_A & \bar{Y}_A + \bar{Y}_C + \bar{Y}_D & -\bar{Y}_C & -\bar{Y}_D \\ -\bar{Y}_B & \bar{Y}_C & \bar{Y}_B + \bar{Y}_C & 0 \\ 0 & -\bar{Y}_D & 0 & \bar{Y}_D \end{bmatrix}$$

The Power Flow Problem

- What we know:
  - Generator real power
  - Generator voltage magnitude
  - Load real and reactive power
- What we don't know:
  - Bus voltage angles
  - Non-generator bus voltage magnitude
  - Generator reactive power
  - Current injections of generators or loads
- Note that the problem is formulated with *power* values, not current, so we cannot use the Y-bus equations directly
- Power flow problem is ***non-linear***

Power Flow Problem Variables

- These two equations must be satisfied for all buses in the system
- Total of 2n equations
- Parameters g and b are known from the Y-bus
- Each bus has four variables:
  - Bus voltage magnitude V_i
  - Bus voltage angle θ_i
  - Bus real power injection P_i
  - Bus reactive power injection Q_i
- So in general, we need to pre-specify 2 additional variables at each bus before we can solve

$$\mathrm{P_i} = \sum_{k=1}^{n} |V_i||V_k| \left(g_{ik} \cos\theta_{ik} + b_{ik} \sin\theta_{ik}\right) = P_{Gi} - P_{Di}$$

$$\mathrm{Q_i} = \sum_{k=1}^{n} |V_i||V_k| \left(g_{ik} \sin\theta_{ik} - b_{ik} \cos\theta_{ik}\right) = Q_{Gi} - Q_{Di}$$

Power Flow Bus Types

Each bus has one of three basic types, and for each one there are two known variables and two unknown variables:

- Load (PQ) bus
  - Assume P/Q are fixed at most buses in the system
  - Known: P and Q, Unknown: V and θ
- Slack bus
  - Only one in the whole system
  - Usually assigned to a large generator bus
  - Known: V and θ, Unknown: P and Q
  - Provides angle reference ($\theta = 0$) and picks up the slack P
- Generator (PV) bus
  - Other generators (5-10% of buses in a large system)
  - Assume generator is controlling power output and voltage
  - Known: V and P, Unknown: θ and Q

Using Newton-Raphson to Solve the Power Flow Problem

- Gauss is no longer in wide use for power flow solutions
- Newton-Raphson is the standard method
- Variables are θ and V for every bus (except slack bus)

$$\mathbf{x} = \begin{bmatrix} \theta_2 \\ \vdots \\ \theta_{\mathrm{n}} \\ |V_2| \\ \vdots \\ |V_n| \end{bmatrix} \quad \mathbf{f}(\mathbf{x}) = \begin{bmatrix} P_2(\mathbf{x}) - P_{G2} + P_{D2} \\ \vdots \\ P_n(\mathbf{x}) - P_{Gn} + P_{Dn} \\ Q_2(\mathbf{x}) - Q_{G2} + Q_{D2} \\ \vdots \\ Q_n(\mathbf{x}) - Q_{Gn} + Q_{Dn} \end{bmatrix}$$

$$\mathrm{P_i} = \sum_{k=1}^{n} |V_i||V_k| \left(g_{ik} \cos\theta_{ik} + b_{ik} \sin\theta_{ik}\right) = P_{Gi} - P_{Di}$$

$$\mathrm{Q_i} = \sum_{k=1}^{n} |V_i||V_k| \left(g_{ik} \sin\theta_{ik} - b_{ik} \cos\theta_{ik}\right) = Q_{Gi} - Q_{Di}$$

Newton-Raphson Power Flow Procedure

Same procedure we have been using for N-R solutions so far

1. Build the Y-bus matrix
2. Write the list the variables and power balance equations
3. Make the power flow Jacobian
4. Make an initial guess of $\mathbf{x}$
5. While $\left\|\mathbf{f}\left(\mathbf{x}^{(v)}\right)\right\| > \epsilon$

$$\mathbf{x}^{(v+1)} = \mathbf{x}^{(v)} - \mathbf{J}(\mathbf{x}^{(v)})^{-1}\mathbf{f}(\mathbf{x}^{(v)})$$

$$v = v + 1$$

Power Flow Jacobian Matrix

The most difficult part of the algorithm is determining and inverting the n by n Jacobian matrix, $\mathbf{J}(\mathbf{x})$

$$\mathbf{J}(\mathbf{x}) = \begin{bmatrix} \frac{\partial f_1(\mathbf{x})}{\partial x_1} & \frac{\partial f_1(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_1(\mathbf{x})}{\partial x_n} \\ \frac{\partial f_2(\mathbf{x})}{\partial x_1} & \frac{\partial f_2(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_2(\mathbf{x})}{\partial x_n} \\ \vdots & \ddots & \ddots & \vdots \\ \frac{\partial f_n(\mathbf{x})}{\partial x_1} & \frac{\partial f_n(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_n(\mathbf{x})}{\partial x_n} \end{bmatrix}$$

Power Flow Jacobian Matrix, Cont.

Jacobian elements are calculated by differentiating each function, $\mathrm{f_i}(\mathbf{x})$, with respect to each variable. For example, if $\mathrm{f_i}(\mathbf{x})$ is the bus i real power equation

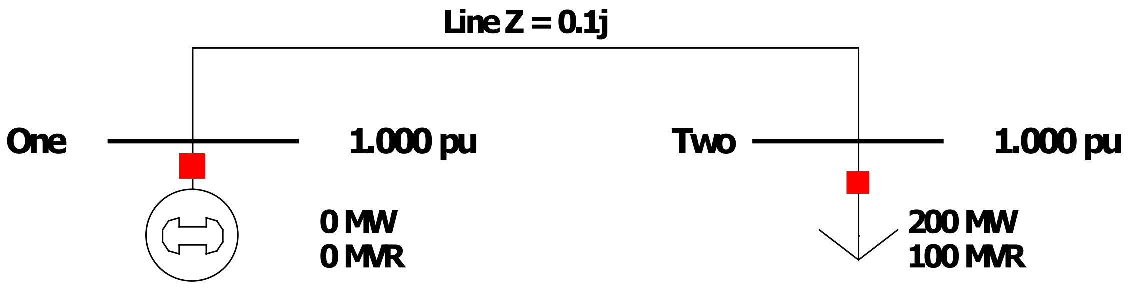
$$\mathrm{f_i}(x) = \sum_{k=1}^{n} |V_i||V_k| \left(g_{ik}\cos\theta_{ik} + b_{ik}\sin\theta_{ik}\right) - P_{Gi} + P_{Di}$$

$$\frac{\partial \mathrm{f_i}(x)}{\partial \theta_i} = \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k| \left(-g_{ik}\sin\theta_{ik} + b_{ik}\cos\theta_{ik}\right)$$

$$\frac{\partial \mathrm{f_i}(x)}{\partial \theta_j} = |V_i|\left|V_j\right|\left(g_{ij}\sin\theta_{ij} - b_{ij}\cos\theta_{ij}\right)\ (j \neq i)$$

Power Flow Example #1

- For the two bus power system shown below, use the Newton-Raphson power flow to determine the voltage magnitude and angle at bus two. Assume that bus one is the slack and SBase = 100 MVA.



$$\mathrm{P_i} = \sum_{k=1}^{n} |V_i||V_k| \left(g_{ik} \cos\theta_{ik} + b_{ik} \sin\theta_{ik}\right) = P_{Gi} - P_{Di}$$

$$\mathrm{Q_i} = \sum_{k=1}^{n} |V_i||V_k| \left(g_{ik} \sin\theta_{ik} - b_{ik} \cos\theta_{ik}\right) = Q_{Gi} - Q_{Di}$$

Power Flow Example #1

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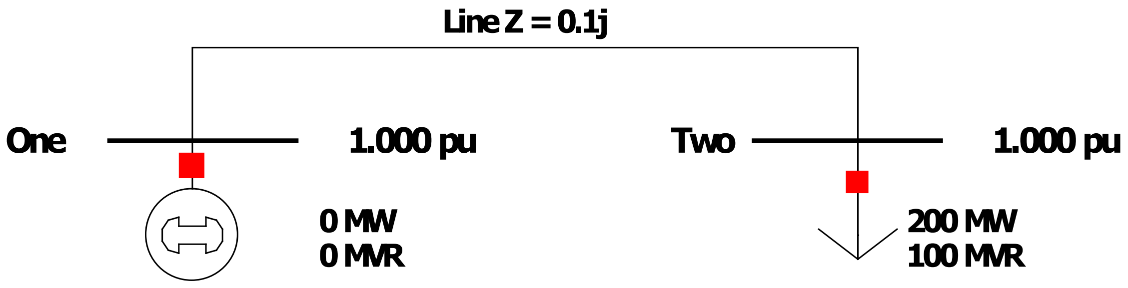
Power Flow Example #1, Solution

- For the two bus power system shown below, use the Newton-Raphson power flow to determine the voltage magnitude and angle at bus two. Assume that bus one is the slack and SBase = 100 MVA.



$$\mathbf{x} = \begin{bmatrix} \theta_2 \\ |V_2| \end{bmatrix} \quad \mathbf{Y}_{bus} = \begin{bmatrix} -j10 & j10 \\ j10 & -j10 \end{bmatrix}$$

General power balance equations

$$\mathrm{P_i} = \sum_{k=1}^{n} |V_i||V_k| \left(G_{ik} \cos\theta_{ik} + B_{ik} \sin\theta_{ik}\right) = P_{Gi} - P_{Di}$$

$$\mathrm{Q_i} = \sum_{k=1}^{n} |V_i||V_k| \left(G_{ik} \sin\theta_{ik} - B_{ik} \cos\theta_{ik}\right) = Q_{Gi} - Q_{Di}$$

Bus two power balance equations

$$|V_2||V_1|(10 \sin\theta_2) + 2.0 = 0$$

$$|V_2||V_1|(-10 \cos\theta_2) + |V_2|^2(10) + 1.0 = 0$$

Now calculate the power flow Jacobian

$$J(\mathbf{x}) = \begin{bmatrix} \dfrac{\partial \mathrm{P}_2(\mathbf{x})}{\partial \theta_2} & \dfrac{\partial \mathrm{P}_2(\mathbf{x})}{\partial |V|_2} \\ \dfrac{\partial \mathrm{Q}_2(\mathbf{x})}{\partial \theta_2} & \dfrac{\partial \mathrm{Q}_2(\mathbf{x})}{\partial |V|_2} \end{bmatrix}$$

$$= \begin{bmatrix} 10|V_2| \cos\theta_2 & 10 \sin\theta_2 \\ 10|V_2| \sin\theta_2 & -10 \cos\theta_2 + 20|V_2| \end{bmatrix}$$

Power Flow Example #1, Solution Cont.

- Set $v = 0$, guess $\mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

  Calculate

  $$f(\mathbf{x}^{(0)}) = \begin{bmatrix} |V_2|(10\sin\theta_2) + 2.0 \\ |V_2|(-10\cos\theta_2) + |V_2|^2(10) + 1.0 \end{bmatrix} = \begin{bmatrix} 2.0 \\ 1.0 \end{bmatrix}$$

  $$\mathbf{J}(\mathbf{x}^{(0)}) = \begin{bmatrix} 10|V_2|\cos\theta_2 & 10\sin\theta_2 \\ 10|V_2|\sin\theta_2 & -10\cos\theta_2 + 20|V_2| \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

  Solve $\mathbf{x}^{(1)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}^{-1} \begin{bmatrix} 2.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0.9 \end{bmatrix}$

- $f(\mathbf{x}^{(1)}) = \begin{bmatrix} 0.9(10\sin(-0.2)) + 2.0 \\ 0.9(-10\cos(-0.2)) + 0.9^2 \times 10 + 1.0 \end{bmatrix} = \begin{bmatrix} 0.212 \\ 0.279 \end{bmatrix}$

  $$\mathbf{J}(\mathbf{x}^{(1)}) = \begin{bmatrix} 8.82 & -1.986 \\ -1.788 & 8.199 \end{bmatrix}$$

  $$\mathbf{x}^{(2)} = \begin{bmatrix} -0.2 \\ 0.9 \end{bmatrix} - \begin{bmatrix} 8.82 & -1.986 \\ -1.788 & 8.199 \end{bmatrix}^{-1} \begin{bmatrix} 0.212 \\ 0.279 \end{bmatrix} = \begin{bmatrix} -0.233 \\ 0.8586 \end{bmatrix}$$

  $f(\mathbf{x}^{(2)}) = \begin{bmatrix} 0.0145 \\ 0.0190 \end{bmatrix}$ $\mathbf{x}^{(3)} = \begin{bmatrix} -0.236 \\ 0.8554 \end{bmatrix}$

  $f(\mathbf{x}^{(3)}) = \begin{bmatrix} 0.0000906 \\ 0.0001175 \end{bmatrix}$ Done! $V_2 = 0.8554\angle -13.52°$