ECEN 460, Spring 2026
Power System Operation and Control

Class 11: Power Flow Network Equations

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Announcements

- Homework #5: book problems 6.30 and 6.31, due Thursday, Feb. 26, 2026
- Homework #6: book problems 6.9, 6.12, 6.18, 6.25, 6.38, due Thursday, Mar 5, 2026

Recall, Transmission Line and Transformer Models

- Transmission lines use the "pi" model

These devices reduce down to impedances

- Transformer non-ideal model is simplified with the per-unit system (and neglecting shunt elements)

Let's Start to Think About a Larger System

- The most common power system analysis tool is the **power flow** – sometimes also called load flow
- Power flow assumes system is in steady state
- The goal is to solve the system state
  - Bus complex voltages $V_i \angle \theta_i$ everywhere
  - From these values we can get other values such as current and power flows

Larger System – Network Equations

Larger System – Network Equations, Cont.

- KCL at bus 1

$$\bar{I}_1 = \bar{I}_{G1} - \bar{I}_{D1}$$
$$= \bar{I}_{12} + \bar{I}_{13}$$
$$= \frac{\bar{V}_1 - \bar{V}_2}{\bar{Z}_A} + \frac{\bar{V}_1 - \bar{V}_3}{\bar{Z}_B} \qquad (\bar{Y}_j = 1/\bar{Z}_j)$$
$$= (\bar{V}_1 - \bar{V}_2) \cdot \bar{Y}_A + (\bar{V}_1 - \bar{V}_3) \cdot \bar{Y}_B$$
$$= (\bar{Y}_A + \bar{Y}_B) \cdot \bar{V}_1 - \bar{Y}_A \cdot \bar{V}_2 - \bar{Y}_B \cdot \bar{V}_3$$

Similarly---

$$\bar{I}_2 = \bar{I}_{21} + \bar{I}_{23} + \bar{I}_{24}$$
$$= -\bar{Y}_A \bar{V}_1 + (\bar{Y}_A + \bar{Y}_C + \bar{Y}_D)\bar{V}_2 - \bar{Y}_C \bar{V}_3 - \bar{Y}_D \bar{V}_4$$

Larger System – Network Equations as a Matrix

Larger System – Network Equations as a Matrix 2

- Put all the KCL equations together (using $\bar{Y}$) and we can make a matrix

$$\mathbf{I} = \mathbf{Y_{bus}}\mathbf{V}$$

$$\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \\ \bar{I}_4 \end{bmatrix} = \begin{bmatrix} \bar{Y}_A + \bar{Y}_B & -\bar{Y}_A & -\bar{Y}_B & 0 \\ -\bar{Y}_A & \bar{Y}_A + \bar{Y}_C + \bar{Y}_D & -\bar{Y}_C & -\bar{Y}_D \\ -\bar{Y}_B & \bar{Y}_C & \bar{Y}_B + \bar{Y}_C & 0 \\ 0 & -\bar{Y}_D & 0 & \bar{Y}_D \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \bar{V}_3 \\ \bar{V}_4 \end{bmatrix}$$

This is the y-bus matrix

Bus Network Admittance Matrix

- Finding the bus network admittance matrix (y-bus) is the first step of the power flow
- It shows the relationship between all the bus current injections **I** (from generation and load) and the bus voltages **V**

$$\mathbf{I} = \mathbf{Y_{bus}}\mathbf{V}$$

$$\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \\ \bar{I}_4 \end{bmatrix} = \begin{bmatrix} \bar{Y}_A + \bar{Y}_B & -\bar{Y}_A & -\bar{Y}_B & 0 \\ -\bar{Y}_A & \bar{Y}_A + \bar{Y}_C + \bar{Y}_D & -\bar{Y}_C & -\bar{Y}_D \\ -\bar{Y}_B & \bar{Y}_C & \bar{Y}_B + \bar{Y}_C & 0 \\ 0 & -\bar{Y}_D & 0 & \bar{Y}_D \end{bmatrix} = \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \bar{V}_3 \\ \bar{V}_4 \end{bmatrix}$$

Building the Y-Bus Matrix

- Here are the steps for building the Y-bus matrix
  - Start with an NxN matrix of all zeros (N is the number of buses)
  - For any shunt elements, add $\bar{Y}$ to the corresponding diagonal elements of the Y-bus
  - For branches between two buses,
    - add $\bar{Y}$ to both diagonal elements of the Y-bus
    - Subtract $\bar{Y}$ from both off-diagonal elements of the Y-bus
- Example: if a transmission line from bus 12 to bus 17 has $\bar{Z} = 0.05 + j0.4$ (per-unit) and $\overline{\mathrm{B}}_{shunt}/2 = 2.1$ then
  - Add the shunt $\bar{Y}_{shunt} = j2.1$ to matrix positions (12, 12) and (17, 17)
  - Add the series $\bar{Y} = \frac{1}{0.05+j0.4} = 0.308 - j2.46$ to matrix positions (12, 12) and (17, 17)
  - Subtract the series $\bar{Y} = 0.308 - j2.46$ to matrix positions (12, 17) and (17, 12)
- We add one network element at a time to the Y-bus matrix!

Two-Bus Example

- Here are the steps for building the Y-bus matrix
  - Start with an NxN matrix of all zeros (N is the number of buses)
  - For any shunt elements, add $\bar{Y}$ to the corresponding diagonal elements
  - For branches,
    - add $\bar{Y}$ to both diagonal elements
    - Subtract $\bar{Y}$ from both off-diagonal elements
- Try it on a two-bus system:

Two-Bus Example 2

- Here are the steps for building the Y-bus matrix
  - Start with an NxN matrix of all zeros (N is the number of buses)
  - For any shunt elements, add $\bar{Y}$ to the corresponding diagonal elements
  - For branches,
    - add $\bar{Y}$ to both diagonal elements
    - Subtract $\bar{Y}$ from both off-diagonal elements
- Try it on a two-bus system:

$$\mathbf{I} = \mathbf{Y_{bus}} \cdot \mathbf{V}$$

$$\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \end{bmatrix} = \mathbf{Y_{bus\ (2\times2)}} \cdot \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \end{bmatrix}$$

$$\mathbf{Y_{bus}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} j0.1 & 0 \\ 0 & j0.1 \end{bmatrix} \rightarrow \begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix}$$

Two-Bus Example, Continued

- For the previous circuit, assume the voltages are $\bar{V}_1 = 1.0\angle 0°$ and $\bar{V}_2 = 0.82\angle -14°$
- What are the power injections of the source and load?

Two-Bus Example, Continued 2

- For the previous circuit, assume the voltages are $\bar{V}_1 = 1.0\angle 0°$ and $\bar{V}_2 = 0.82\angle - 14°$
- What are the power injections of the source and load?

$$\begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.8 - j0.2 \end{bmatrix} = \begin{bmatrix} 5.60 - j0.70 \\ -5.58 + j0.88 \end{bmatrix}$$

Therefore the power injected at bus 1 is

$$S_1 = V_1 I_1^* = 1.0 \times (5.60 + j0.70) = 5.60 + j0.70$$

$$S_2 = V_2 I_2^* = (0.8 - j0.2) \times (-5.58 - j0.88) = -4.64 + j0.41$$

Two-Bus Example, Continued 3

- For the previous circuit, assume the current injections are $\bar{I}_1 = 5.0\angle 0°$ and $\bar{I}_2 = 4.8\angle 180°$
- What are the voltages and the power injections of the source and load?

Two-Bus Example, Continued 4

- For the previous circuit, assume the current injections are $\bar{I}_1 = 5.0\angle 0°$ and $\bar{I}_2 = 4.8\angle 180°$
- What are the voltages and the power injections of the source and load?

$$\begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix}^{-1} \begin{bmatrix} 5.0 \\ -4.8 \end{bmatrix} = \begin{bmatrix} 0.0738 - j0.902 \\ -0.0738 - j1.098 \end{bmatrix}$$

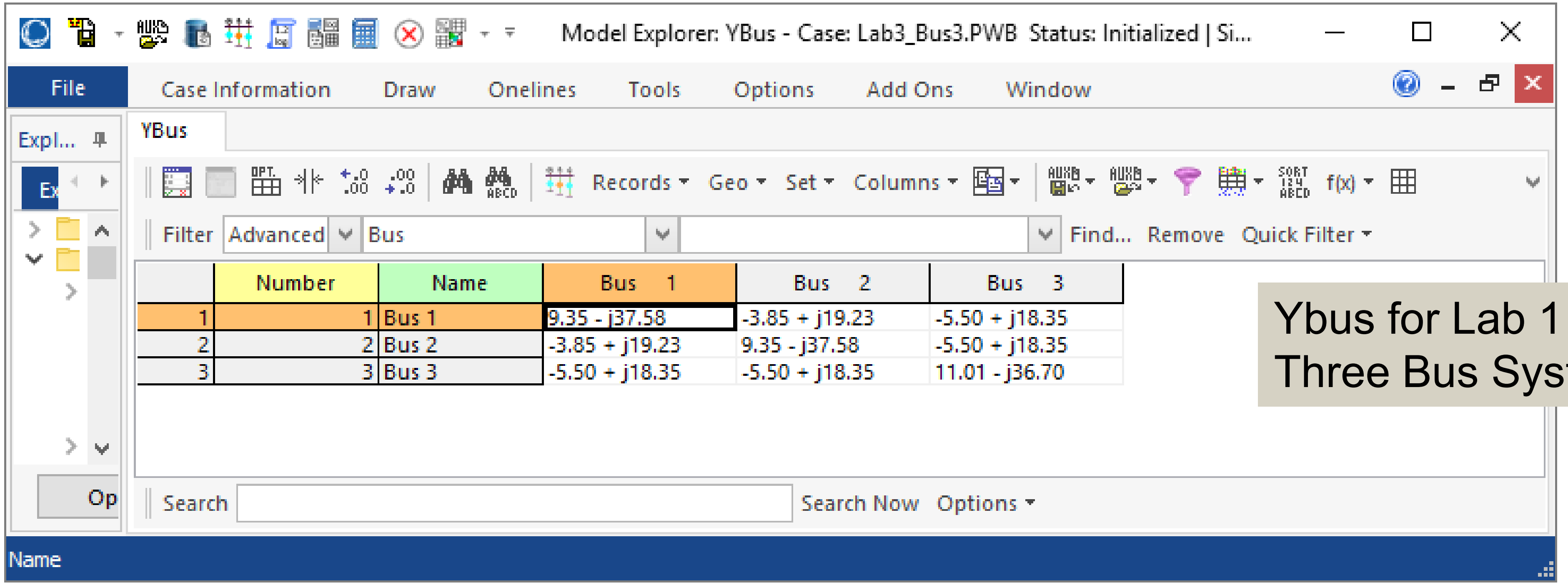
Therefore the power injected is

$$S_1 = V_1 I_1^* = (0.0738 - j0.902) \times 5 = 0.37 - j4.51$$

$$S_2 = V_2 I_2^* = (-0.0738 - j1.098) \times (-4.8) = 0.35 + j5.27$$

Y-bus in PowerWorld

- To see the Y_{bus} in PowerWorld, select Case Information, Solution Details, Ybus
- For large systems most of the Y_{bus} elements are zero, giving what is known as a sparse matrix



	Number	Name	Bus 1	Bus 2	Bus 3
1	1	Bus 1	9.35 - j37.58	-3.85 + j19.23	-5.50 + j18.35
2	2	Bus 2	-3.85 + j19.23	9.35 - j37.58	-5.50 + j18.35
3	3	Bus 3	-5.50 + j18.35	-5.50 + j18.35	11.01 - j36.70

Ybus for Lab 1
Three Bus System

Modeling Generators and Loads for Power Flow

Generators

- Engineering models depend upon application
- Generators are usually synchronous machines
- For generators we will use two different models:
  - a short term model treating the generator as a constant voltage source behind a possibly time-varying reactance (used in Lab and earlier in class)
  - a steady-state model, treating the generator as a **constant power source operating at a fixed voltage**; this model will be used for power flow and economic analysis

Loads

- Ultimate goal is to supply loads with electricity at constant frequency and voltage
- Electrical characteristics of individual loads matter, but usually they can only be estimated
  - Actual loads are constantly changing, consisting of a large number of individual devices
  - Only limited network observability of load characteristics
- Aggregate models are typically used for analysis
- Two common models
  - **Constant power**: Si = Pi + jQi
  - Constant impedance: Si = |V|2 / Zi

The Power Flow Problem

- What we know:
  - Generator real power
  - Generator voltage magnitude
  - Load real and reactive power
- What we don't know:
  - Bus voltage angles
  - Non-generator bus voltage magnitude
  - Generator reactive power
  - Current injections of generators or loads
- Note that the problem is formulated with *power* values, not current, so we cannot use the Y-bus equations directly

Linear vs. Non-Linear Systems

A function **H** is linear if

$$\mathbf{H}(a_1\mathbf{m}_1 + a_2\mathbf{m}_2) = a_1\mathbf{H}(\mathbf{m}_1) + a_2\mathbf{H}(\mathbf{m}_2)$$

That is

1) the output is proportional to the input
2) the principle of superposition holds

Linear Example: $\mathbf{y} = \mathbf{H}(\mathbf{x}) = c\,\mathbf{x}$

$$\mathbf{y} = c(\mathbf{x}_1+\mathbf{x}_2) = c\mathbf{x}_1 + c\,\mathbf{x}_2$$

Nonlinear Example: $\mathbf{y} = \mathbf{H}(\mathbf{x}) = c\,\mathbf{x}^2$

$$\mathbf{y} = c(\mathbf{x}_1+\mathbf{x}_2)^2 \neq (c\mathbf{x}_1)^2 + (c\,\mathbf{x}_2)^2$$

Power Flow Problem is Non-Linear

- The Y-Bus equations alone are very linear—can solve with superposition

$$\bar{I}_1 = (\bar{Y}_A + \bar{Y}_B) \cdot \bar{V}_1 - \bar{Y}_A \cdot \bar{V}_2 - \bar{Y}_B \cdot \bar{V}_3$$

- Because of the way we model generators and loads with *constant power* characteristics, the problem becomes non-linear

$$\bar{S}_1 = \bar{V}_1 \bar{I}_1^* = \bar{V}_1 \cdot \left((\bar{Y}_A + \bar{Y}_B) \cdot \bar{V}_1 - \bar{Y}_A \cdot \bar{V}_2 - \bar{Y}_B \cdot \bar{V}_3\right)^*$$

- Non-linear problems can be quite difficult to solve, and usually require an iterative approach.

Something to Think About

- How do we solve this DC circuit for different values of P?

P	V_L	
0 W		
1 W		
2 W		
4 W		
5 W		
6 W		
10 W		

Possibly 0, 1, or Multiple Solutions to Non-Linear Problems

- How do we solve this DC circuit for different values of P?

P	Load voltage and current	Alternative, low-voltage solution
0 W	10 V, 0 A	0 V, 2 A
1 W	9.47 V, 0.11 A	0.53 V, 1.89 A
2 W	8.87 V, 0.23 A	1.13 V, 1.77 A
4 W	7.24 V, 0.55 A	2.76 V, 1.45 A
5 W	5 V, 1 A	
6 W	No Solution	
10 W	No Solution	