

- A **phasor** is a complex number that represents a cosine-valued AC function
- The Root Mean Square (RMS) for cosine is found by dividing the maximum value by $\sqrt{2}$
- In polar form, $R\angle\theta$, a phasor represents the RMS voltage or current and phase angle
 - $R\angle\theta \rightarrow \sqrt{2} R \cos(2\pi ft + \theta)$
- Conversions to rectangular form: $a+jb$, and back can be done with these identities:
 - $R = \sqrt{a^2 + b^2} \quad \theta = \text{atan}\left(\frac{b}{a}\right) \quad a = R \cos \theta \quad b = R \sin \theta$
- Complex number addition can be done in rectangular form, and complex number multiplication can be done in polar form.
- Phasor diagrams have the real part on the x axis and imaginary part on the y axis.
- The angular frequency is $\omega = 2\pi f$.
- KVL, KCL, and Ohm's law all apply with AC phasor analysis exactly as with DC.

- The effect of resistors, inductors, and capacitors upon phasors is handled with **impedance (Z)**, which acts like complex resistance. $V = I \cdot Z$
- The impedance of inductors and capacitors depends on frequency
- Instantaneous power from the time signal $p(t) = v(t)i(t)$

Element	Time Domain	Phasor Domain	Z (impedance)
Resistor	$v(t) = Ri(t)$	$V = IR$	R
Inductor	$v(t) = L \frac{di(t)}{dt}$	$V = j\omega LI$	$j\omega L$
Capacitor	$i(t) = C \frac{dv(t)}{dt}$	$V = \frac{1}{j\omega C} I$	$\frac{1}{j\omega C}$

- **Complex power:** $S = VI^* = |S|\angle\theta_s = P + jQ$ Don't forget the conjugate!! (*)
- Average power or active power or real power: $\text{Re}[S] = P = |S| \cos \theta_s$
 - This is what's normally thought of as "power". Units are W.
 - It's also what you get if you take the average value of the instantaneous power
- Reactive power: $\text{Im}[S] = Q = |S| \sin \theta_s$ Units are "var".
- Apparent power: $|S| = |V| \cdot |I|$ Units are "VA".
- Power factor angle: θ_s is the angle of S or $\theta_v - \theta_i$
- **Power factor:** $\cos(\theta_s) = P/|S|$. It must be indicated as "leading" (negative θ_s) or "lagging" (positive θ_s). A unity (1) power factor indicates zero reactive power and is neither leading nor lagging.
- **At every node in the system, both active (real) and reactive power are conserved**
- Inductors only absorb reactive power, capacitors only produce reactive power
- Capacitor banks are used for power factor correction by supplying reactive power locally
- A three-phase system is **balanced** if (1) all voltages are equal in magnitude and shifted in phase by 120° , (2) loads are equal on each phase, (3) impedances are equal on each phase.
- Line-to-line voltages are related to line-to-neutral (phase) voltages as: $V_{phase} = \frac{V_{line}}{\sqrt{3}\angle 30^\circ}$
- Delta-connected loads can be replaced with wye-connected loads: $Z_Y = \frac{1}{3}Z_\Delta$
- Delta-connected sources can be replaced by wye-connected sources $V_{phase} = \frac{V_{line}}{\sqrt{3}\angle 30^\circ}$
- Per-phase analysis works for balanced systems if there is no mutual inductance between phases. Steps: (1) convert delta sources and loads to equivalent wye (2) solve phase "a" circuit independent of other phases (3) total system power is $3V_a I_a^*$ (4) if needed, go back to original circuit to find "b" or "c" values and internal Δ values.

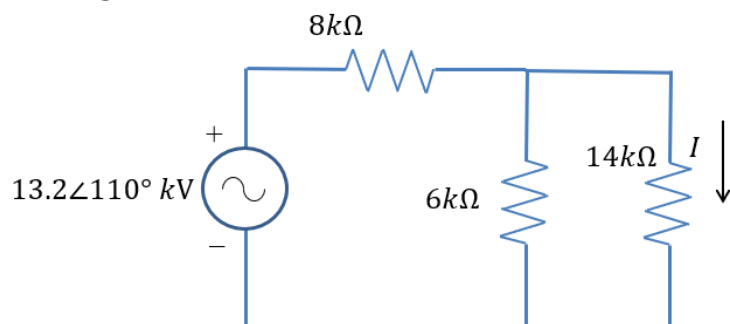
Homework #1 due Tuesday, Jan 20th

For more help, read Chapters 1 and 2 in the textbook, view the slides on the website, and take advantage of office hours of TAs and the instructor.

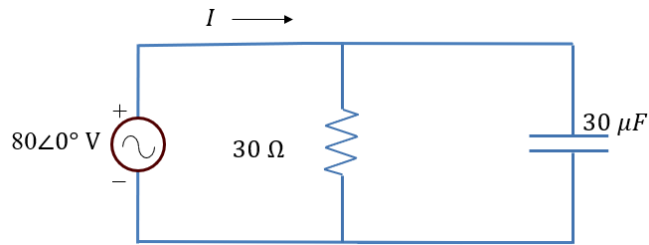
1. Practice problems for phasor conversion

- Convert $5\angle 12^\circ$ A to rectangular form.
- Convert $14\angle 20^\circ$ V to the cosine time function, assuming a frequency of 14 kHz.
- Find the polar form phasor for $20 \cos(377t - 40^\circ)$ kV.
- Convert the phasor $12-j3$ A to polar form.
- Sketch a time plot of the phasor $18\angle 12^\circ$ mA, assuming a frequency of 100 MHz.
- Draw a phasor diagram for the phasor $35\angle -110^\circ$ V.
- Convert $24\angle -60^\circ$ A to rectangular form.
- Convert $30\angle 0^\circ$ V to the cosine time function, assuming a frequency of 50 Hz.
- Find the rectangular form phasor for $20 \cos((6.28 \times 10^6)t + 18^\circ)$ kV.
- Convert the phasor $30+j30$ kA to polar form.
- Sketch a time plot of the phasor $1.32\angle 10^\circ$ MV, assuming a frequency of 60 Hz.
- Draw a phasor diagram for the two phasors $3.5\angle 10^\circ$ A and $2.7\angle 40^\circ$ A.
- Convert $16\angle -90^\circ$ A to rectangular form.
- Convert $100.5\angle 0^\circ$ V to the cosine time function, assuming a frequency of 400 Hz.
- Find the polar form phasor for $200 \sin(377t)$ kV.
- Convert the phasor $j5$ V to polar form.
- Sketch a time plot of the phasor $300\angle -90^\circ$ V, assuming a frequency of 10 Hz.
- Draw a phasor diagram for the phasor $3.25\angle 0^\circ$ V.
- Convert $2.0\angle 90^\circ$ MV to rectangular form.
- Convert $74.5\angle 14^\circ$ V to the cosine time function, assuming a frequency of 2500 Hz.
- Find the rectangular form phasor for $55 \cos(10^9t - 108^\circ)$ V. What is the frequency?
- Convert the phasor $10-j30000$ kA to polar form.
- Sketch a time plot of the phasor $100\angle 90^\circ$ A, assuming a frequency of 6000 Hz.
- Draw a phasor diagram for the two phasors $90\angle 90^\circ$ A and $90\angle -90^\circ$ A.

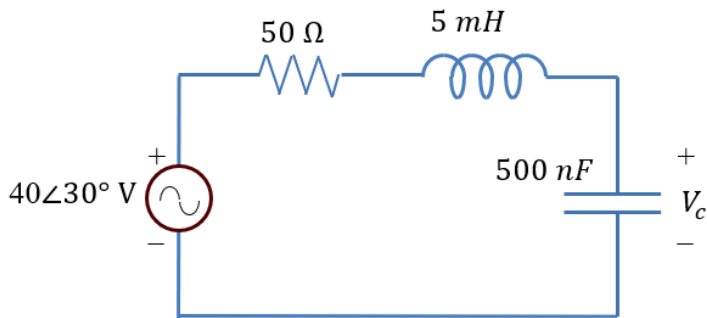
2. Solve for I as a phasor using any method. Assuming the frequency is 60 Hz, write the time signals for V and I .



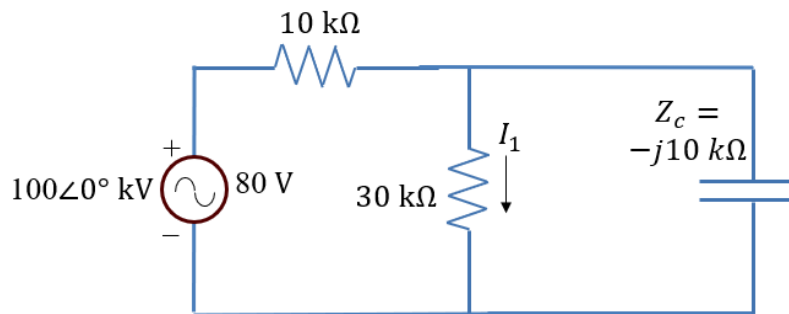
3. Find the current phasor I for this 1 kHz circuit



4. Solve for the phasor V_c for this circuit if it is operated at 400 Hz

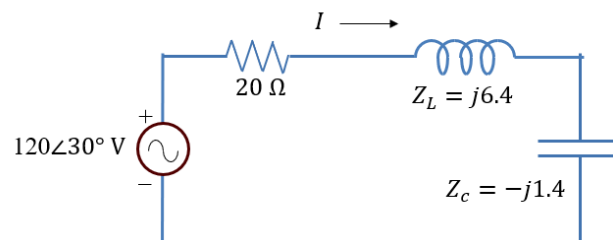


5. Solve for the phasor I_1 . The impedance for Z_c is given so you don't need the frequency.



6. Calculate for the voltage source

1. Current I
2. Complex power S
3. Active power P
4. Reactive power Q
5. Apparent power $|S|$
6. Power factor angle θ_s
7. Power factor



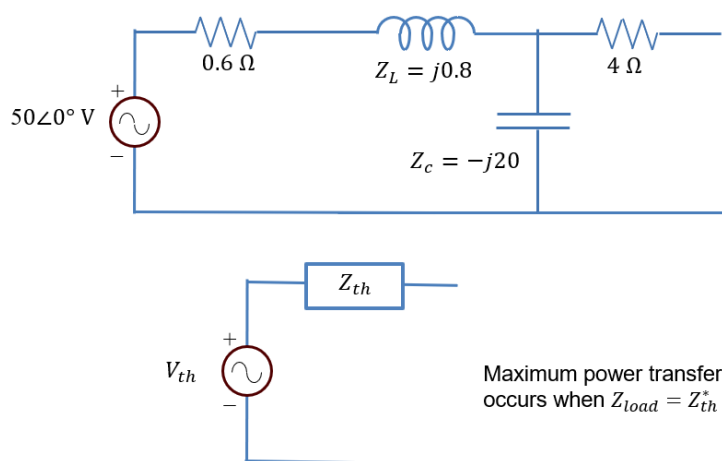
7. A factory is acting like a $250\ \Omega$ resistor in parallel with a 500 mH inductor. A single-phase power distribution line supplying the factory from the substation can be modeled as a 65 mH inductor. At the substation, the voltage is $12\angle 6.5^\circ$ kV.

- Draw this circuit and find the impedance of the circuit elements. The substation can be modeled as a voltage source. The frequency is 60 Hz.
- What is the voltage at the factory?
- How much active and reactive power is the factory absorbing?
- What is the factory's power factor?
- If a $50\ \mu\text{F}$ capacitor is installed in parallel with the factory, how do the voltage and power factor change?
- What should the value of the capacitor be so that the power factor is 0.95 lagging?

8.

We want to find

- The Thevenin equivalent for this circuit, as shown.
- The maximum power that could be delivered to a load with impedance Z_{load}



Homework #2 due Tuesday, Jan 27th

Textbook problems: 2.9, 2.22, 2.28, 2.43, 2.48