

- A **phasor** is a complex number that represents a cosine-valued AC function
- The Root Mean Square (RMS) for cosine is found by dividing the maximum value by  $\sqrt{2}$
- In polar form,  $R\angle\theta$ , a phasor represents the RMS voltage or current and phase angle
  - $R\angle\theta \rightarrow \sqrt{2} R \cos(2\pi ft + \theta)$
- Conversions to rectangular form:  $a+jb$ , and back can be done with these identities:
  - $R = \sqrt{a^2 + b^2}$     $\theta = a \tan\left(\frac{b}{a}\right)$     $a = R \cos \theta$     $b = R \sin \theta$
- Complex number addition can be done in rectangular form, and complex number multiplication can be done in polar form.
- Phasor diagrams have the real part on the x axis and imaginary part on the y axis.
- The angular frequency is  $\omega = 2\pi f$ .
- KVL, KCL, and Ohm's law all apply with AC phasor analysis exactly as with DC.
- The effect of resistors, inductors, and capacitors upon phasors is handled with **impedance (Z)**, which acts like complex resistance.  $V = I \cdot Z$
- The impedance of inductors and capacitors depends on frequency
- Instantaneous power from the time signal  $p(t) = v(t)i(t)$
- **Complex power:**  $S = VI^* = |S|\angle\theta_s = P + jQ$       Don't forget the conjugate!! (\*)
- Average power or active power or real power:  $\text{Re}[S] = P = |S| \cos \theta_s$ 
  - This is what's normally thought of as "power". Units are W.
  - It's also what you get if you take the average value of the instantaneous power
- Reactive power:  $\text{Im}[S] = Q = |S| \sin \theta_s$       Units are "var".
- Apparent power:  $|S| = |V| \cdot |I|$       Units are "VA".
- Power factor angle:  $\theta_s$  is the angle of  $S$  or  $\theta_v - \theta_i$
- **Power factor:**  $\cos(\theta_s) = P/|S|$ . It must be indicated as "leading" (negative  $\theta_s$ ) or "lagging" (positive  $\theta_s$ ). A unity (1) power factor indicates zero reactive power and is neither leading nor lagging.
- **At every node in the system, both active (real) and reactive power are conserved**
- Inductors only absorb reactive power, capacitors only produce reactive power
- Capacitor banks are used for power factor correction by supplying reactive power locally
- A three-phase system is **balanced** if (1) all voltages are equal in magnitude and shifted in phase by  $120^\circ$ , (2) loads are equal on each phase, (3) impedances are equal on each phase.
- Line-to-line voltages are related to line-to-neutral (phase) voltages as:  $V_{phase} = \frac{V_{line}}{\sqrt{3}\angle 30^\circ}$
- Delta-connected loads can be replaced with wye-connected loads:  $Z_Y = \frac{1}{3}Z_\Delta$
- Delta-connected sources can be replaced by wye-connected sources  $V_{phase} = \frac{V_{line}}{\sqrt{3}\angle 30^\circ}$
- Per-phase analysis works for balanced systems if there is no mutual inductance between phases. Steps: (1) convert delta sources and loads to equivalent wye (2) solve phase "a" circuit independent of other phases (3) total system power is  $3V_a I_a^*$  (4) if needed, go back to original circuit to find "b" or "c" values and internal  $\Delta$  values.

**Homework #1 due Tuesday, Jan 20th**

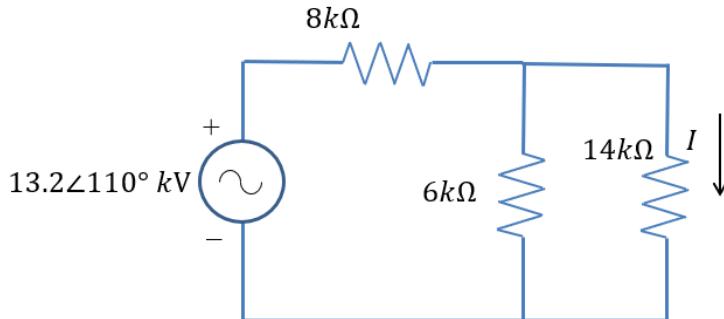
For more help, read Chapters 1 and 2 in the textbook, view the slides on the website, and take advantage of office hours of TAs and the instructor.

1. Practice problems for phasor conversion

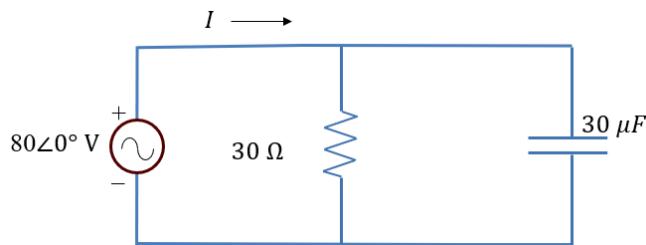
- a. Convert  $5\angle 12^\circ$  A to rectangular form.
- b. Convert  $14\angle 20^\circ$  V to the cosine time function, assuming a frequency of 14 kHz.
- c. Find the polar form phasor for  $20 \cos(377t - 40^\circ)$  kV.
- d. Convert the phasor  $12-j3$  A to polar form.
- e. Sketch a time plot of the phasor  $18\angle 12^\circ$  mA, assuming a frequency of 100 MHz.
- f. Draw a phasor diagram for the phasor  $35\angle -110^\circ$  V.
- g. Convert  $24\angle -60^\circ$  A to rectangular form.
- h. Convert  $30\angle 0^\circ$  V to the cosine time function, assuming a frequency of 50 Hz.
- i. Find the rectangular form phasor for  $20 \cos((6.28 \times 10^6)t + 18^\circ)$  kV.
- j. Convert the phasor  $30+j30$  kA to polar form.
- k. Sketch a time plot of the phasor  $1.32\angle 10^\circ$  MV, assuming a frequency of 60 Hz.
- l. Draw a phasor diagram for the two phasors  $3.5\angle 10^\circ$  A and  $2.7\angle 40^\circ$  A.
- m. Convert  $16\angle -90^\circ$  A to rectangular form.
- n. Convert  $100.5\angle 0^\circ$  V to the cosine time function, assuming a frequency of 400 Hz.
- o. Find the polar form phasor for  $200 \sin(377t)$  kV.
- p. Convert the phasor  $j5$  V to polar form.
- q. Sketch a time plot of the phasor  $300\angle -90^\circ$  V, assuming a frequency of 10 Hz.
- r. Draw a phasor diagram for the phasor  $3.25\angle 0^\circ$  V.
- s. Convert  $2.0\angle 90^\circ$  MV to rectangular form.
- t. Convert  $74.5\angle 14^\circ$  V to the cosine time function, assuming a frequency of 2500 Hz.
- u. Find the rectangular form phasor for  $55 \cos(10^9 t - 108^\circ)$  V. What is the frequency?
- v. Convert the phasor  $10-j30000$  kA to polar form.
- w. Sketch a time plot of the phasor  $100\angle 90^\circ$  A, assuming a frequency of 6000 Hz.
- x. Draw a phasor diagram for the two phasors  $90\angle 90^\circ$  A and  $90\angle -90^\circ$  A.

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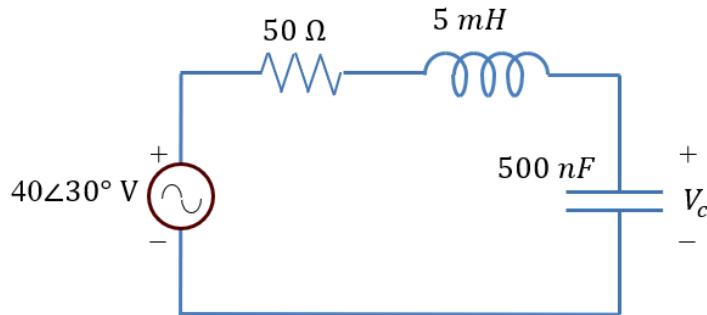
2. Solve for  $I$  as a phasor using any method. Assuming the frequency is 60 Hz, write the time signals for  $V$  and  $I$ .



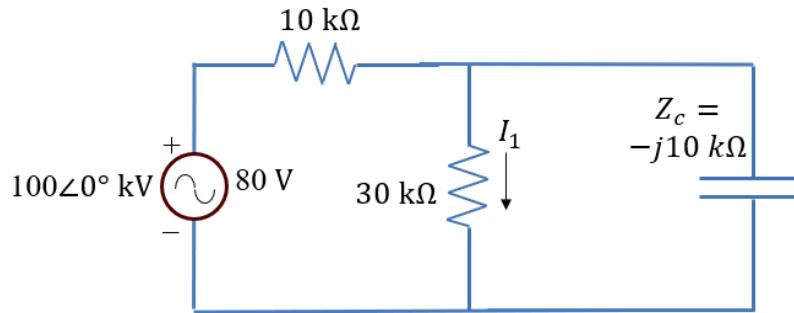
3. Find the current phasor  $I$  for this 1 kHz circuit



4. Solve for the phasor  $V_c$  for this circuit if it is operated at 400 Hz

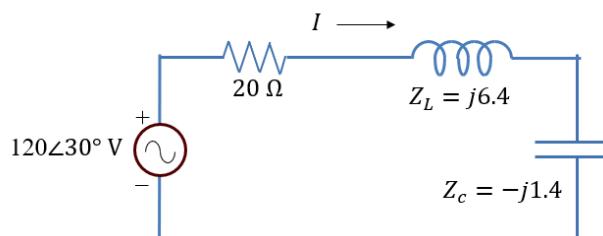


5. Solve for the phasor  $I_1$ . The impedance for  $Z_c$  is given so you don't need the frequency.



6.  
 Calculate for the voltage source

1. Current I
2. Complex power S
3. Active power P
4. Reactive power Q
5. Apparent power  $|S|$
6. Power factor angle  $\theta_s$
7. Power factor



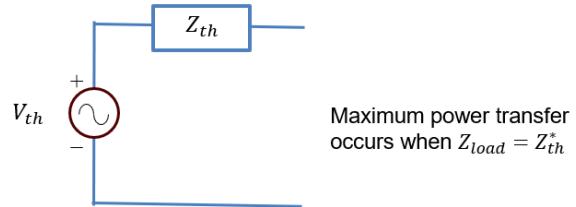
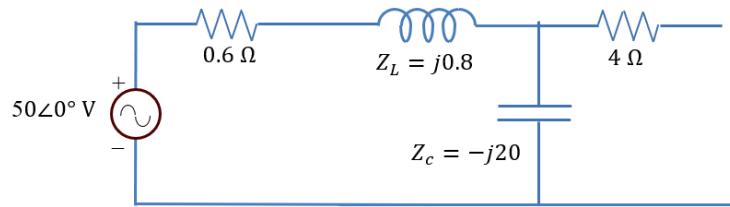
7. A factory is acting like a  $250 \Omega$  resistor in parallel with a  $500 \text{ mH}$  inductor. A single-phase power distribution line supplying the factory from the substation can be modeled as a  $65 \text{ mH}$  inductor. At the substation, the voltage is  $12\angle 6.5^\circ \text{ kV}$ .

- Draw this circuit and find the impedance of the circuit elements. The substation can be modeled as a voltage source. The frequency is 60 Hz.
- What is the voltage at the factory?
- How much active and reactive power is the factory absorbing?
- What is the factory's power factor?
- If a  $50 \mu\text{F}$  capacitor is installed in parallel with the factory, how do the voltage and power factor change?
- What should the value of the capacitor be so that the power factor is 0.95 lagging?

8.

We want to find

- The Thevenin equivalent for this circuit, as shown.
- The maximum power that could be delivered to a load with impedance  $Z_{load}$



**Homework #2 due Tuesday, Jan 27th**

Textbook problems: 2.9, 2.22, 2.28, 2.43, 2.48