ECEN 460, Spring 2026
Power System Operation and Control

Class 6: Transformers, Part 2

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Homework 3 and 4

- No homework on generators. Make sure you understand the lecture notes and labs 2 and 3 (starting this week).
- Homework 3 on transformers: book problems 3.4, 3.5, 3.23, due Feb. 3rd.
- Homework 4 on transmission lines: book regular problems 4.10, 4.11, 4.20, and 4.41, 5.14 (a,b), 5.38, and 5.41 (a,b), due Feb. 10th.

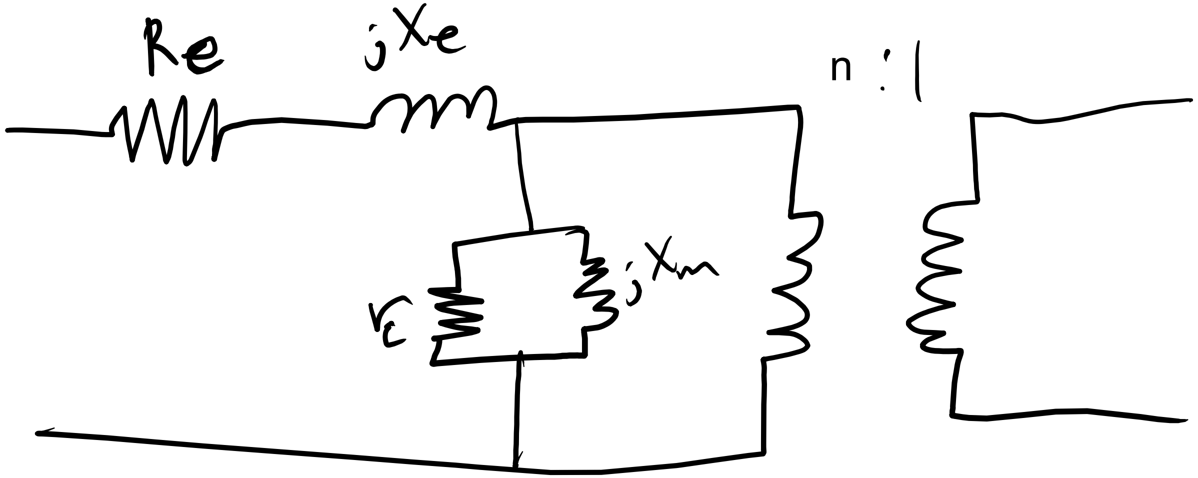
Non-Ideal Transformer Example

- Example: A single phase, 15 MVA, 35/13 kV transformer has the following test data:

  open circuit: 14 amps, with 3 kW losses

  short circuit: 6 kV, with 200 kW losses

- Determine the model parameters.



Transformer Example, Cont'd

From the short circuit test

$$I_{sc} = \frac{15MVA}{35kV} = 429\ A, |\mathrm{R_e} + jX_e| = \frac{6\ kV}{429\ A} = 14\ \Omega$$

$$\mathrm{P_{SC}} = R_e I_{sc}^2 = 500\ \mathrm{kW}\ \rightarrow \mathrm{R_e} = 1.09\ \Omega,$$

Hence $\mathrm{X_e} = \sqrt{14^2 - 1^2} = 14\ \Omega$

From the open circuit test

$$R_c = \frac{35\ kV^2}{3\ kW} = 0.408\ M\Omega$$

$$|\mathrm{R_e} + jX_e + jX_m| = \frac{35\ kV}{14\ A} = 2500\ \Omega \quad X_m = 2500\ \Omega$$

Per Unit Change of MVA Base

- Parameters for equipment are often given using power rating of equipment as the MVA base
- To analyze a system all per unit data must be on a common power base

$$Z_{pu}^{OriginalBase} \rightarrow Z_{actual} \rightarrow Z_{pu}^{NewBase}$$

$$\text{Hence } Z_{pu}^{OriginalBase} \times \frac{V_{base}^2}{S_{Base}^{OriginalBase}} / \frac{V_{base}^2}{S_{Base}^{NewBase}} = Z_{pu}^{NewBase}$$

$$Z_{pu}^{OriginalBase} \times \frac{S_{Base}^{NewBase}}{S_{Base}^{OriginalBase}} = Z_{pu}^{NewBase}$$

Per Unit Change of Base Example

- A 54 MVA transformer has a leakage reactance of 3.69%. What is the reactance on a 100 MVA base?

$$X_e = 0.0369 \times \frac{100}{54} = 0.0683 \text{ p.u.}$$

Transformer Reactance

- Transformer reactance is often specified as a percentage, say 10%. This is a per unit value (divide by 100) on the power base of the transformer.
- Example: A 350 MVA, 230/20 kV transformer has leakage reactance of 10%. What is p.u. value on 100 MVA base? What is value in ohms (230 kV)?

$$X_e = 0.10 \times \frac{100}{350} = 0.0286 \text{ p.u.}$$

$$0.0286 \times \frac{230^2}{100} = 15.1\ \Omega$$

Three Phase Per-Unit

- Procedure is very similar to 1 phase except we use a 3 phase VA base, and use line to line voltage bases
- Pick a 3ϕ VA base for the entire system $S_{\mathrm{B}}^{3\varphi}$
- Pick a voltage base for each different voltage level, V_B. Voltages are line to line.
- Calculate the impedance base

$$Z_B = \frac{V_{B,LL}^2}{S_B^{3\phi}} = \frac{(\sqrt{3}\ V_{B,LN})^2}{3S_B^{1\phi}} = \frac{V_{B,LN}^2}{S_B^{1\phi}}$$

Exactly the same impedance bases as with single phase!

Three Phase Per-Unit, Cont'd

- Calculate the current base, I_B

$$\mathrm{I}_{\mathrm{B}}^{3\phi} = \frac{S_B^{3\phi}}{\sqrt{3}\ V_{B,LL}} = \frac{3\ S_B^{1\phi}}{\sqrt{3}\ \sqrt{3}\ V_{B,LN}} = \frac{S_B^{1\phi}}{V_{B,LN}} = \mathrm{I}_{\mathrm{B}}^{1\phi}$$

Exactly the same current bases as with single phase!
But, be careful in using 3ph bases to calculate it (need a root 3)

- Convert actual values to per unit

Three Phase Per-Unit Example

- Solve for the current, load voltage and load power in the previous circuit, assuming a 3ϕ power base of 300 MVA, and line to line voltage bases of 13.8 kV,138 kV and 27.6 kV (square root of 3 larger than the 1f example voltages). Also assume the generator is Y-connected so its line to line voltage is 13.8 kV.

Convert to per unit as before. Note the system is exactly the same!

3ϕ Per-Unit Example, Cont'd

- $I = \frac{1.0\angle 0°}{3.91 + j2.327} = 0.22\angle - 30.8°$ p.u. (not amps)

$$V_L = 1.0\angle 0° - 0.22\angle - 30.8° \times 2.327\angle 90°$$
$$= 0.859\angle - 30.8° \text{ p.u.}$$
$$S_L = V_L I_L^* = \frac{|V_L|^2}{Z} = 0.189 \text{ p.u.}$$
$$S_G = 1.0\angle 0° \times 0.22\angle 30.8° = 0.22\angle 30.8° \text{ p.u.}$$

Again, analysis is exactly the same!

3ϕ Per-Unit Example, Cont'd 2

Differences appear when we convert back to actual values

$$V_L^{\text{Actual}} = 0.859\angle -30.8° \times 27.6 \text{ kV} = 23.8\angle -30.8° \text{kV}$$

$$S_L^{\text{Actual}} = 0.189\angle 0° \times 300 \text{ MVA} = 56.7\angle 0° \text{ MVA}$$

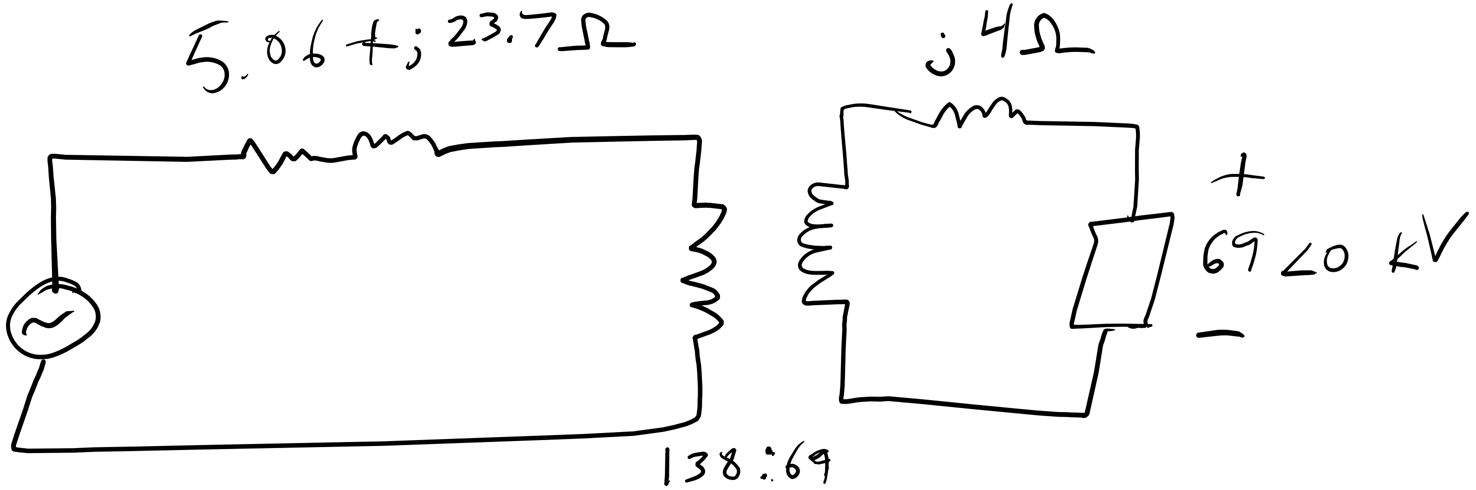
$$S_G^{\text{Actual}} = 0.22\angle 30.8° \times 300 \text{ MVA} = 66.0\angle 30.8° \text{ MVA}$$

$$I_B^{\text{Middle}} = \frac{300 \text{ MVA}}{\sqrt{3}\ 138 \text{ kV}} = 1250 \text{ Amps} \quad \text{(same current!)}$$

$$I_{\text{Middle}}^{\text{Actual}} = 0.22\angle -30.8° \times 1250 \text{ Amps} = 275\angle -30.8° \text{ A}$$

3ϕ Per-Unit Example 2

- Assume a 3ϕ load of 100+j50 MVA with V_{LL} of 69 kV is connected to a source through the below network:



What is the supply current and complex power?

Answer: I=467 amps, S = 103.3 + j76.0 MVA