

# ECEN 460, Spring 2026

## Power System Operation and Control

### Class 5: Transformers, Part 1

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TEXAS A&M  
UNIVERSITY

# Reminder

- PPE Needed for Machines Labs 2 and 3



## PROPER LAB PPE

	<b>DO</b> wear clear safety glasses with side shields		<b>Don't</b> wear sunglasses or only your eye glasses.
	<b>DO</b> pull back long hair		<b>Don't</b> leave long hair down.
	<b>DO</b> wear shirts with sleeves		<b>Don't</b> wear sleeveless shirts
	<b>DO</b> wear long pants		<b>Don't</b> wear skirts or shorts, or pants with holes
	<b>DO</b> wear closed-toe and closed-heel shoes that cover the whole foot		<b>Don't</b> wear slip-on shoes or sandals
	<b>DO</b> wear your pant leg over the whole opening of your shoe		



# Homework 3 and 4

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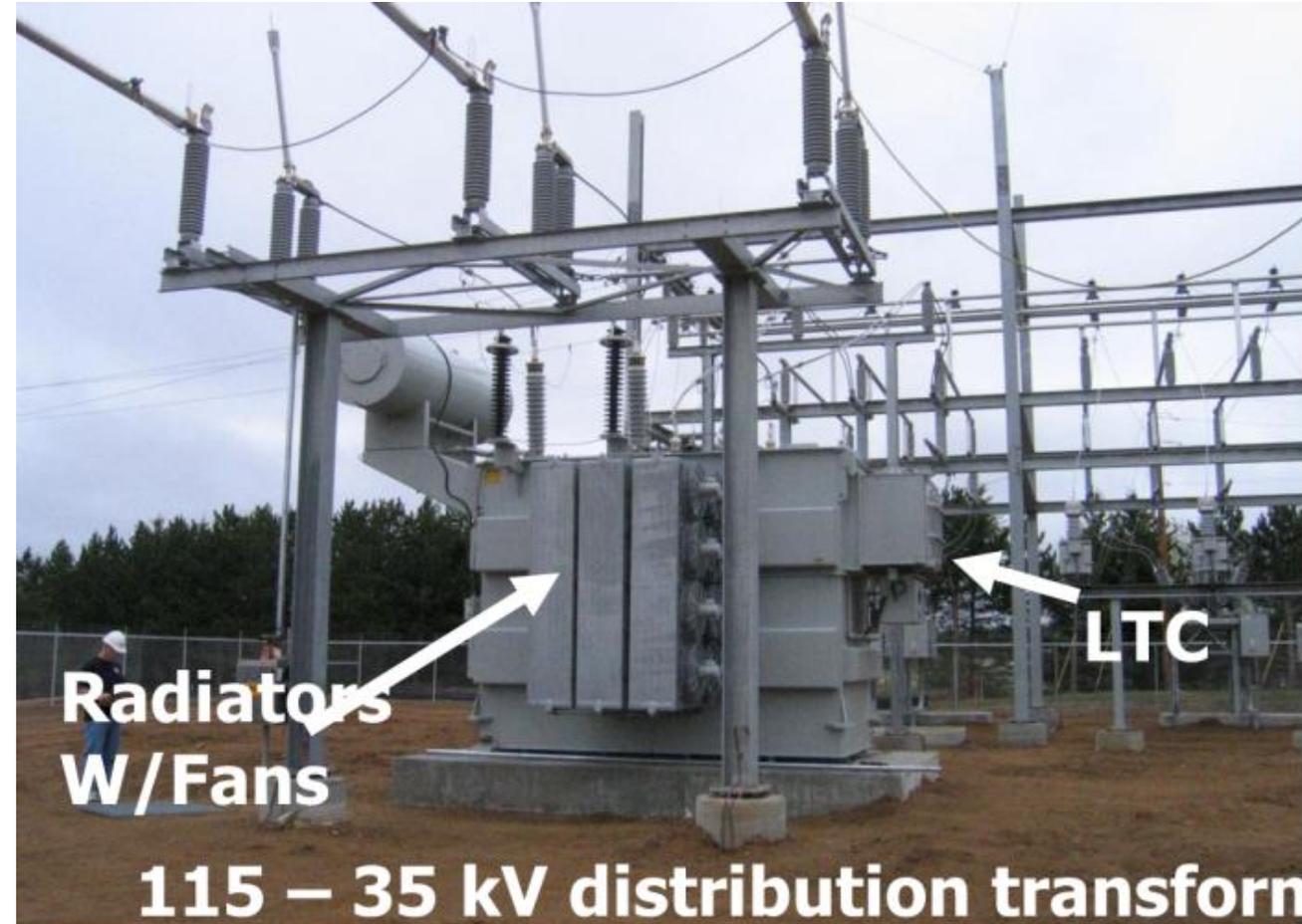


- No homework on generators. Make sure you understand the lecture notes and labs 2 and 3 (starting this week).
- Homework 3 on transformers: book problems 3.4, 3.5, 3.23, due Feb. 3rd.
- Homework 4 on transmission lines: book regular problems 4.10, 4.11, 4.20, and 4.41, 5.14 (a,b), 5.38, and 5.41 (a,b), due Feb. 10th.

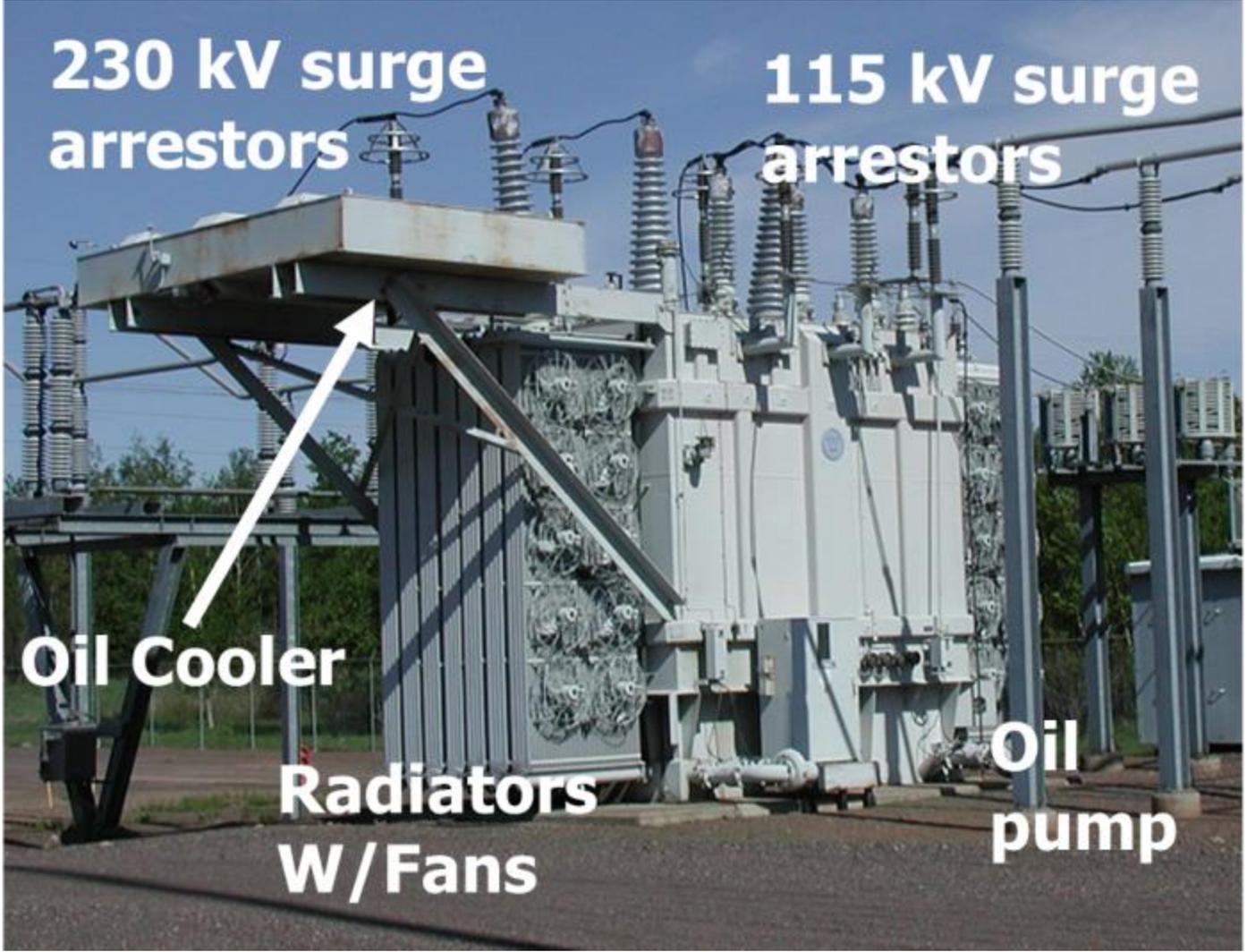
# Transformers Overview



- Power systems are characterized by many different voltage levels, ranging from 765 kV down to 240/120 volts.
- Transformers are used to transfer power between different voltage levels.
- The ability to inexpensively change voltage levels is a key advantage of ac systems over dc systems.
- In this section we'll development models for the transformer and discuss various ways of connecting three phase transformers.



# Transmission Level Transformer



# Ideal Transformer

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- First, we review the voltage/current relationships for an ideal transformer
  - no real power losses
  - magnetic core has infinite permeability
  - no leakage flux
- We'll define the “primary” side of the transformer as the side that usually takes power, and the secondary as the side that usually delivers power.
  - primary is usually the side with the higher voltage but may be the low voltage side on a generator step-up transformer.

# XF Theory

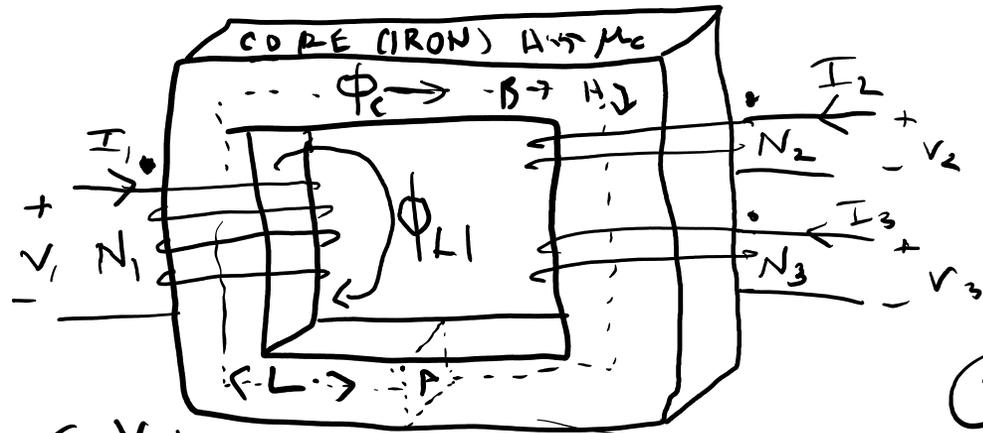


$$i(t) = I_{max} \cos(\omega t + \theta)$$

$$\frac{di}{dt} = -I_{max} \omega \sin(\omega t + \theta)$$

$$\left(\frac{I_{max}}{\sqrt{2}} \angle \theta\right) = I$$

$$-\frac{I_{max}}{\sqrt{2}} \angle \theta + 90^\circ = j\omega I$$



Winding

	1	2	3
Turns	$N_1$	$N_2$	$N_3$
Voltage	$\bar{V}_1$	$\bar{V}_2$	$\bar{V}_3$
Current	$\bar{I}_1$	$\bar{I}_2$	$\bar{I}_3$

$\mu_c$  Permeability of core  
(high)

$\phi_c$  Magnetic flux in core  
 $H$  Magnetic field  
 $B$  Flux density

$A$  cross-section area of core  
 $L$  length of core path

① Voltages

Faraday's Law

$$\mathcal{E} = -\frac{d\phi(t)}{dt}$$

$$V_1 = -N_1 \left( \frac{d\phi_c}{dt} + \cancel{\frac{d\phi_L}{dt}} \right)$$

$$\frac{V_1}{N_1} = -\frac{d\phi_c}{dt} = \frac{V_2}{N_2} = \frac{V_3}{N_3}$$

$$\frac{V_1}{N_1} = \frac{V_2}{N_2} = \frac{V_3}{N_3}$$

$\mathcal{E}$  ideal:  
assume  
leakage  
 $\phi_L = 0$

② Currents Ampere's Law

$$\oint \vec{H} \cdot d\vec{l} = i_{enc}$$

$$H \cdot L = N_1 i_1 + N_2 i_2 + N_3 i_3$$

$$B = \mu_0 \mu_c H \quad \phi = B A$$

$$\frac{\phi L}{\mu_0 \mu_c} = N_1 i_1 + N_2 i_2 + N_3 i_3$$

$$\phi = \frac{\mu_0 \mu_c}{L} (N_1 i_1 + N_2 i_2 + N_3 i_3)$$

$$\frac{V_1}{N_1} = -\frac{d\phi}{dt} = \frac{\mu_0 \mu_c \omega}{L} (N_1 I_1 + N_2 I_2 + N_3 I_3)$$

$$\frac{V_1 L}{\mu_0 \mu_c} = N_1 I_1 + N_2 I_2 + N_3 I_3$$

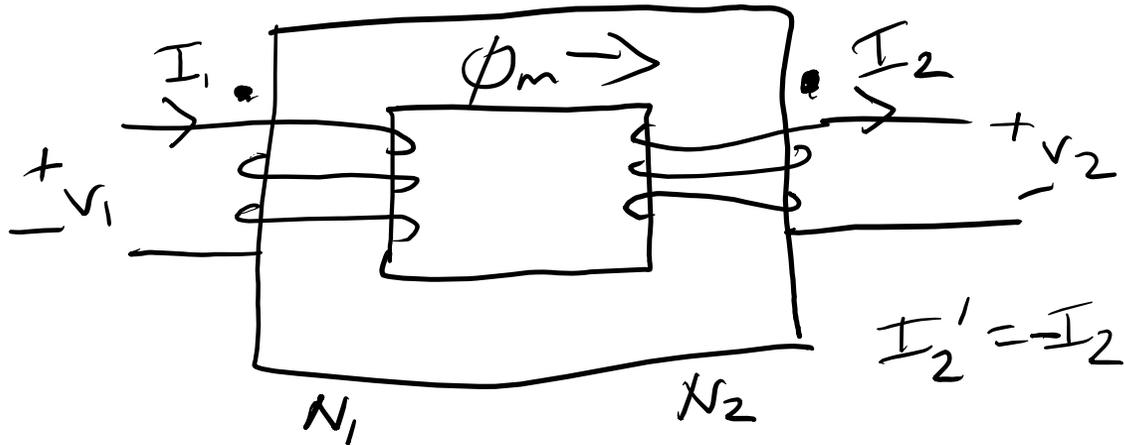
Magnetizing current

$\mathcal{E}$  ideal,  $\mu = \infty$

$$0 = N_1 \bar{I}_1 + N_2 \bar{I}_2 + N_3 \bar{I}_3$$

~~$$N_1 \bar{I}_1 + N_2 \bar{I}_2 = 0$$~~

# Ideal Transformer Relationships



Assume we have flux  $\phi_m$  in magnetic material. Then

$$\lambda_1 = N_1 \phi_m \quad \lambda_2 = N_2 \phi_m$$

$$v_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\phi_m}{dt} \quad v_2 = \frac{d\lambda_2}{dt} = N_2 \frac{d\phi_m}{dt}$$

$$\frac{d\phi_m}{dt} = \frac{v_1}{N_1} = \frac{v_2}{N_2} \rightarrow \frac{v_1}{v_2} = \frac{N_1}{N_2} = a = \text{turns ratio}$$

# Current and Voltage Relationships



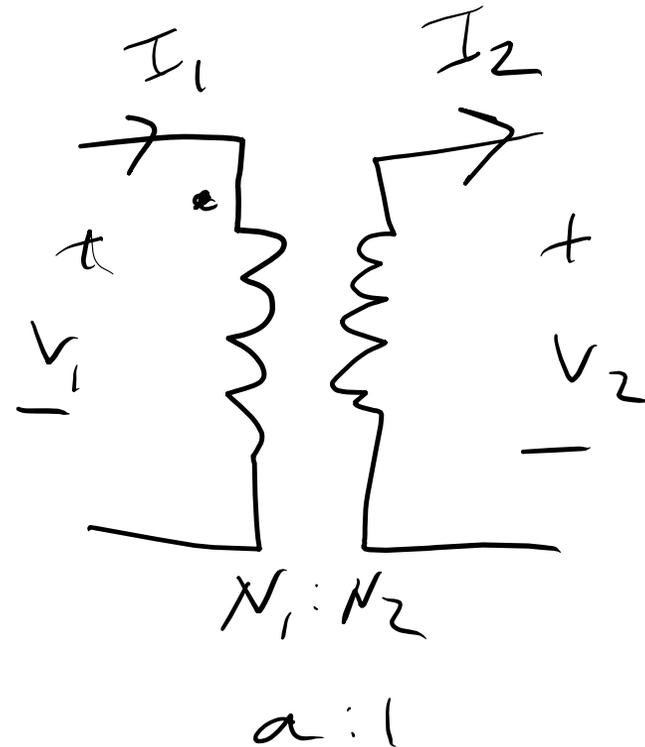
If  $\mu$  is infinite then 0

$= N_1 i_1 + N_2 i'_2$ . Hence

$$\frac{i_1}{i'_2} = -\frac{N_2}{N_1} \text{ or } \frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{a}$$

Then

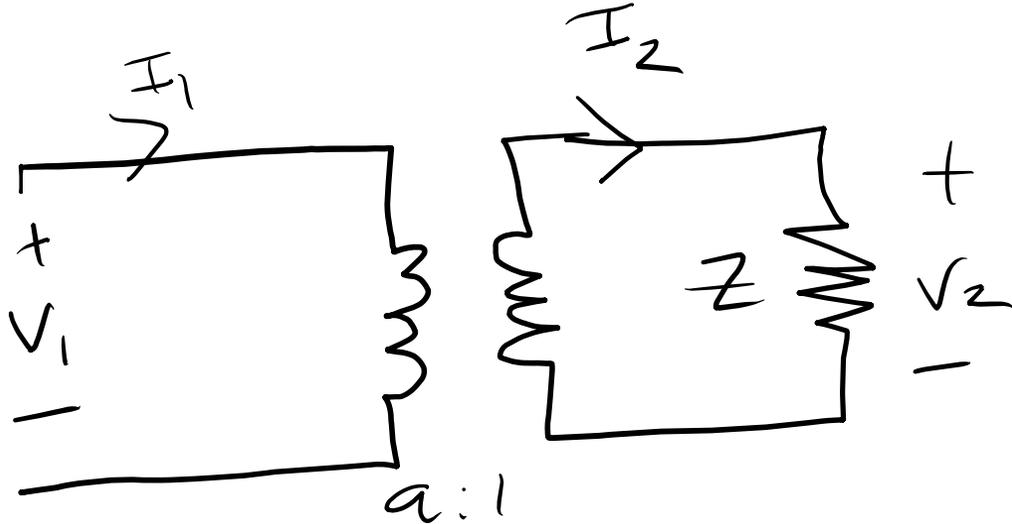
$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & \frac{1}{a} \end{bmatrix} \begin{bmatrix} v_2 \\ i_2 \end{bmatrix}$$



# Impedance Transformation Example



- Example: Calculate the primary voltage and current for an impedance load on the secondary



$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & \frac{1}{a} \end{bmatrix} \begin{bmatrix} v_2 \\ \frac{v_2}{Z} \end{bmatrix}$$

$$v_1 = av_2 \quad i_1 = \frac{1}{a} \frac{v_2}{Z} \quad \frac{v_1}{i_1} = a^2 Z$$

# Real Transformers

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- Real transformers
  - have losses
  - have leakage flux
  - have finite permeability of magnetic core
  
- Real power losses
  - resistance in windings ( $i^2 R$ )
  - core losses due to eddy currents and hysteresis

# Transformer Core Losses



Eddy currents arise because of changing flux in core.

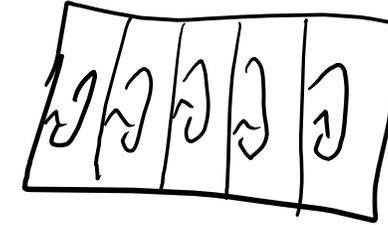
Eddy currents are reduced by laminating the core

Hysteresis losses are proportional to area of BH curve and the frequency

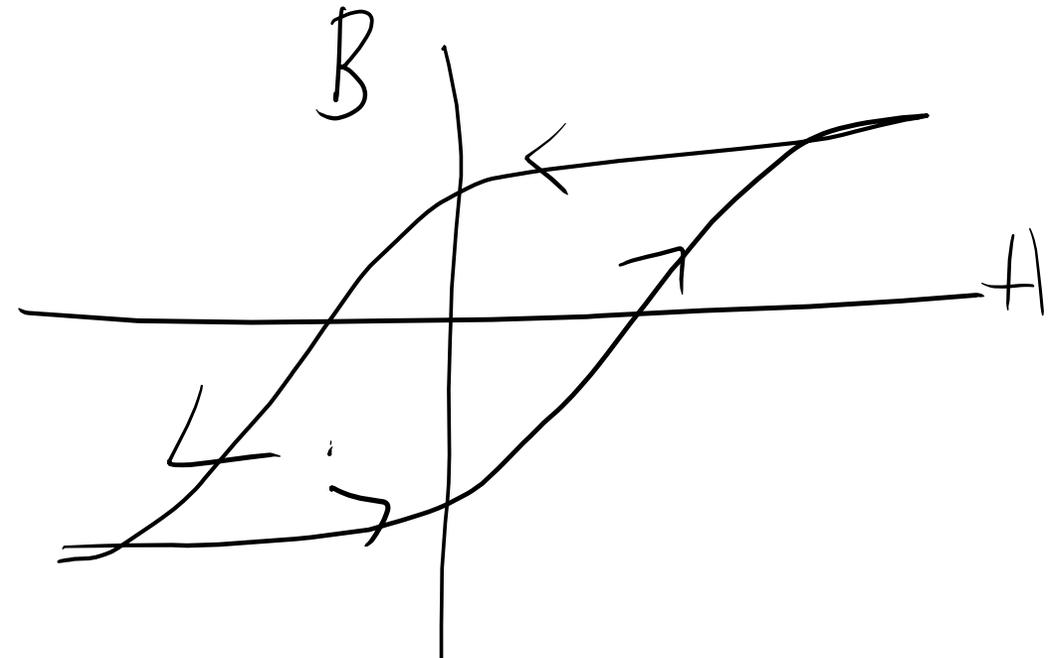
These losses are reduced by using material with a thin BH curve



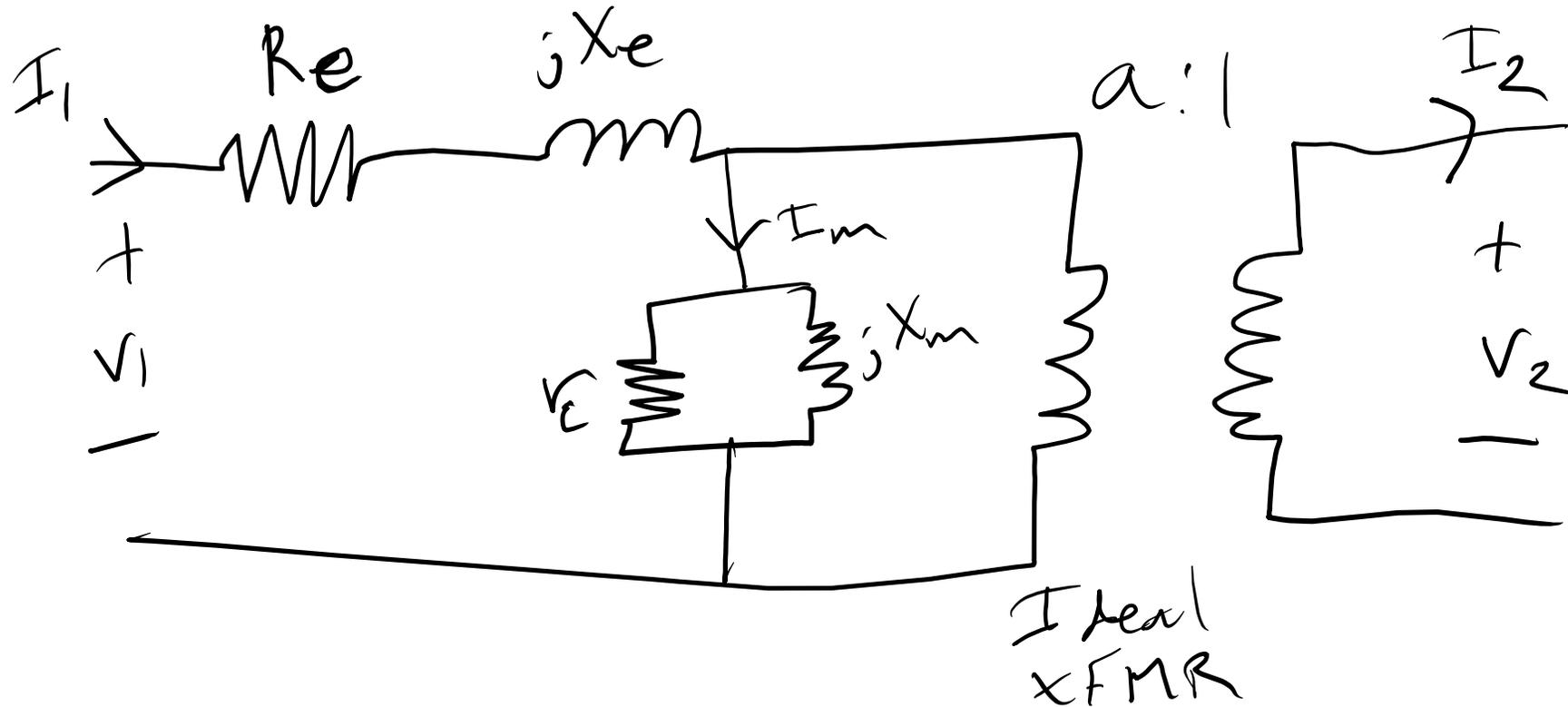
No lamination, high eddy current



With lamination, low eddy current



# Simplified Equivalent Circuit



Often the shunt elements are assumed to have such high impedance that they are ignored

# Calculation of Model Parameters

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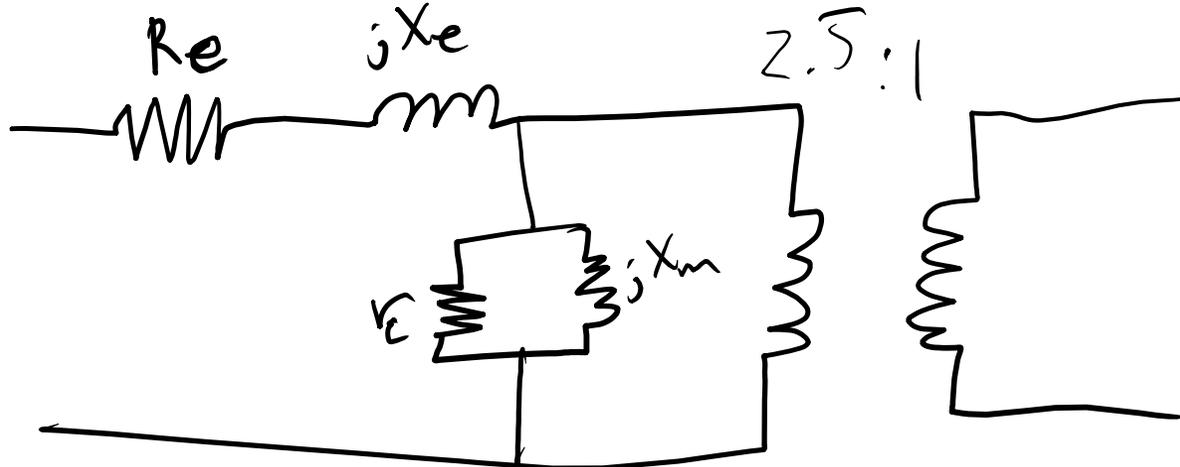


- The parameters of the model are determined based upon
  - nameplate data: gives the rated voltages and power
  - open circuit test: rated voltage is applied to primary with secondary open; measure the primary current and losses (the test may also be done applying the voltage to the secondary, calculating the values, then referring the values back to the primary side).
  - short circuit test: with secondary shorted, apply voltage to primary to get rated current to flow; measure voltage and losses.

# Transformer Example



- Example: A single phase, 100 MVA, 200/80 kV transformer has the following test data:
  - open circuit: 20 amps, with 10 kW losses
  - short circuit: 30 kV, with 500 kW losses
- Determine the model parameters.



# Transformer Example, Cont'd



**From the short circuit test**

$$I_{sc} = \frac{100MVA}{200kV} = 500 \text{ A}, |R_e + jX_e| = \frac{30 \text{ kV}}{500 \text{ A}} = 60 \ \Omega$$

$$P_{sc} = R_e I_{sc}^2 = 500 \text{ kW} \rightarrow R_e = 2 \ \Omega,$$

$$\text{Hence } X_e = \sqrt{60^2 - 2^2} = 60 \ \Omega$$

**From the open circuit test**

$$R_c = \frac{200 \text{ kV}^2}{10 \text{ kW}} = 4M\Omega$$

$$|R_e + jX_e + jX_m| = \frac{200 \text{ kV}}{20 \text{ A}} = 10,000 \ \Omega X_m = 10,000 \ \Omega$$

# Residential Distribution Transformers



Single phase transformers are commonly used in residential distribution systems. Most distribution systems are 4 wire, with a multi-grounded, common neutral.

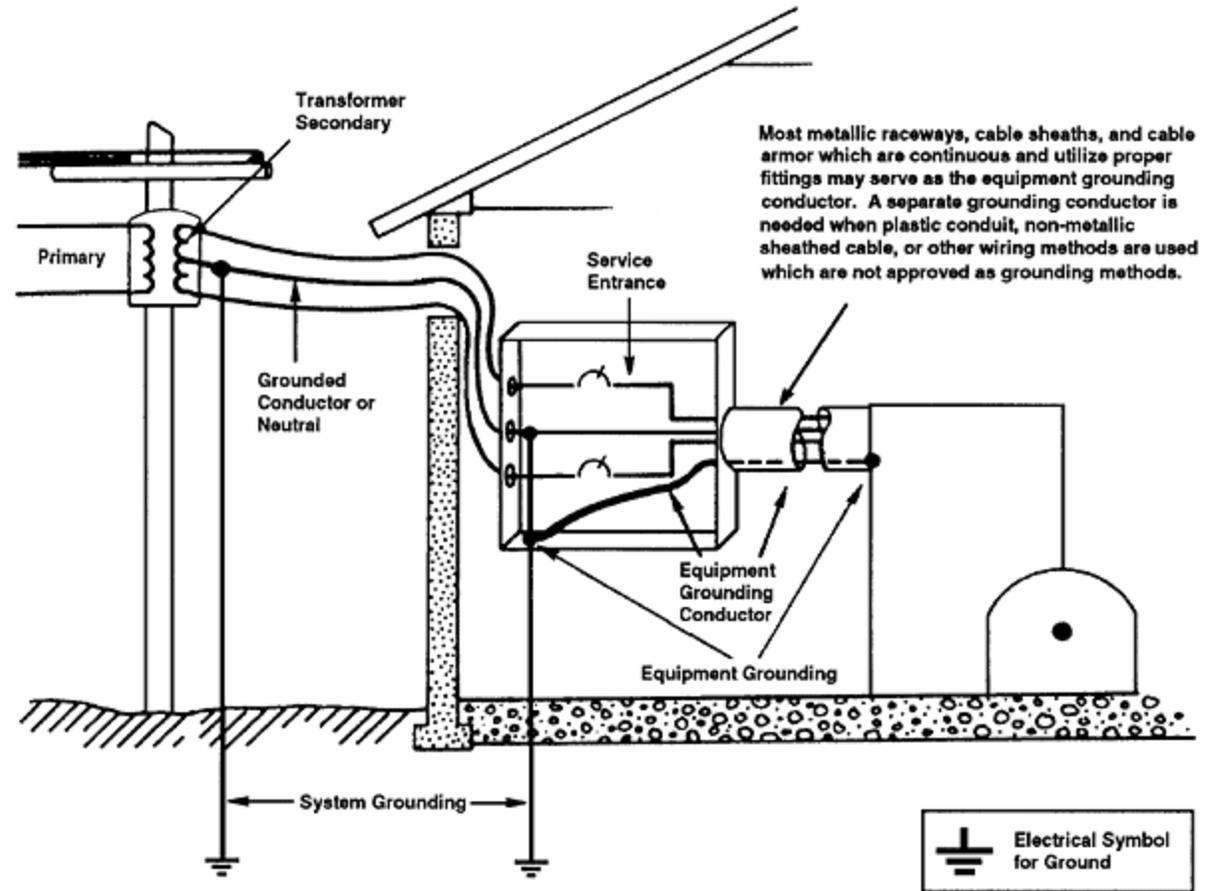
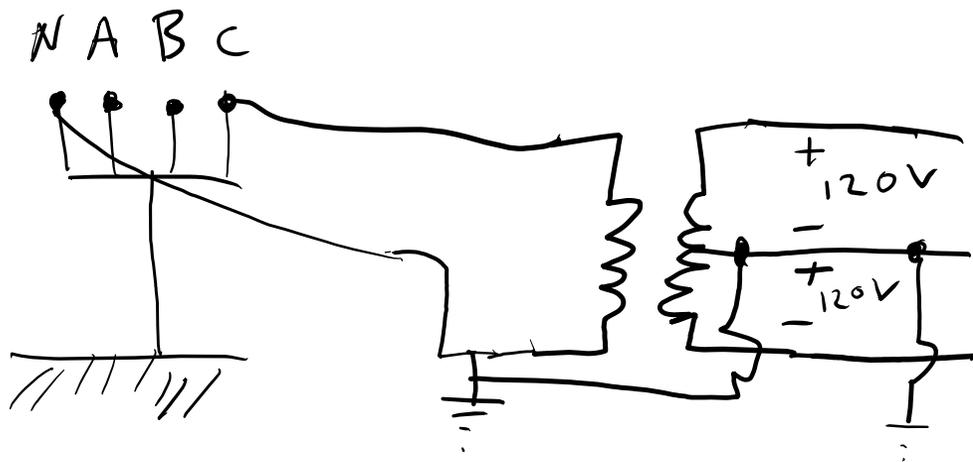


Image: [www.osha.gov/dte/library/electrical/electrical\\_10.gif](http://www.osha.gov/dte/library/electrical/electrical_10.gif)

# How Do We Deal with Transformers on a Large System?

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- With many transformers, the equations of the form  $v_1 = a \cdot v_2$  would be everywhere
- The per-unit system was designed to make hand calculations and early computer calculations easier.
  - Divide all numerical values by a "base" value, i.e., normalize them
  - Instead of voltages in V, currents in A, impedance in Ohms, all are given in per-unit.
  - Ideal transformers go away in the per-unit system
  - Per-unit system has other advantages as well, and is extremely common in industry
  - Next lecture we will discuss the per-unit system in detail

# Per Unit Calculations

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- A key problem in analyzing power systems is the large number of transformers.
  - It would be very difficult to continually have to refer impedances to the different sides of the transformers
- This problem is avoided by a normalization of all variables.
- This normalization is known as per unit analysis.

$$\text{quantity in per unit} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

# Per Unit Conversion Procedure, Single-Phase

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- Pick a single-phase VA base for the entire system,  $S_b$
- Pick a voltage base for each different voltage level,  $V_b$ . Voltage bases are related by transformer turns ratios. Voltages are line to neutral.
- Calculate the impedance base,  $Z_b = V_b^2 / S_b$
- Calculate the current base,  $I_b = S_b / V_b$
- Convert actual values to per unit

Note, per unit conversion only affects magnitudes, not the angles. Also, per unit quantities no longer have units (i.e., a voltage is 1.0 p.u., not 1 p.u. volts)

# Per Unit Solution Procedure

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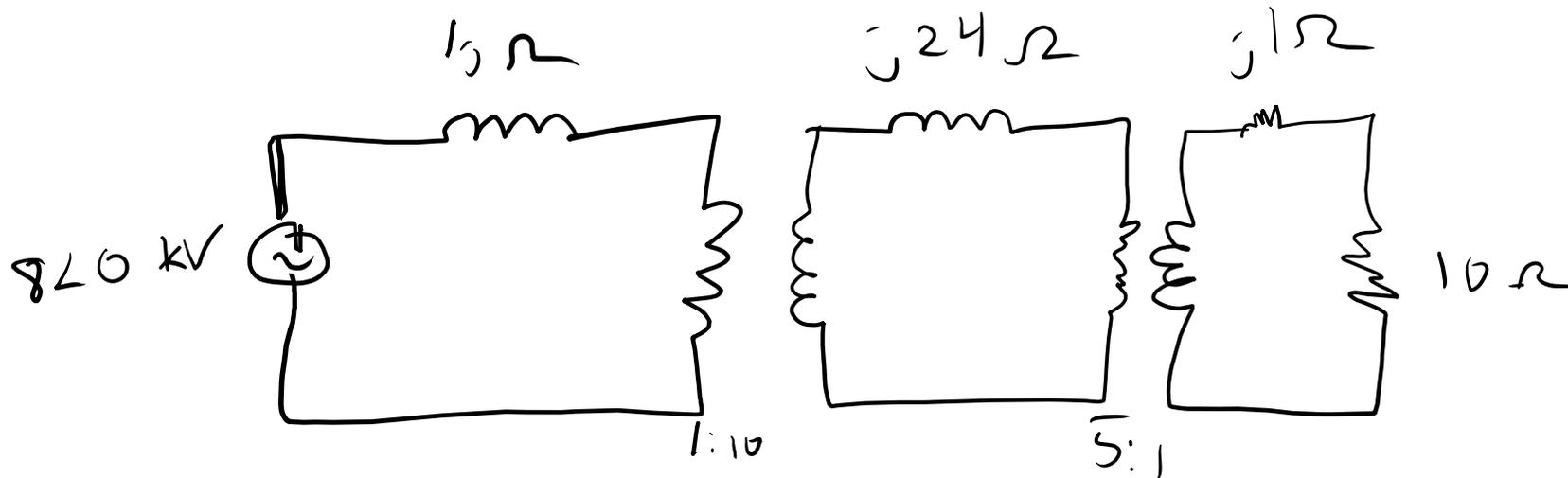


1. Convert to per unit (p.u.) (many problems are already in per unit)
2. Solve
3. Convert back to actual as necessary

# Per Unit Example



Solve for the current, load voltage and load power in the circuit shown below using per unit analysis with an  $S_B$  of 100 MVA, and voltage bases of 8 kV, 80 kV and 16 kV.



Original Circuit

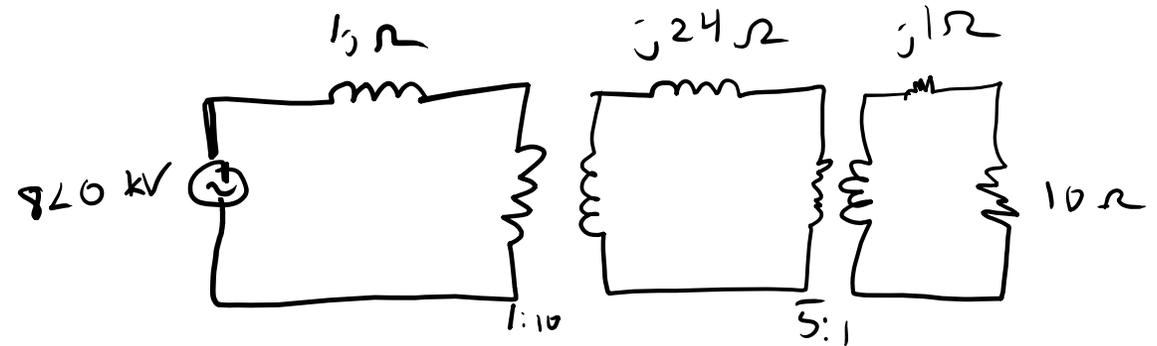
# Per Unit Example, Cont'd



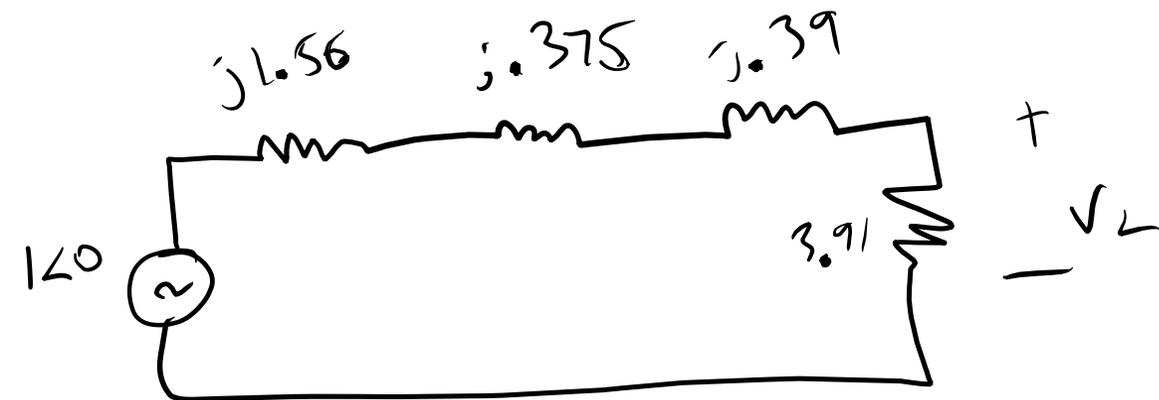
$$Z_B^{Left} = \frac{8kV^2}{100MVA} = 0.64\Omega$$

$$Z_B^{Middle} = \frac{80kV^2}{100MVA} = 64\Omega$$

$$Z_B^{Right} = \frac{16kV^2}{100MVA} = 2.56\Omega$$



Same circuit, with values expressed in per unit.



# Per Unit Example, Cont'd 2



$$I = \frac{1.0 \angle 0^\circ}{3.91 + j2.327}$$

$$= 0.22 \angle -30.8^\circ \text{ p.u. (not amps)}$$

$$V_L = 1.0 \angle 0^\circ - 0.22 \angle -30.8^\circ$$

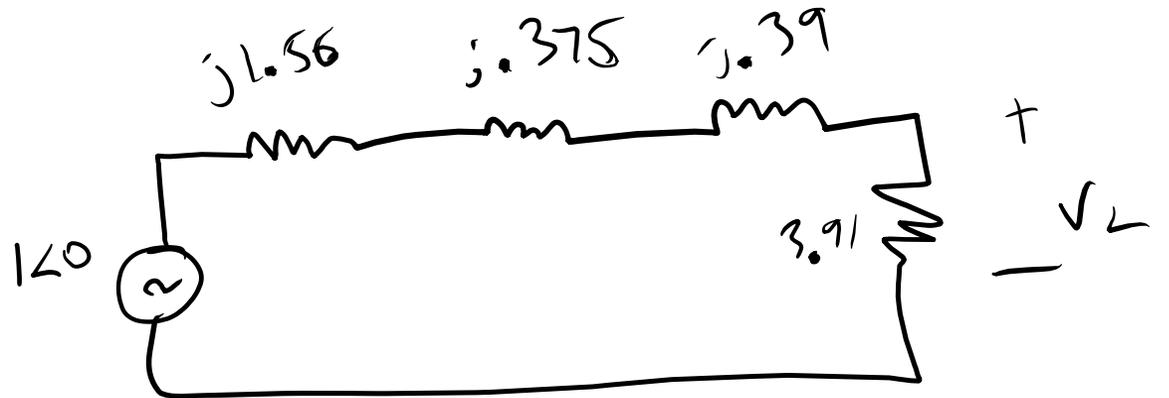
$$\times 2.327 \angle 90^\circ$$

$$= 0.859 \angle -30.8^\circ \text{ p.u.}$$

$$S_L = V_L I_L^* = \frac{|V_L|^2}{Z} = 0.189 \text{ p.u.}$$

$$S_G = 1.0 \angle 0^\circ \times 0.22 \angle 30.8^\circ$$

$$= 0.22 \angle 30.8^\circ \text{ p.u.}$$



## Per Unit Example, Cont'd 3



To convert back to actual values just multiply the per unit values by their per unit base

$$V_L^{\text{Actual}} = 0.859 \angle -30.8^\circ \times 16 \text{ kV} = 13.7 \angle -30.8^\circ \text{ kV}$$

$$S_L^{\text{Actual}} = 0.189 \angle 0^\circ \times 100 \text{ MVA} = 18.9 \angle 0^\circ \text{ MVA}$$

$$S_G^{\text{Actual}} = 0.22 \angle 30.8^\circ \times 100 \text{ MVA} = 22.0 \angle 30.8^\circ \text{ MVA}$$

$$I_B^{\text{Middle}} = \frac{100 \text{ MVA}}{80 \text{ kV}} = 1250 \text{ Amps}$$

$$I_{\text{Middle}}^{\text{Actual}} = 0.22 \angle -30.8^\circ \times 1250 \text{ Amps} = 275 \angle -30.8^\circ \text{ A}$$

# Three Phase Per-Unit



- Procedure is very similar to 1 phase except we use a 3 phase VA base, and use line to line voltage bases
- Pick a  $3\phi$  VA base for the entire system  $S_B^{3\phi}$
- Pick a voltage base for each different voltage level,  $V_B$ . Voltages are line to line.
- Calculate the impedance base

$$Z_B = \frac{V_{B,LL}^2}{S_B^{3\phi}} = \frac{(\sqrt{3} V_{B,LN})^2}{3S_B^{1\phi}} = \frac{V_{B,LN}^2}{S_B^{1\phi}}$$

Exactly the same impedance bases as with single phase!

# Three Phase Per-Unit, Cont'd



- Calculate the current base,  $I_B$

$$I_B^{3\phi} = \frac{S_B^{3\phi}}{\sqrt{3} V_{B,LL}} = \frac{3 S_B^{1\phi}}{\sqrt{3} \sqrt{3} V_{B,LN}} = \frac{S_B^{1\phi}}{V_{B,LN}} = I_B^{1\phi}$$

Exactly the same current bases as with single phase!

But, be careful in using 3ph bases to calculate it (need a root 3)

- Convert actual values to per unit

# Three Phase Transformers

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- There are 4 different ways to connect  $3\phi$  transformers: Y-Y, D-D, Y-D, D-Y
  - The reasons have to do with grounding and harmonics, which are outside of the ECEN 460 scope
  - Only Y connections can be grounded
  - Mixing Y and D introduces a 30-degree phase shift
- Most high voltage generator step-up transformers (GSUs) are D on the generator side, grounded Y on the transmission side
- Most transmission to distribution is D on the transmission side, grounded Y on the distribution side

# Autotransformers

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- Autotransformers are transformers in which the primary and secondary windings are coupled magnetically and electrically.
- This results in lower cost, and smaller size and weight.
- Most transmission level transformers are autotransformers, connected Y-Y with the low side grounded
- The key disadvantage is loss of electrical isolation between the voltage levels; not used when  $a$  is large. For example, in stepping down 7160/240 V we do not ever want 7160 on the low side!

# Three Winding Transformers

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- Many high voltage transformers have a third winding, called the tertiary winding; called three winding transformers
- There are a number of benefits in having 3 windings
  - Tertiary can be used to provide lower voltage electric service, including providing substation service for remote transmission substations; sometimes capacitors are connected to the tertiary
  - Helps with fault protection by reducing the zero sequence current providing higher zero sequence currents (beyond ECEN 460 scope)
  - When D-connected helps to reduce unbalanced and third harmonic issues (again beyond ECEN 460 scope)

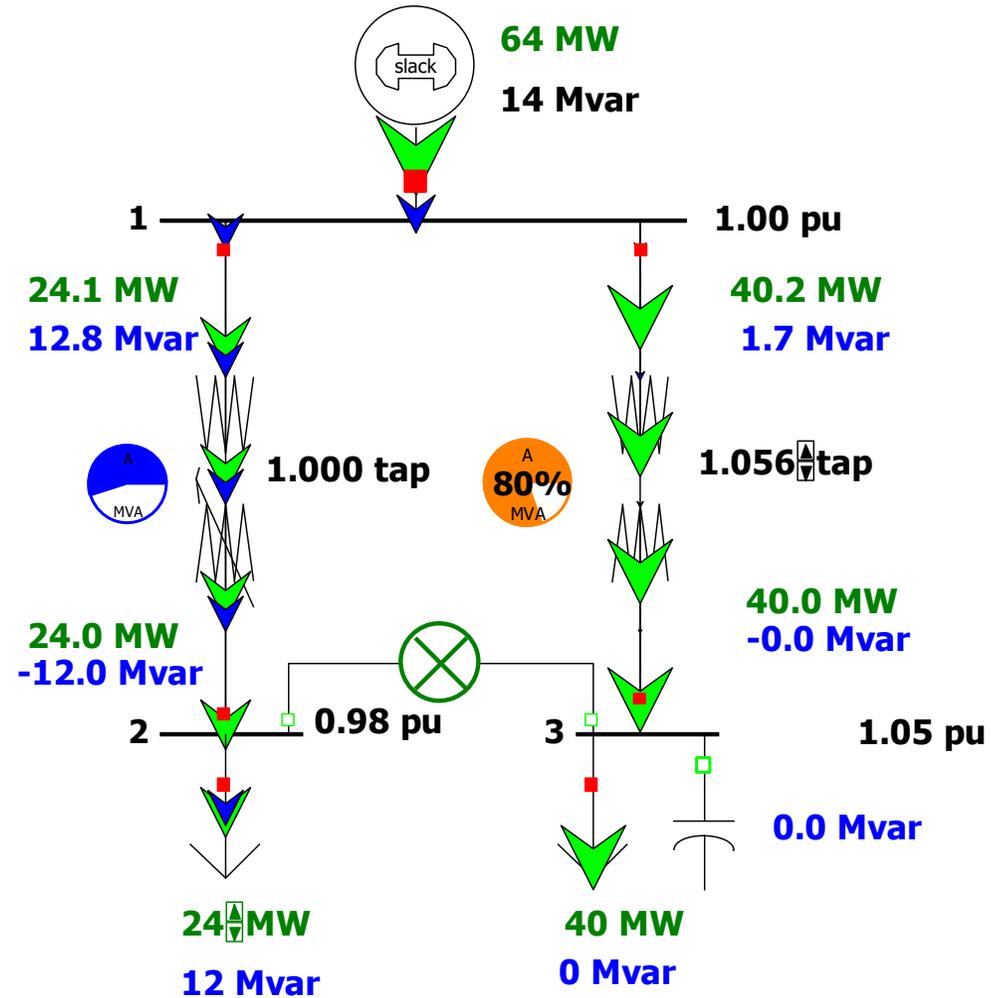
# Load Tap-Changing Transformers

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- LTC transformers have tap ratios that can be varied to regulate bus voltages
  - Sometimes called on-load tap-changing transformers (OLTCs) or under-load tap-changing transformers (ULTCs)
- The typical range of variation is  $\pm 10\%$  from the nominal values, usually in 33 discrete steps (0.0625% per step).
- Because tap changing is a mechanical process, LTC transformers usually have a time delay deadband to avoid repeated changes (e.g., 30 seconds)
- Unbalanced tap positions can cause “circulating vars”

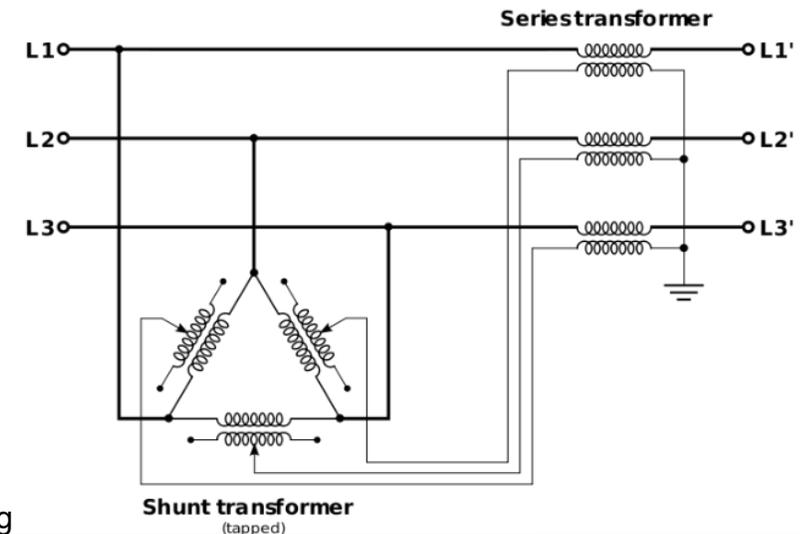
# LTCs and Circulating Vars



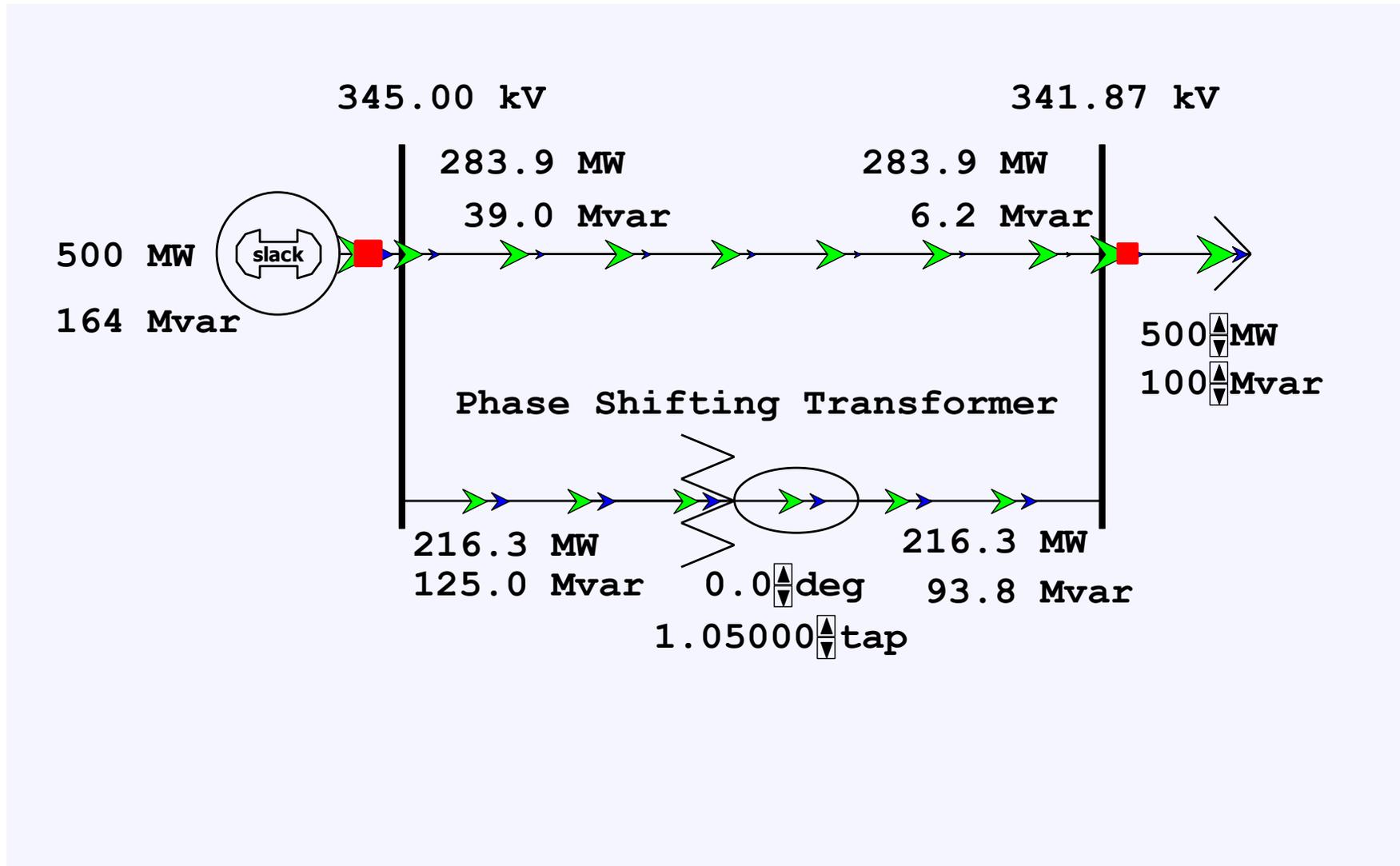
# Phase Shifting Transformers



- Phase shifting transformers are used to control the phase angle across the transformer
  - Also called phase angle regulators (PARs) or quadrature booster transformers (British usage)
- Since power flow through the transformer depends upon phase angle, this allows the transformer to regulate the power flow through the transformer
- Phase shifters can be used to prevent inadvertent "loop flow" and to prevent line overloads.



# Phase Shifter Example 3.13

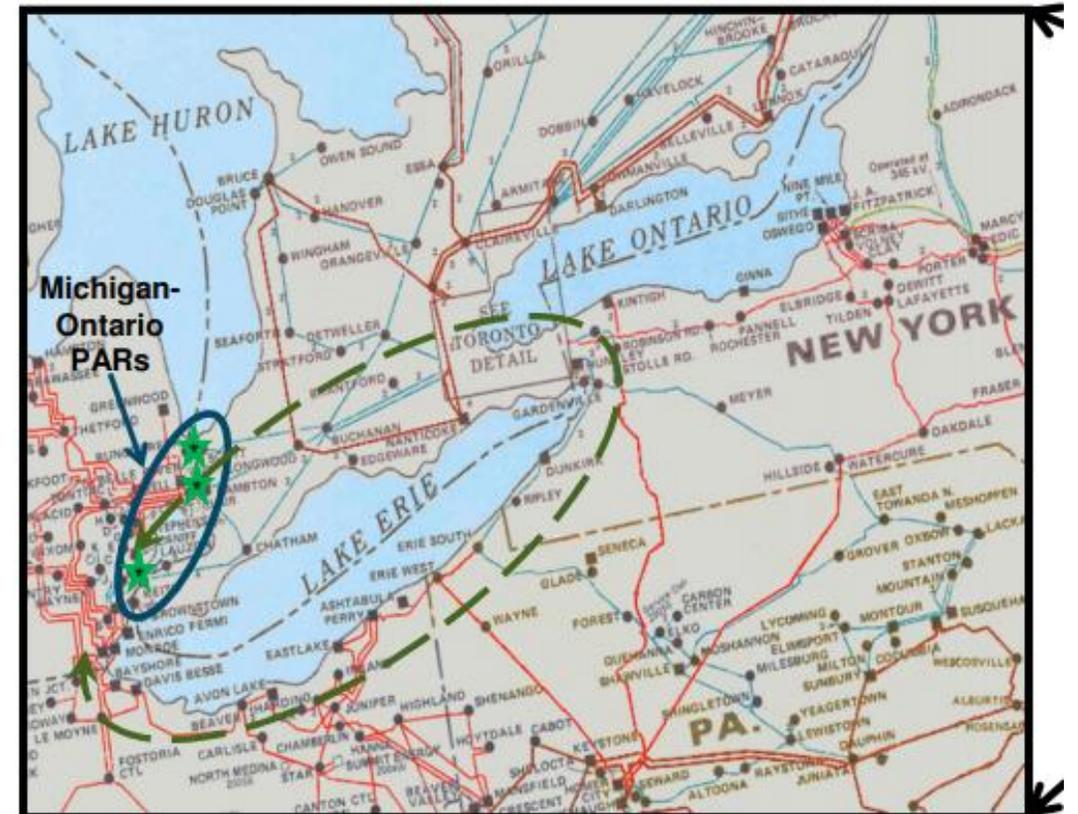


# Phase Shifter Example: Lake Erie Loop Flow



- There are five phase shifters (called phase angle regulators [PARs] here) at the border between Michigan and Ontario. They are used to control how much electricity travels between the US and Canada
- By adjusting the PARs, the flow of electricity that “loops” around Lake Erie can be controlled

## Michigan Ontario PARs



Source: MISO Board of Directors Market Committee Update, 9/30/13