ECEN 460, Spring 2026
Power System Operation and Control

Class 1: Three-Phase AC Power Calculations

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TEXAS A&M UNIVERSITY

Energy and Power Group (EPG) at Texas A&M

- Texas A&M is a leader in power and energy research!
- 14+ Faculty members in various expertise areas of power systems, power electronics, and electric machines
- Get connected!

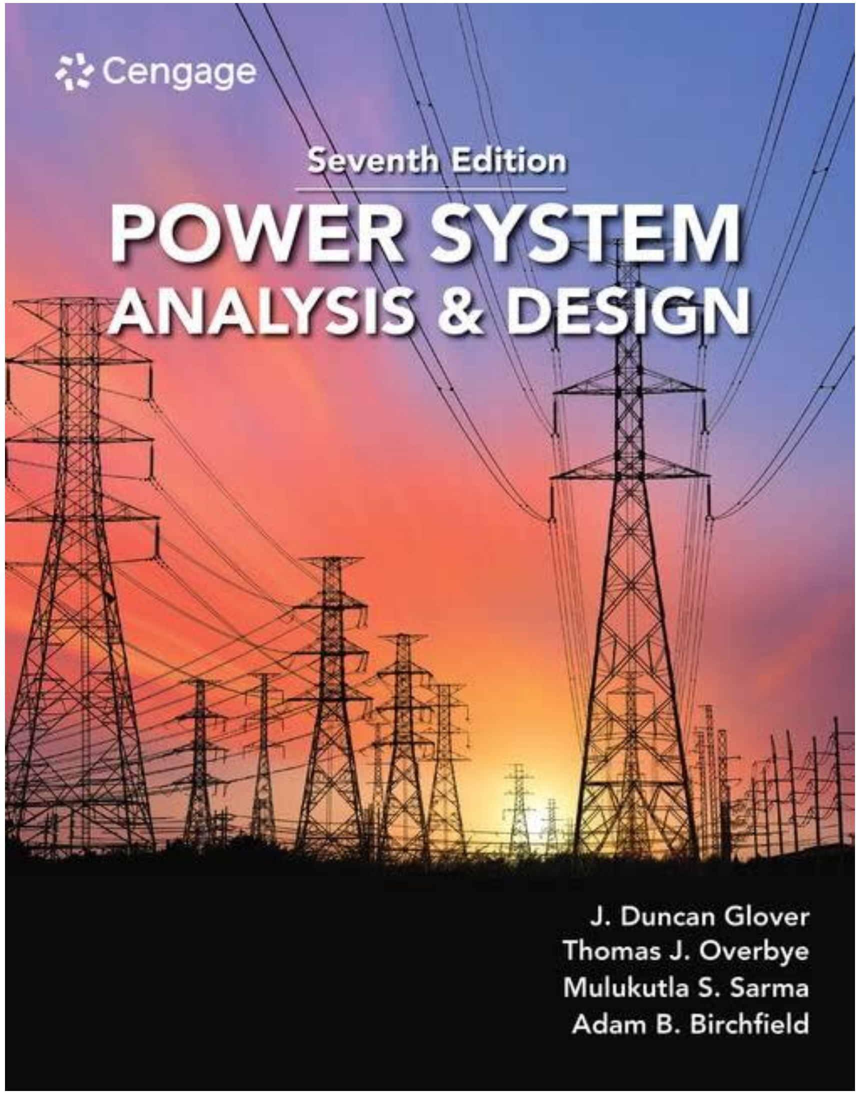
Smart Grid Research Facility

EPG Spring 2025 Picnic at Dr. Overbye's house

Texas Power and Energy Conference (**Student Run!!**)

Welcome to ECEN 460!

- Syllabus and other important information, including slides will be on the website: https://birchfield.engr.tamu.edu/460s26/
- Canvas is used for lab reports and grades
- Textbook: Power System Analysis and Design
- Evaluation
  - 20% First Exam, Tentatively Feb 17
  - 20% Second Exam, Tentatively April 2
  - 20% Final, May 5
  - 20% In-class quizzes
  - 20% Lab – starts next week
- If you need accommodation for a disability, please let me know as soon as you can



Motivation for this Class

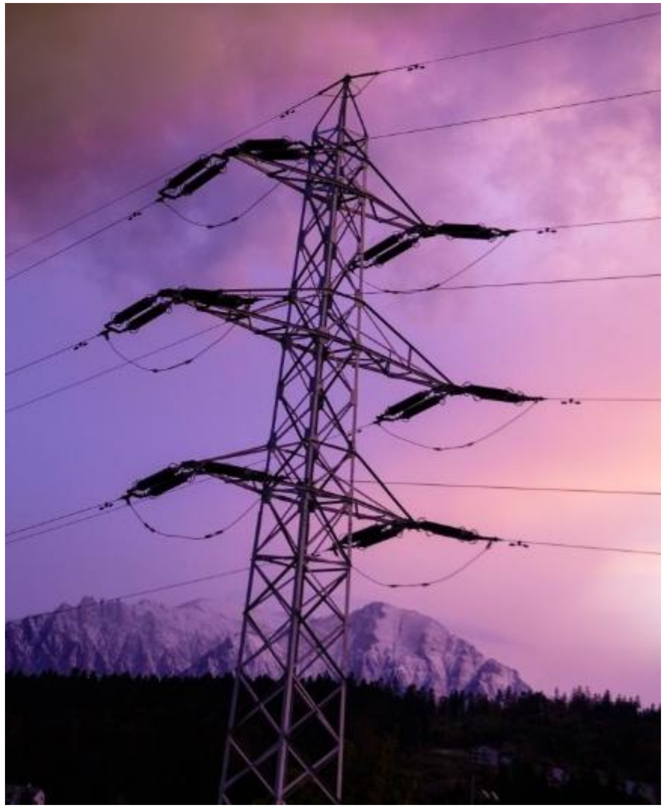
- Electrification has changed the world, and power systems are now critical to our way of life
  - Lighting, HVAC, manufacturing
  - Refrigeration, medical technology, agriculture
  - Communications, computing, transportation
- Now is a time of rapid transformation for electric grids worldwide
  - Major additions of wind, solar, and batteries
  - EVs and smart/efficient consumer devices
  - Cyber and physical security concerns
  - AI's consumption of power and using AI for grid control
- In 460, we want to develop future power engineers who can tackle difficult problems either directly or indirectly related to the grid

Some Things to Think About – For Next Class

- What is energy? What is power?
- Why is the bulk of the energy we use transmitted at some point by electricity?
- Why is it important to have very high voltages in power systems?
- Why do large electric power systems exclusively use AC at 50 or 60 Hz?
- Why are large electric power systems exclusively three-phase?
- Why are power grids connected across continents? What are the advantages or disadvantages of connecting to neighbors?

What this Class Will Cover

1. Review of three-phase AC power calculations (today)
2. Structure, history, and design of power systems
3. Generator and load modeling
4. Transmission line and transformer modeling
5. The power flow problem and solution methods
6. Economic power system operation
7. Dynamics and stability of power systems
8. Emerging topics



First Assignment – Review Three-Phase AC Power Calculations

- Main topics you need to know
  1. Complex number math – by hand and calculator
  2. Phasors – representing AC voltage and current
  3. Impedance – for AC circuit solving
  4. Complex power – including power factor, reactive power
  5. Three phase – wye/delta, line-to-line/line-to-neutral voltage, per-phase analysis
- Today we're going to review these
- Homework #1 and #2 are practice problems on these topics
  - Homework #1 due next Tuesday (Jan 20) and Homework #2 due Tuesday Jan 27
- Take advantage of the textbook, and office hours (me and TAs)
- We're going to have quizzes most classes to review.

Review of Phasors – Representing AC Voltage and Current

- The goal of phasor analysis is to simplify the analysis of constant frequency ac systems
- We represent ac voltage and current as complex numbers

$$v(t) = V_{\max} \cos(\omega t + \theta_v)$$
$$i(t) = I_{\max} \cos(\omega t + \theta_i)$$

- Euler's Identity: $e^{j\theta} = \cos\theta + j\sin\theta$
- Phasor notation is developed by rewriting using Euler's identity

$$v(t) = \sqrt{2}\,|V| \cos(\omega t + \theta_V)$$
$$v(t) = \sqrt{2}\,|V|\, \text{Re}\left[e^{j(\omega t + \theta_V)}\right]$$

- (Note: |V| is the RMS voltage)

Root Mean Square (RMS) voltage of sinusoid

$$\sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt} = \frac{V_{\max}}{\sqrt{2}}$$

Phasors, Cont.

- The RMS, cosine-referenced voltage phasor is:
- $\boldsymbol{V} \quad = \quad |V|e^{j\theta_V} = |V|\angle\theta_V$
- $v(t) = \quad \mathrm{Re}\,[\sqrt{2}\,V e^{j\omega t}\, e^{j\theta_V}]$
- $\boldsymbol{V} \quad = \quad |V|\cos\theta_V + j|V|\sin\theta_V$
- $\boldsymbol{I} \quad = \quad |I|\cos\theta_I + j|I|\sin\theta_I$

(Note: Some texts use boldface type for complex numbers; some use overbars)

Impedance – For AC Circuit Solutions

Device	Time Analysis	Phasor
Resistor	$v(t) = Ri(t)$	$\boldsymbol{V} = R\boldsymbol{I}$
Inductor	$v(t) = L\frac{di(t)}{dt}$	$\boldsymbol{V} = j\omega L\boldsymbol{I}$
Capacitor	$\frac{1}{C}\int_0^t i(t)dt + v(0)$	$\boldsymbol{V} = \frac{1}{j\omega C}\boldsymbol{I}$

$\boldsymbol{Z}$ = Impedance $= R + jX = |Z|\angle\phi$

R = Resistance

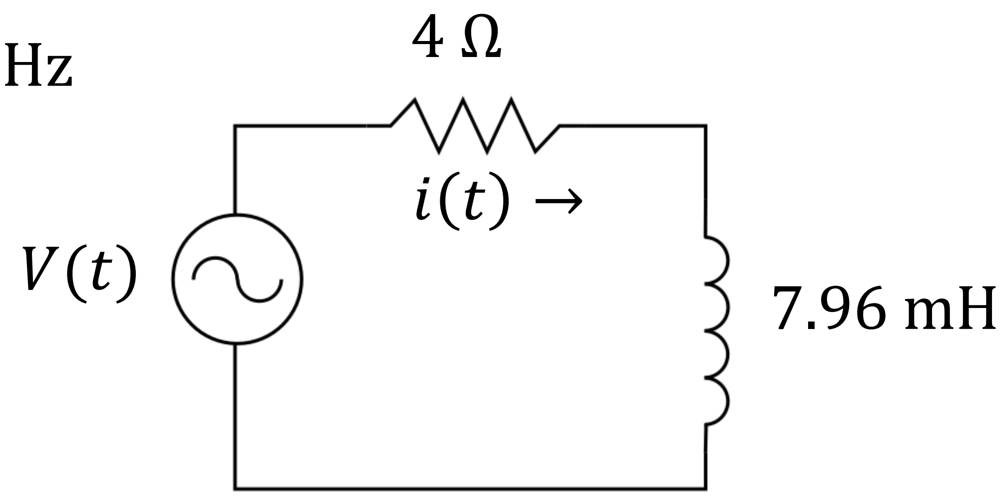
X = Reactance

$|Z| = \sqrt{R^2 + X^2}$ $\qquad$ $\phi = \arctan\left(\frac{X}{R}\right)$

(Note: **Z** is a complex number but not a phasor)

Impedance – RL Circuit Example

$f = 60\text{ Hz}$



Given $V(t) = \sqrt{2} \cdot 100\cos(\omega t + 30°)$, calculate current time function.

Answer: $Z = R + jX$

$$= R + \omega L = 4 + j3\ \Omega$$

$$|Z| = \sqrt{4^2 + 3^2} = 5\ \Omega$$

$$\phi = 36.9°$$

$$I = \frac{V}{Z} = \frac{100\angle 30°}{5\angle 36.9°}$$

$$= 20\angle -6.9°\ \text{Amps}$$

$$i(t) = 20\sqrt{2}\cos(\omega t - 6.9°)$$

Complex Power

Power

$p(t) = v(t) \cdot i(t)$

$v(t) = V_{\max} \cos(\omega t + \theta_v)$

$i(t) = I_{\max} \cos(\omega t + \theta_I)$

$\cos\alpha \cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$

$p(t) = \frac{1}{2} V_{\text{Max}} I_{\text{Max}}[\cos(\theta_V - \theta_I) + \cos(2\omega t + \theta_V + \theta_I)]$

$P_{avg} = \frac{1}{T}\int_0^T p(t)dt$

$= \frac{1}{2} V_{\max} I_{\max} \cos(\theta_V - \theta_I)$

$= |V||I| \cos(\theta_V - \theta_I)$

Power Factor Angle $= \phi = \theta_V - \theta_I$

Complex Power, Cont’d

$$S = |V||I|[\cos(\theta_V - \theta_I) + j\sin(\theta_V - \theta_I)]$$
$$= P + j\,Q$$
$$= V\,I^*$$

$P =$ Real Power (W, kW, MW)

$Q =$ Reactive Power (var, kvar, Mvar)

$S =$ Complex power (VA, kVA, MVA)

Power Factor (pf) $= \cos\phi$

If current leads voltage then pf is leading

If current lags voltage then pf is lagging

(Note: S is a complex number but not a phasor)

Complex Power, Cont'd 2

- Relationships between real, reactive, and complex power

$$P = |S| \cos \phi$$

$$Q = |S| \sin \phi = \pm |S| \cdot pf$$

- Example: A load draws 100 kW with a leading pf of 0.85. What are ϕ (power factor angle), Q and $|S|$?

$$\phi = -\cos^{-1} 0.85 = -31.8°$$

$$|S| = \frac{100\text{kW}}{0.85} = 117.6 \text{ kVA}$$

$$Q = 117.6 \sin(-31.8°) = -62.0 \text{ kVar}$$

Conservation of Power

- At every node (bus) in the system:
  - Sum of real power into node must equal zero
  - Sum of reactive power into node must equal zero
- This is a direct consequence of Kirchhoff's current law, which states that the total current into each node must equal zero.
  - Conservation of power follows since $S = VI^*$

Power Consumption in Devices

Resistors only consume real power

$$\mathrm{P}_{\text{Resistor}} = |I_{\text{Resistor}}|^2 R$$

Inductors only consume reactive power

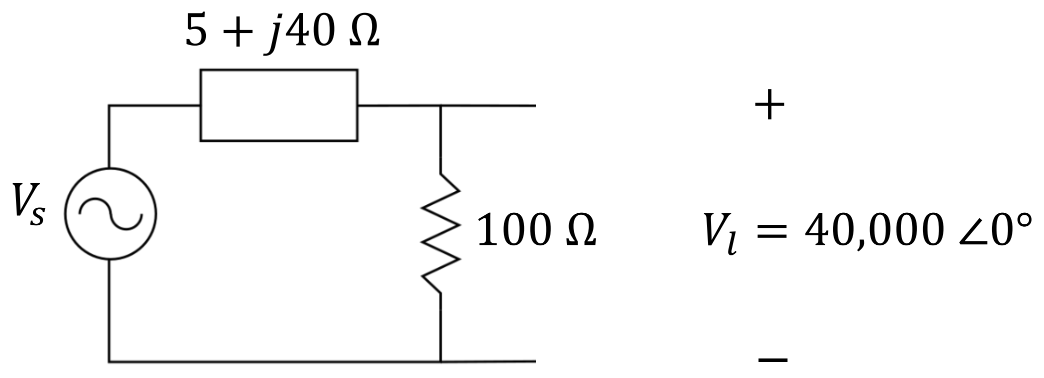
$$\mathrm{Q}_{\text{Inductor}} = |I_{\text{Inductor}}|^2 X_{\mathrm{L}}$$

Capacitors only generate reactive power

$$\mathrm{Q}_{\text{Capacitor}} = -\left|I_{\text{Capacitor}}\right|^2 X_{\mathrm{C}} \quad X_C = \frac{1}{\omega C}$$

$$\mathrm{Q}_{\text{Capacitor}} = -\frac{\left|V_{\text{Capacitor}}\right|^2}{X_{\mathrm{C}}}$$ (Note−some define $\mathrm{X_C}$ negative)

Example



First solve basic circuit

$$I = \frac{40000\angle 0°\ V}{100\angle 0°\ \Omega} = 400\angle 0°\ \text{Amps}$$

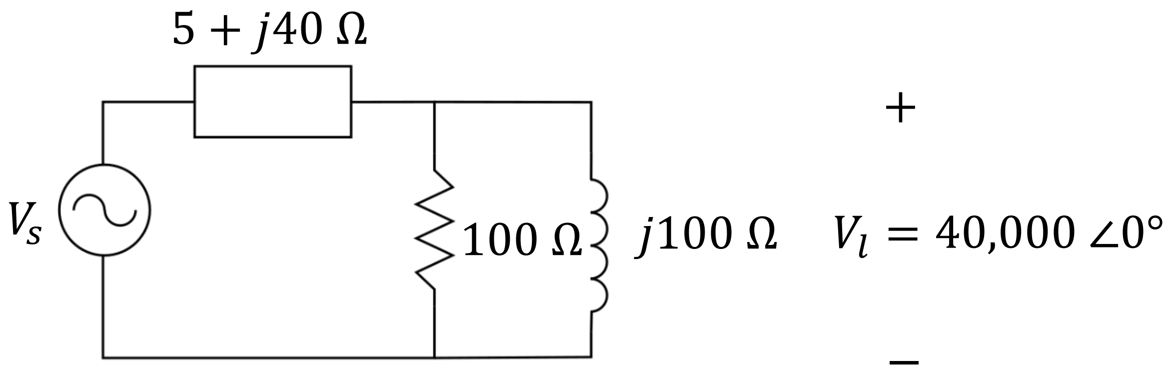
$$V = 40000\angle 0° + (5 + j40)\ 400\angle 0°$$

$$= 42000 + j16000 = 44.9\angle 20.8°\ \text{kV}$$

$$\text{S} = VI^* = 44.9\text{k}\angle 20.8° \times 400\angle 0°$$

$$= 17.98\angle 20.8°\ \text{MVA} = 16.8 + j6.4\ \text{MVA}$$

Example, Cont'd

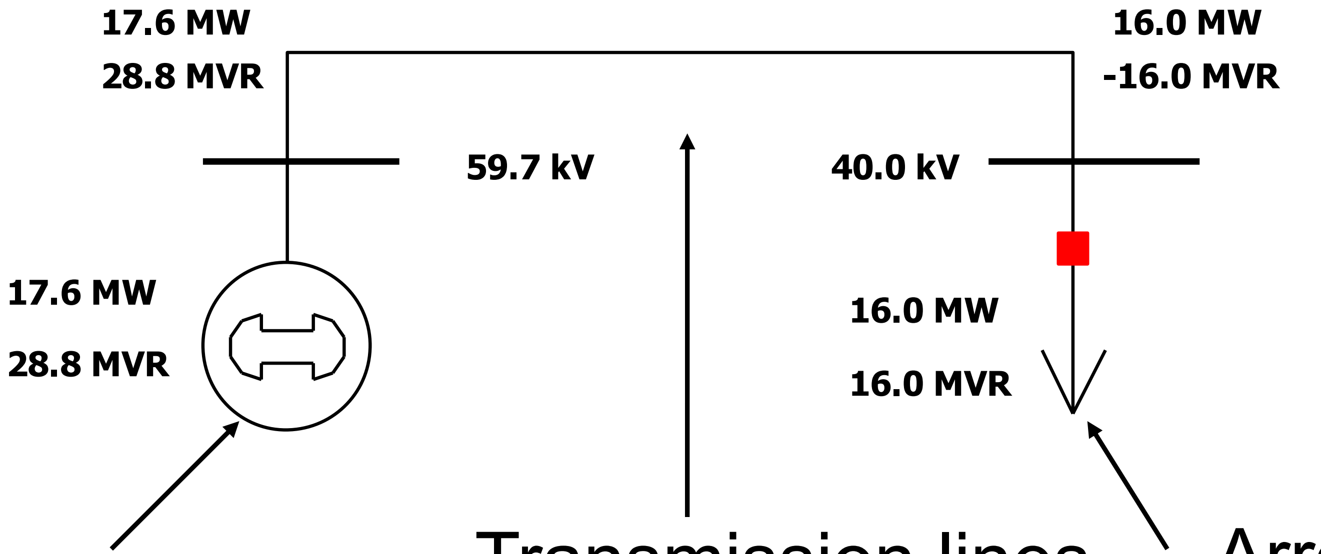


Now add additional reactive power load and resolve

$$Z_{Load} = 70.7\angle 45° \quad pf = 0.7 \text{ lagging}$$
$$I = 564\angle -45° \text{ Amps}$$
$$V = 59.7\angle 13.6° \text{ kV}$$
$$S = 33.7\angle 58.6° \text{ MVA} = 17.6 + j28.8 \text{ MVA}$$

Power System Notation

Power system components are usually shown as “one-line diagrams.” Previous circuit redrawn



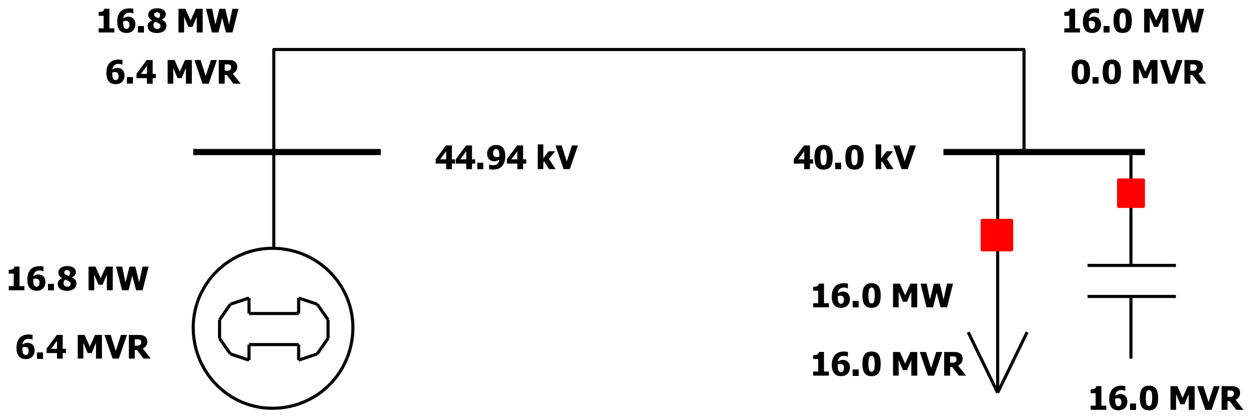
Generators are shown as circles

Transmission lines are shown as a single line

Arrows are used to show loads

Reactive Compensation

- Key idea of reactive compensation is to supply reactive power locally. In the previous example this can be done by adding a 16 Mvar capacitor at the load
- Reactive compensation decreased the line flow from 564 Amps to 400 Amps. This has advantages
  - Lines losses, which are equal to I2 R decrease
  - Lower current allows utility to use small wires, or alternatively, supply more load over the same wires
  - Voltage drop on the line is less



Reactive Compensation, Cont'd

- Reactive compensation is used extensively by utilities
- Capacitors can be used to "correct" a load's power factor to an arbitrary value.
- Distribution-level capacitor shown on right

Power Factor Correction Example

- Assume we have 100 kVA load with pf=0.8 lagging, and would like to correct the pf to 0.95 lagging

$$S = 80 + j60 \text{ kVA} \quad \phi = \cos^{-1} 0.8 = 36.9°$$

PF of 0.95 requires $\phi_{\text{desired}} = \cos^{-1} 0.95 = 18.2°$

$$S_{\text{new}} = 80 + j(60 - Q_{\text{cap}})$$

$$\frac{60 - Q_{\text{cap}}}{80} = \tan 18.2° \quad \Rightarrow \quad 60 - Q_{\text{cap}} = 26.3 \text{ kvar}$$

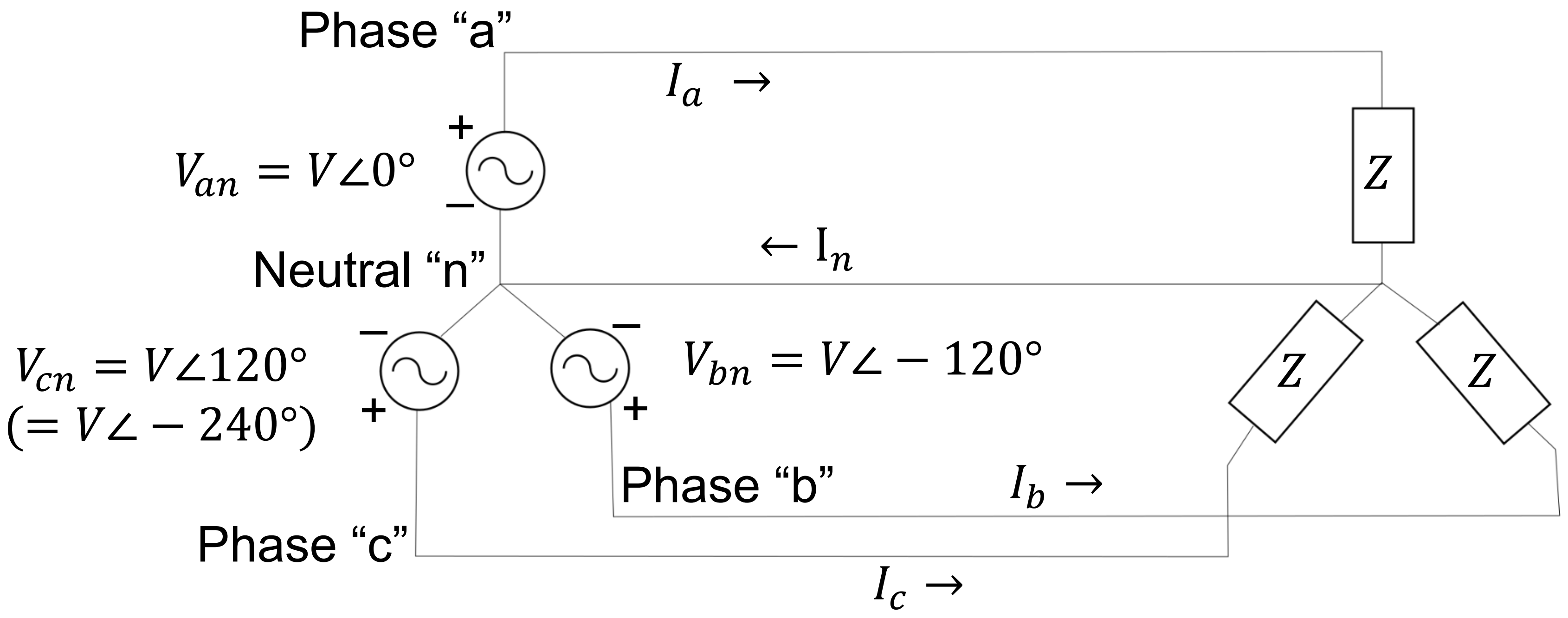
$$Q_{\text{cap}} = 33.7 \text{ kvar}$$

Similar example is given in video linked to website.

Balanced Three-Phase (ϕ) Systems

- A balanced three-phase (ϕ) system has
  - three voltage sources with equal magnitude, but with an angle shift of 120°
  - equal loads on each phase
  - equal impedance on the lines connecting the generators to the loads
- Bulk power systems are almost exclusively 3ϕ
- Single-phase is used primarily only in low voltage, low power settings, such as residential and some commercial
- Advantages include transmitting more power for same amount of wire, and easier starting, better design, and constant torque for machines

Balanced 3ϕ -- No Neutral Current



$$I_n = I_a + I_b + I_c$$

$$I_n = \frac{V}{Z}(1\angle 0° + 1\angle -120° + 1\angle 120°) = 0$$

$$S = V_{an}I_{an}^* + V_{bn}I_{bn}^* + V_{cn}I_{cn}^* = 3V_{an}I_{an}^*$$

Three-Phase Wye Connection

- There are two ways to connect 3ϕ systems
  - Wye (Y)
  - Delta (Δ)

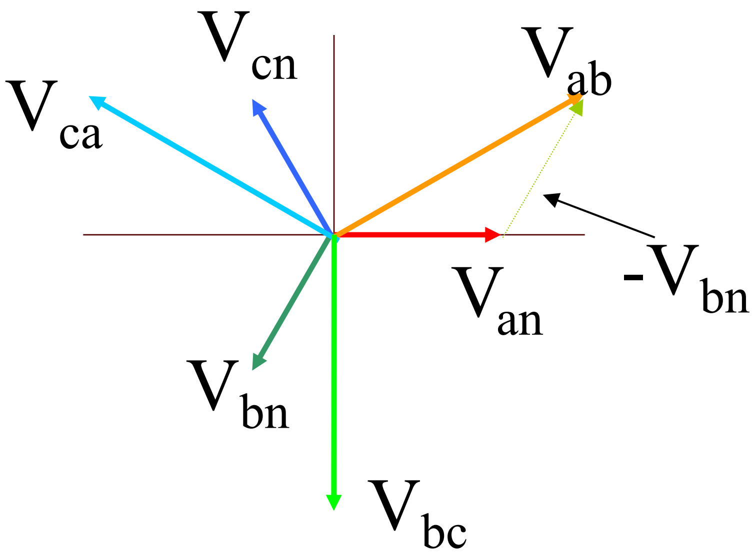
Wye Connection Voltages

$$V_{an} = |V|\angle\alpha^\circ$$

$$V_{bn} = |V|\angle\alpha^\circ - 120^\circ$$

$$V_{cn} = |V|\angle\alpha\angle + 120^\circ$$

Wye Connection Line Voltages



$$V_{ab} = V_{an} - V_{bn} = |V|(1\angle\alpha - 1\angle\alpha + 120°)$$
$$= \sqrt{3}\,|V|\angle\alpha + 30°$$
$$V_{bc} = \sqrt{3}\,|V|\angle\alpha - 90°$$
$$V_{ca} = \sqrt{3}\,|V|\angle\alpha + 150°$$

(α = 0 in this case)

$$V_{ab} = V_{an} - V_{bn} = |V|(1\angle\alpha - 1\angle\alpha + 120°)$$
$$= \sqrt{3}|V|\angle\alpha + 30°$$
$$V_{bc} = \sqrt{3}|V|\angle\alpha + 30°$$
$$V_{ca} = \sqrt{3}|V|$$

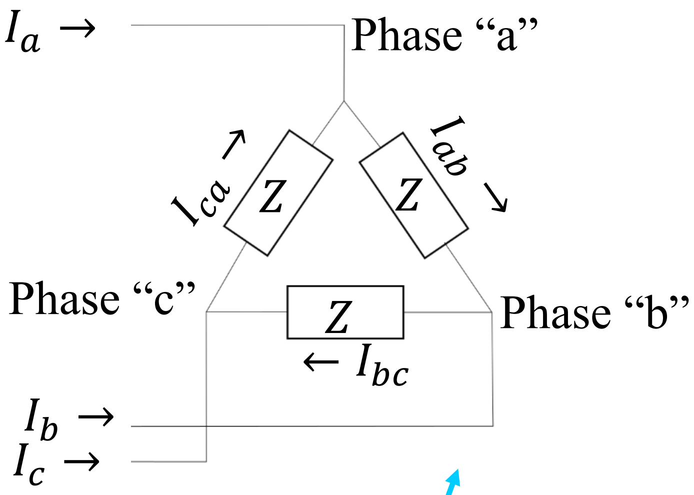
Line to line voltages are also balanced

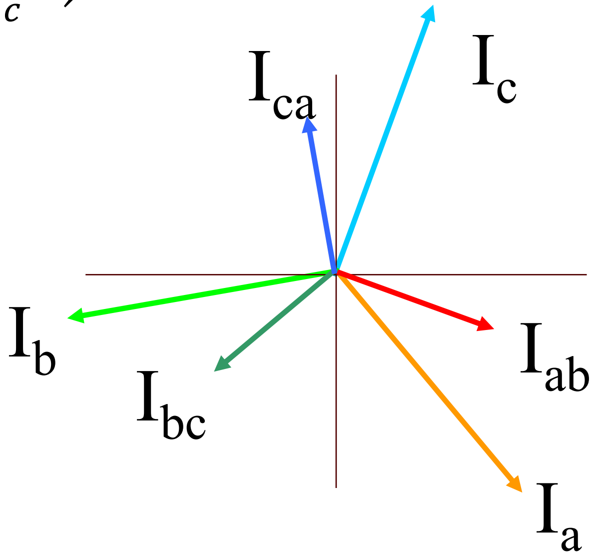
Wye Connection, cont'd

- Phase voltage: across wye-connected device
- Phase current: through wye-connected device
- Line voltage: voltage from one line to another
- Line current: current through a line

$$V_{Line} = \sqrt{3}\ V_{Phase}\ 1\angle 30° = \sqrt{3}\ V_{Phase}\ e^{\frac{j\pi}{6}}$$
$$I_{Line} = I_{Phase}$$
$$S_{3\varphi} = 3\ V_{Phase}\ I^{*}_{Phase}$$

Delta Connection





For the Delta

phase voltages equal

line voltages

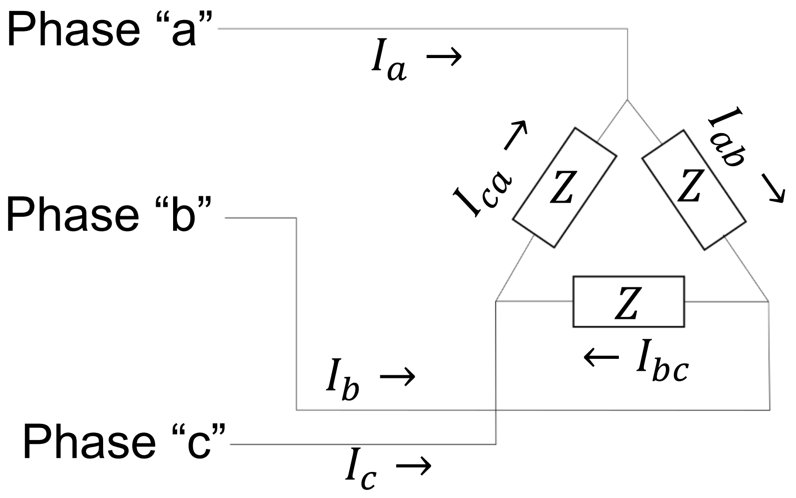
For currents

$$
\begin{aligned}
I_a &= I_{ab} - I_{ca} \\
&= \sqrt{3}\, I_{ab} \angle -30^\circ \\
I_b &= I_{bc} - I_{ab} \\
I_c &= I_{ca} - I_{bc}
\end{aligned}
$$

$$S_{3\phi} = 3\, V_{Phase}\, I^{*}_{Phase}$$

Three-Phase Example

- Assume a Δ-connected load is supplied from a 3ϕ 13.8 kV (L-L) source with Z = 100∠20°Ω



$$V_{ab} = 13.8\angle 0^\circ\ kV$$
$$V_{bc} = 13.8\angle -120^\circ\ kV$$
$$V_{ca} = 13.8\angle 120^\circ\ kV$$

$$I_{ab} = \frac{13.8\angle 0^\circ\ kV}{100\angle 20^\circ\ \Omega} = 138\angle -20^\circ\ amps$$

$$I_{bc} = 138\angle -140^\circ\ amps \qquad I_{ca} = 138\angle 100^\circ\ amps$$

Three-Phase Example, cont'd

$$I_a = I_{ab} - I_{ca} = 138\angle -20° - 138\angle 100°$$
$$= 239\angle -50° \text{ amps}$$
$$I_b = 239\angle -170° \text{ amps} \quad I_c = 239\angle 70° \text{ amps}$$

$$S = 3 \times V_{ab} I_{ab}^{*} = 3 \times 13.8\angle 0° \text{kV} \times 138\angle 20° \text{ amps}$$
$$= 5.7\angle 20° \text{ MVA}$$
$$= 5.37 + j1.95 \text{ MVA}$$
$$\text{pf} = \cos 20° = 0.94 \text{ lagging}$$

Delta-Wye Transformation

To simplify analysis of balanced three-phase systems:

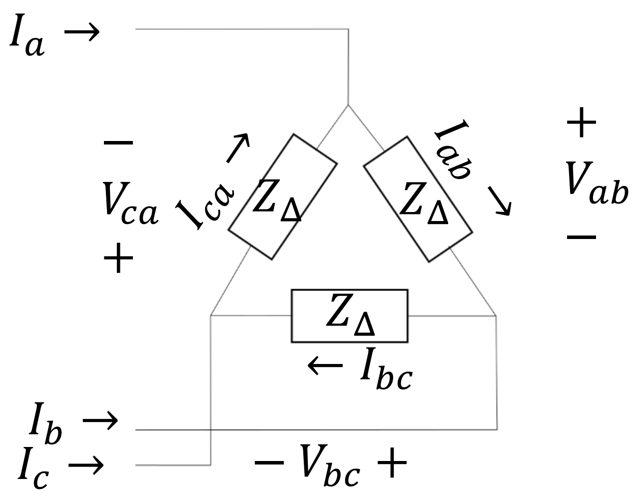
1) Delta connected loads can be replaced by wye-connected loads

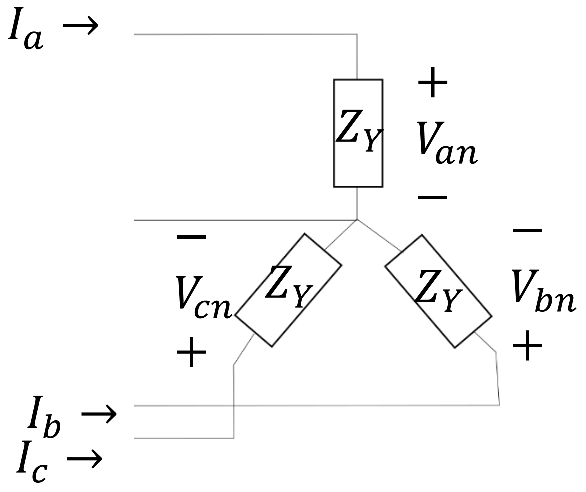
$$Z_Y = \frac{1}{3} Z_\Delta$$

2) Delta-connected sources can be replaced by wye-connected sources with

$$V_{phase} = \frac{V_{line}}{\sqrt{3}\angle 30°}$$

Delta-Wye Transformation Proof





From the Δ side we get

$$I_a = \frac{V_{ab}}{Z_\Delta} - \frac{V_{ca}}{Z_\Delta} = \frac{V_{ab} - V_{ca}}{Z_\Delta}$$

Hence $Z_\Delta = \dfrac{V_{ab} - V_{ca}}{I_a}$

Delta-Wye Transformation, cont'd

From the Y side we get

$$V_{ab} = Z_Y(I_a - I_b) \quad V_{ca} = Z_Y(I_c - I_a)$$

$$V_{ab} - V_{ca} = Z_Y(2I_a - I_b - I_c)$$

Since $\mathrm{I_a} + I_b + I_c = 0 \Rightarrow I_a = -I_b - I_c$

Hence $V_{ab} - V_{ca} = 3\ Z_Y I_a$

$$3\ Z_Y = \frac{V_{ab} - V_{ca}}{I_a} = Z_\Delta$$

Therefore $Z_Y = \frac{1}{3} Z_\Delta$

Per Phase Analysis

- Per phase analysis allows analysis of balanced three-phase systems with the same effort as for a single phase system
- Balanced Three-Phase Theorem: For a balanced three-phase system with
  - All loads and sources Y connected
  - No mutual Inductance between phases
- Then
  - All neutrals are at the same potential
  - All phases are completely decoupled
  - All system values are the same sequence as sources. The sequence order we've been using (phase b lags phase a and phase c lags phase a) is known as "positive" sequence (negative and zero sequence systems are mostly covered in ECEN 459)

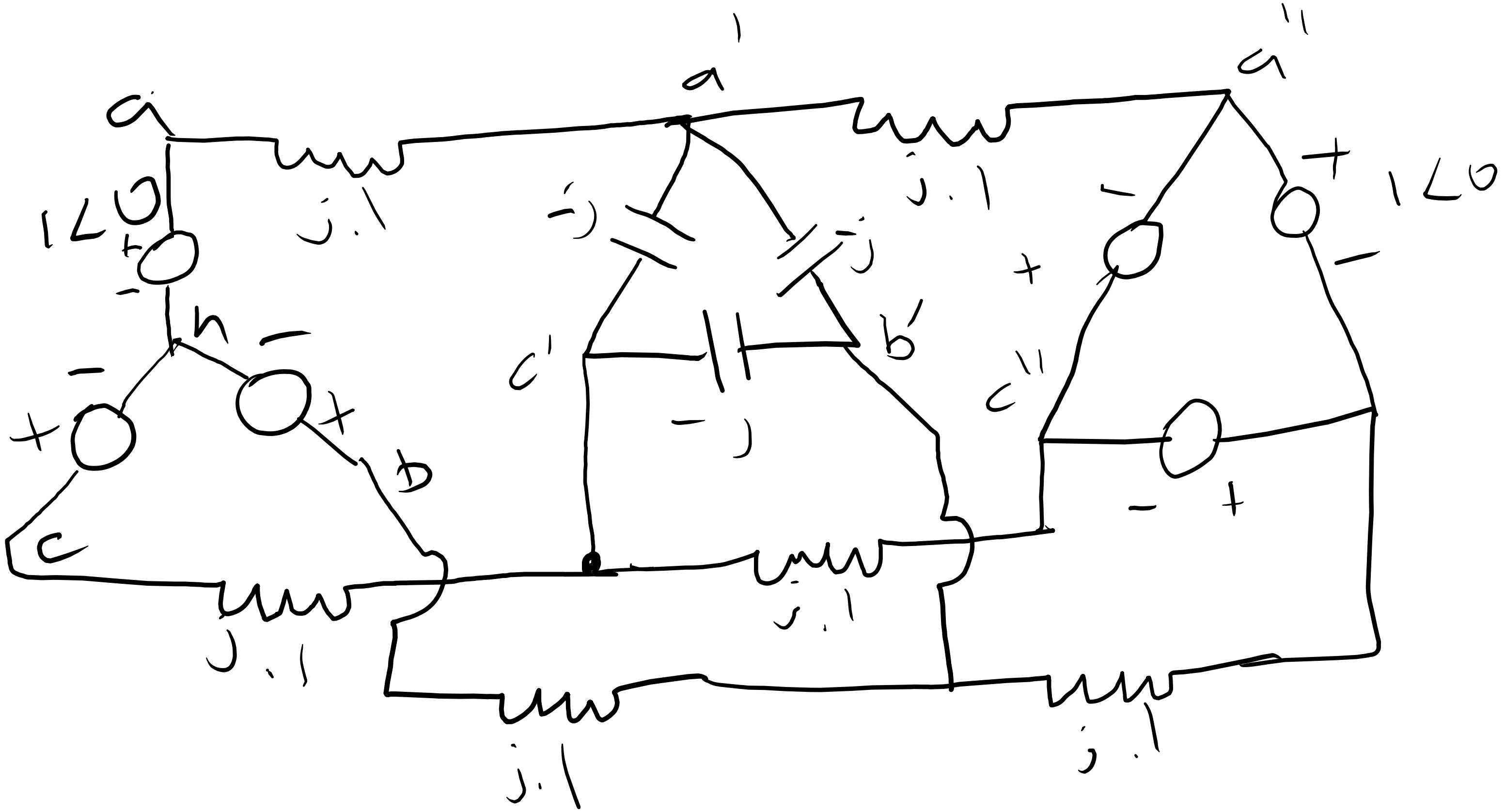
Per Phase Analysis Procedure

- To do per phase analysis
  - Convert all Δ load/sources to equivalent Y's
  - Solve phase "a" independent of the other phases
  - Total system power $S = 3 V_a I_a^*$
  - If desired, phase "b" and "c" values can be determined by inspection (i.e., ±120° degree phase shifts)
  - If necessary, go back to original circuit to determine line-line values or internal Δ values.

Per Phase Example

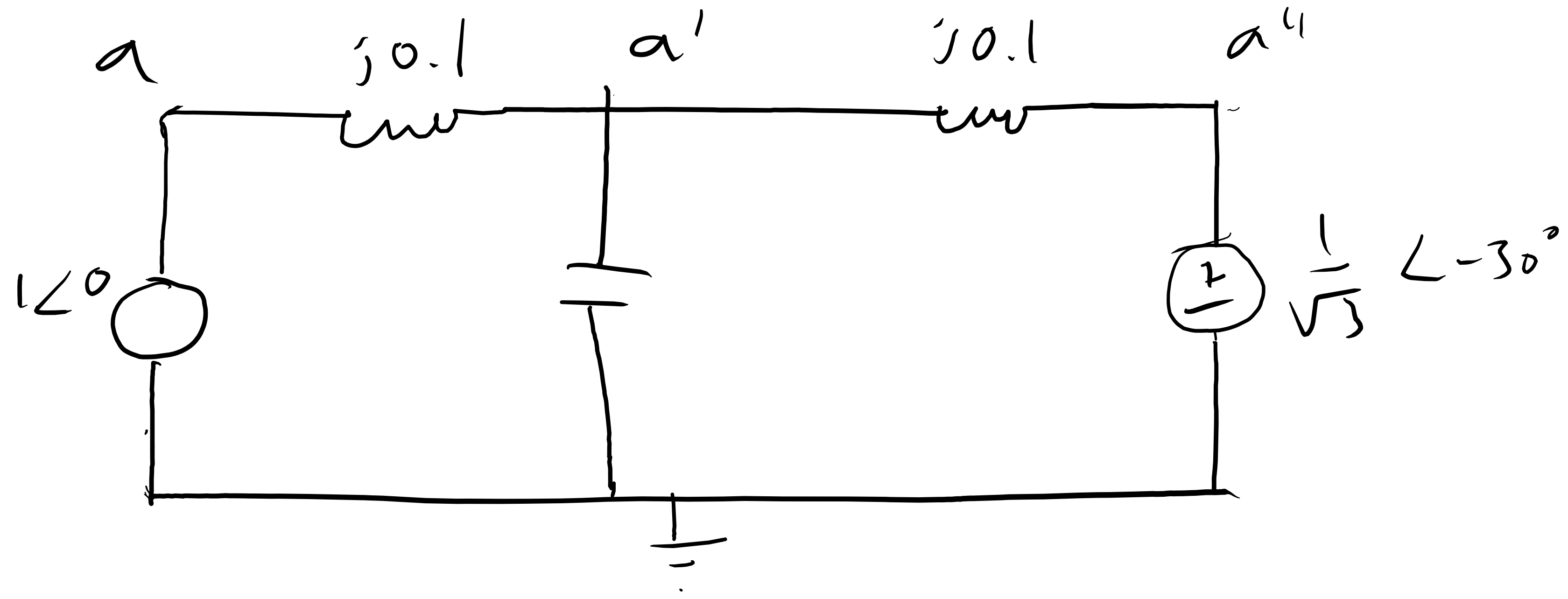
- Assume a 3ϕ, Y-connected generator with Van = 1$\angle$0° volts supplies a Δ-connected load with ZΔ = -jΩ through a transmission line with impedance of j0.1Ω per phase. The load is also connected to a Δ-connected generator with Va"b" = 1$\angle$0° through a second transmission line which also has an impedance of j0.1Ω per phase.
- Find
- 1. The load voltage Va'b'
- 2. The total power supplied by each generator, SY and SΔ

Per Phase Example, cont’d



First convert the delta load and source to equivalent Y values and draw just the "a" phase circuit

Per Phase Example, cont'd 2



To solve the circuit, write the KCL equation at a′

$$(V_a' - 1\angle 0)(-10j) + V_a'(3j) + \left(V_a' - \frac{1}{\sqrt{3}}\angle -30°\right)(-10j) = 0$$

Per Phase Example, cont'd 3

To solve the circuit, write the KCL equation at a′

$$(V_a' - 1\angle 0)(-10j) + V_a'(3j) + (V_a' - \frac{1}{\sqrt{3}}\angle -30°)(-10j) = 0$$

$$(10j + \frac{10}{\sqrt{3}}\angle 60°) = V_a'(10j - 3j + 10j)$$

$V_a' = 0.9\angle -10.9°$ volts $V_b' = 0.9\angle -130.9°$ volts

$V_c' = 0.9\angle 109.1°$ volts $V_{ab}' = 1.56\angle 19.1°$ volts

Per Phase Example, cont'd 4

$$S_{ygen} = 3V_a I_a^* = 3V_a \left(\frac{V_a - V_a'}{j0.1}\right)^*$$

$$= 5.1 + j3.5 \text{ VA}$$

$$S_{\Delta gen} = 3V_a'' \left(\frac{V_a'' - V_a'}{j0.1}\right)^* = -5.1 - j4.7 \text{ VA}$$

Reminders

- Review these topics
  1. Complex number math – by hand and calculator
  2. Phasors – representing AC voltage and current
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  4. Complex power – including power factor, reactive power
  5. Three phase – wye/delta, line-to-line/line-to-neutral voltage, per-phase analysis
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Aggie Core Values and Aggie Honor Code

- Aggie Core Values
  - Respect
  - Excellence
  - Leadership
  - Loyalty
  - Integrity
  - Selfless Service
- Aggie Honor Code: An Aggie does not lie, cheat, or steal, or tolerate those who do.

Reveille and I hope you have a great semester!