

ECEN 667

Power System Stability

Lecture 22: Voltage Stability

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Announcements



- By Nov 20th read the following two papers
 - K. Huang, M. Xiong, Y. Liu and K. Sun, "A Heterogeneous Multiscale Method for Efficient Simulation of Power Systems With Inverter-Based Resources," in *IEEE Transactions on Power Systems*, vol. 40, no. 5, pp. 4292-4306, Sept. 2025.
 - M. Xiong *et al.*, "EMT-TS Hybrid Simulation for Large Power Grids Considering IBR-Driven Dynamics," *IECON 2024 - 50th Annual Conference of the IEEE Industrial Electronics Society*, Chicago, IL, USA, 2024, pp. 1-6.
- HW #5 is on the website, due Nov 20th at 8 AM.
- **Exam 2 will be Tuesday, December 2nd, 2025**

Power System Voltage Stability



- **Voltage Stability:** The ability to maintain system voltage so that both power and voltage are controllable. System voltage responds as expected (i.e., an increase in load causes proportional decrease in voltage).
- **Voltage Instability:** Inability to maintain system voltage. System voltage and/or power become uncontrollable. System voltage does not respond as expected.
- **Voltage Collapse:** Process by which voltage instability leads to unacceptably low voltages in a significant portion of the system. Typically results in loss of system load.

Power System Stability Terms

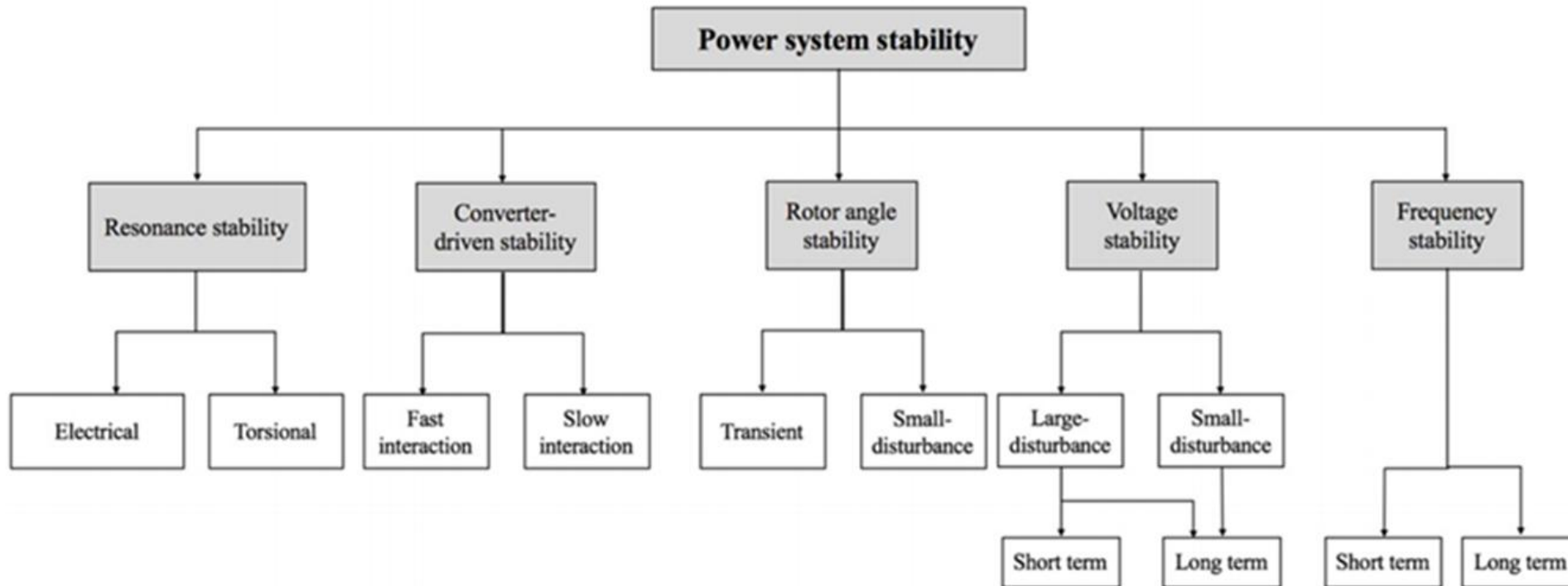


Fig. 4. Classification of power system stability

[a] IEEE/PES Power System Dynamic Performance Committee, “Stability definitions and characterization of dynamic behavior in systems with high penetration of power electronic interfaced technologies”, PES-TR77, April 2020

Voltage Stability

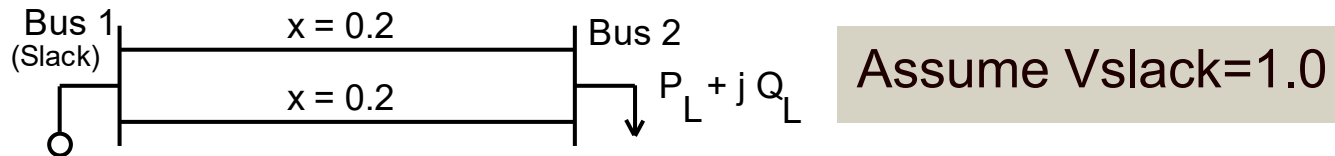


- Two good references are
 - P. Kundur, et. al., “Definitions and Classification of Power System Stability,” *IEEE Trans. on Power Systems*, pp. 1387-1401, August 2004.
 - T. Van Cutsem, “Voltage Instability: Phenomena, Countermeasures, and Analysis Methods,” *Proc. IEEE*, February 2000, pp. 208-227.
- Classified by either size of disturbance or duration
 - Small or large disturbance: small disturbance is just perturbations about an equilibrium point (power flow)
 - Short-term (several seconds) or long-term (many seconds to minutes)

Small Disturbance Voltage Stability



- Small disturbance voltage stability can be assessed using a power flow (maximum loadability)
- Depending on the assumed load model, the power flow can have multiple (or no solutions)
- PV curve is created by plotting power versus voltage



$$P_L - BV \sin \theta = 0$$

$$Q_L + BV \cos \theta - BV^2 = 0$$

Where B is the line susceptance = -10,
 $V \angle \theta$ is the load voltage

Small Disturbance Voltage Stability



- Question: how do the power flow solutions vary as the load is changed?
- A Solution: Calculate a series of power flow solutions for various load levels and see how they change
- Power flow Jacobian

$$\mathbf{J}(\theta, V) = \begin{bmatrix} -BV \cos \theta & -B \sin \theta \\ -BV \sin \theta & B \cos \theta - 2BV \end{bmatrix}$$

$$\det \mathbf{J}(\theta, V) = VB^2(2V \cos \theta - \cos^2 \theta - \sin^2 \theta)$$

$$\text{Singular when } (2V \cos \theta - 1) = 0$$

Maximum Loadability: Singular Power Flow Jacobian



- An important paper considering this was by Sauer and Pai from *IEEE Trans. Power Systems* in Nov 1990, “Power system steady-state stability and the load-flow Jacobian”
- Other earlier papers were looking at the characteristics of multiple power flow solutions
- Work with the power flow optimal multiplier around the same time had shown that optimal multiplier goes to zero as the power flow Jacobian becomes singular
- The power flow Jacobian depends on the assumed load model (we’ll see the impact in a few slides)

Voltage Stability and the Power Flow Jacobian



- The Sauer/Pai paper related system stability to the power flow Jacobian by noting the system dynamics could be written as a set of differential algebraic equations

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y}, \mathbf{p})$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{y}, \mathbf{p})$$

Linearizing about an equilibrium gives

$$\begin{bmatrix} \Delta \dot{\mathbf{x}} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \frac{\partial \mathbf{f}}{\partial \mathbf{x}} & \frac{\partial \mathbf{f}}{\partial \mathbf{y}} \\ \frac{\partial \mathbf{g}}{\partial \mathbf{x}} & \frac{\partial \mathbf{g}}{\partial \mathbf{y}} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{x} \\ \Delta \mathbf{y} \end{bmatrix}$$

Voltage Stability and the Power Flow Jacobian



- Then

Assuming $\frac{\partial \mathbf{g}}{\partial \mathbf{y}}$ is nonsingular then

$$\Delta \dot{\mathbf{x}} = \left[\frac{\partial \mathbf{f}}{\partial \mathbf{x}} - \frac{\partial \mathbf{f}}{\partial \mathbf{y}} \left[\frac{\partial \mathbf{g}}{\partial \mathbf{y}} \right]^{-1} \frac{\partial \mathbf{g}}{\partial \mathbf{x}} \right] \Delta \mathbf{x}$$

- What Sauer and Pai show is if $\frac{\partial \mathbf{g}}{\partial \mathbf{y}}$ is singular then the system is unstable; if $\frac{\partial \mathbf{g}}{\partial \mathbf{y}}$ is nonsingular then the system may or may not be stable
- Hence it provides an upper bound on stability

Bifurcations

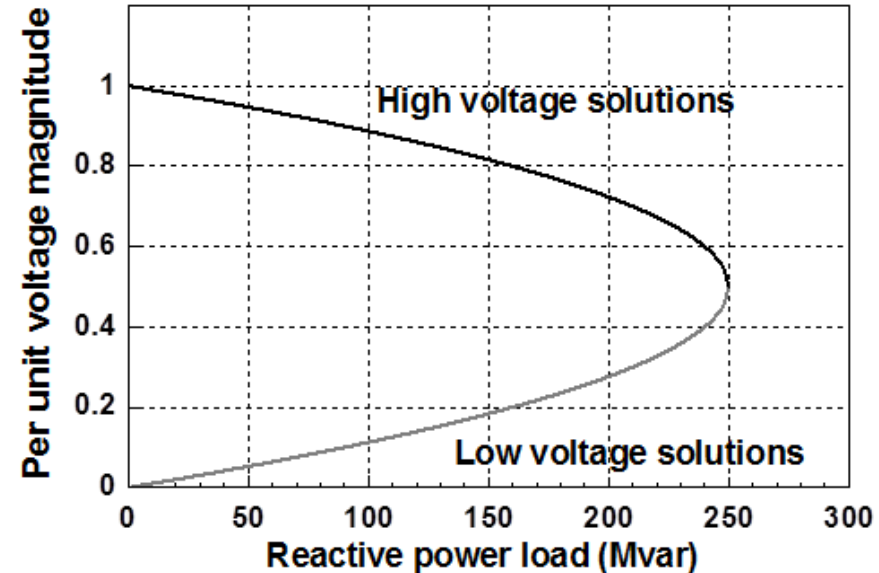
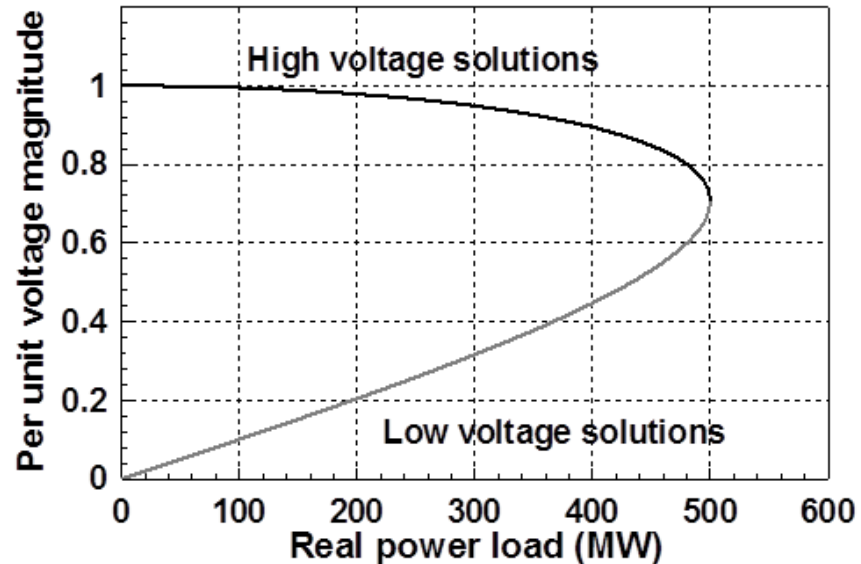


- In general, bifurcation is the division of something into two branches or parts
- For a dynamic system, a bifurcation occurs when small changes in a parameter cause a new quality of motion of the dynamic system
- Two types of bifurcation are considered for voltage stability
 - Saddle node bifurcation is the disappearance of an equilibrium point for parameter variation; for voltage stability it is two power flow solutions coalescing with parameter variation
 - Hopf bifurcation is caused by two eigenvalues crossing into the right-half plane

PV and QV Curves



- PV curves can be traced by plotting the voltage as the real power is increased; QV curves as reactive power is increased
 - At least for the upper portion of the curve
- Two bus example PV and QV curves

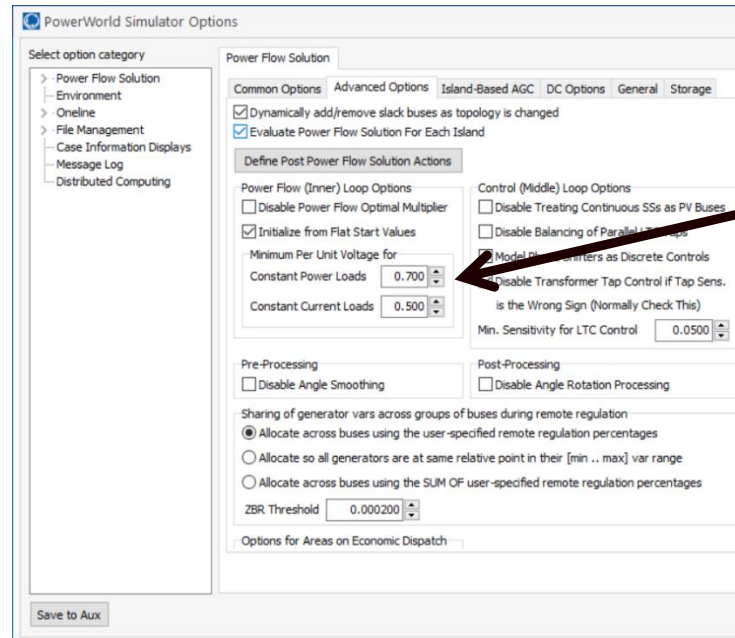
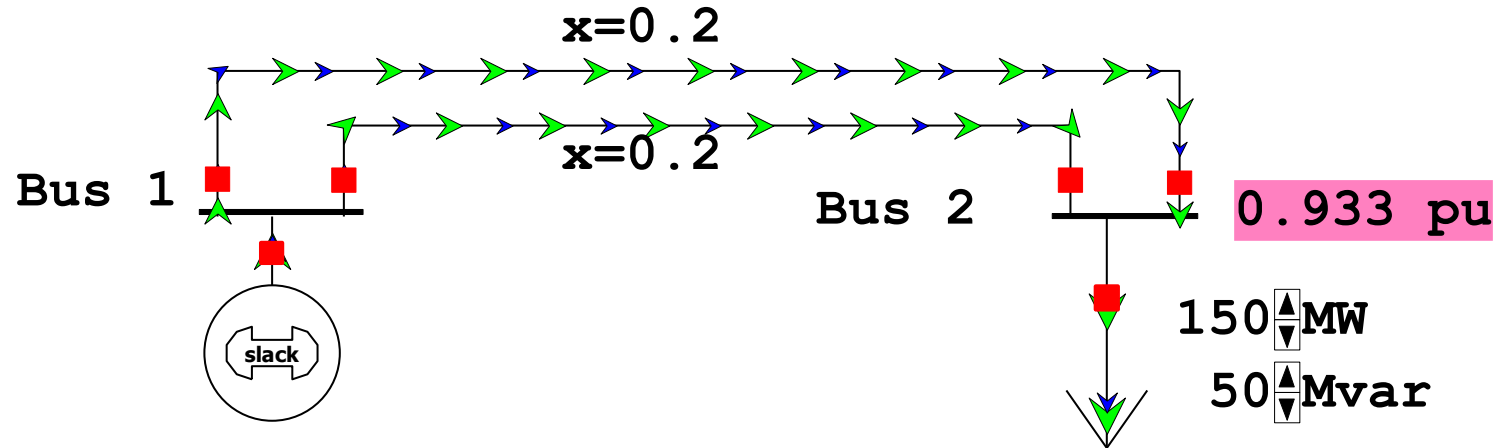


Small Disturbance Voltage Collapse



- At constant frequency (e.g., 60 Hz) the complex power transferred down a transmission line is $S=VI^*$
 - V is phasor voltage, I is phasor current
 - This is the reason for using a high voltage grid
- Line real power losses are given by RI^2 and reactive power losses by XI^2
 - R is the line's resistance, and X its reactance; for a high voltage line $X \gg R$
- Increased reactive power tends to drive down the voltage, which increases the current, which further increases the reactive power losses

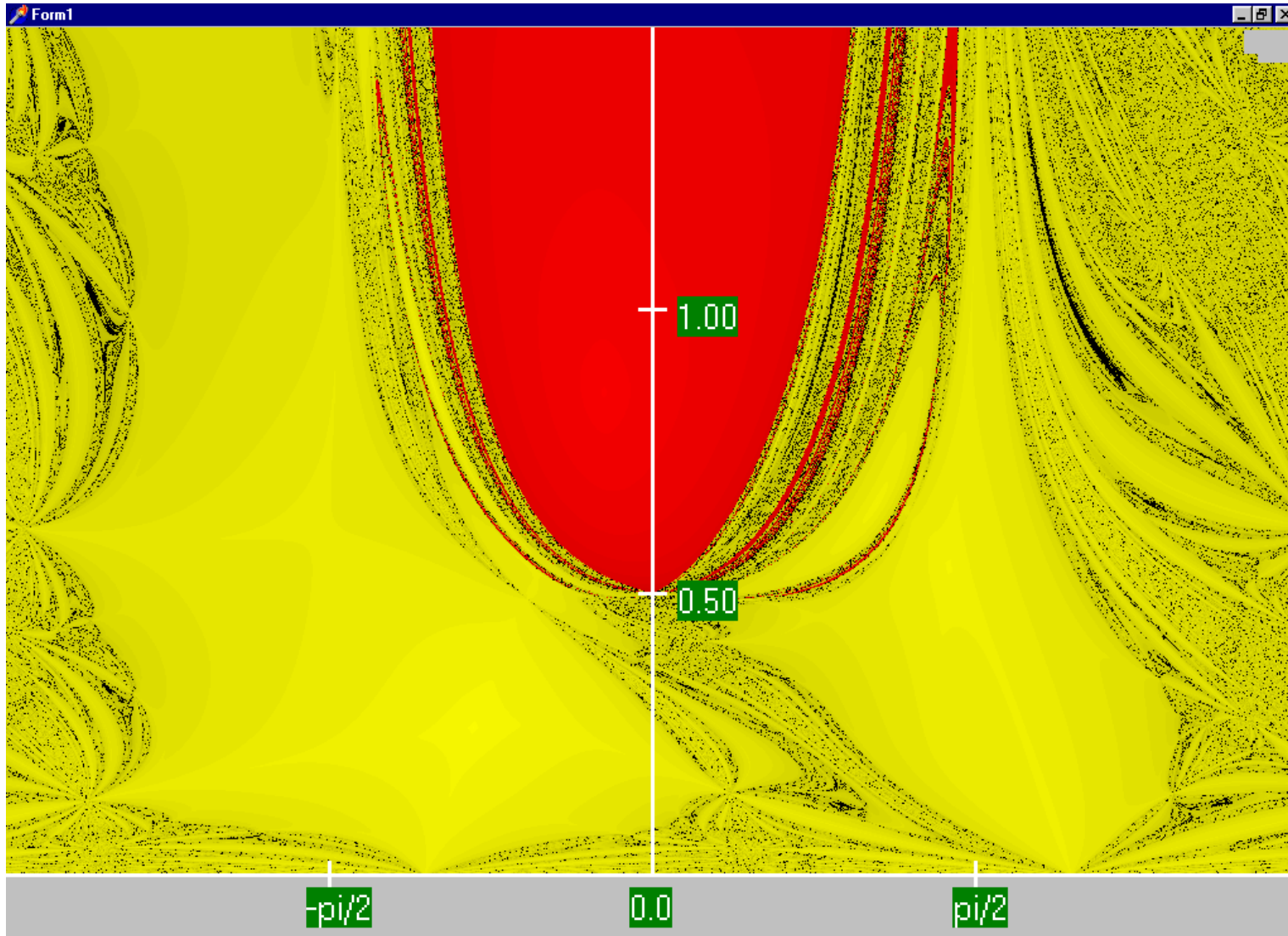
PowerWorld Two Bus Example



Commercial power flow software usually auto converts constant power loads at low voltages; set these fields to zero to disable this conversion

Case is Bus2_PV

Power Flow Region of Convergence



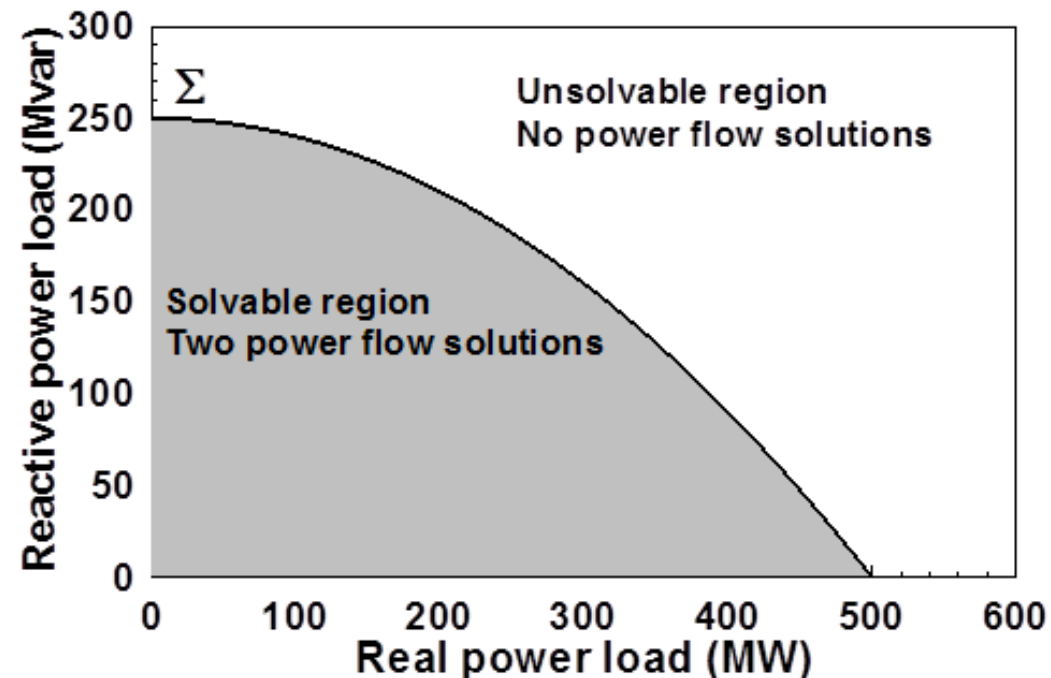
Convergence
regions with
 $P=100$ MW,
 $Q=0$ Mvar

Load Parameter Space Representation



- With a constant power model there is a maximum loadability surface, Σ
 - Defined as point in which the power flow Jacobian is singular
 - For the lossless two bus system it can be determined as

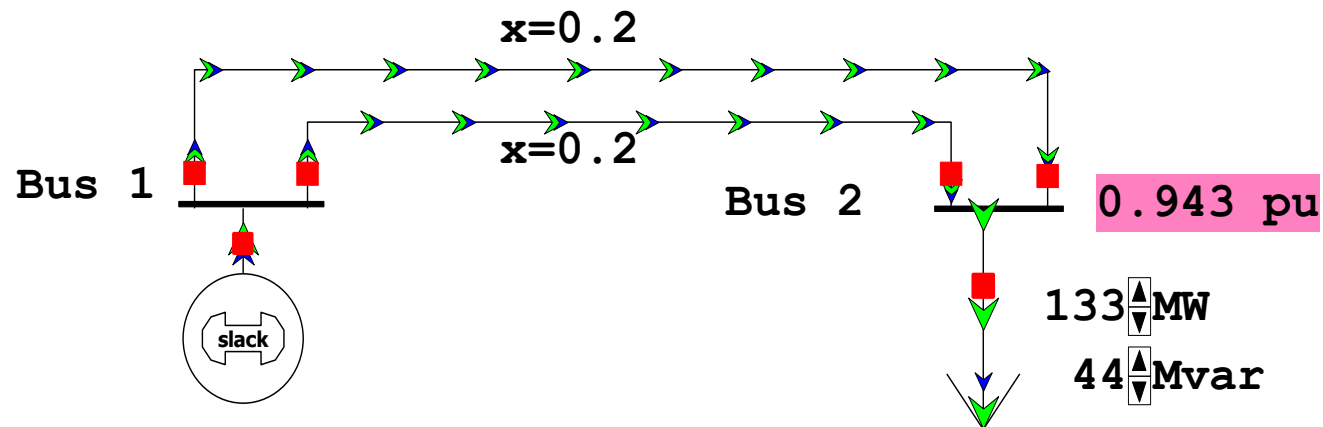
$$-\frac{P_L^2}{B} + Q_L + \frac{1}{4}B = 0$$



Load Model Impact



- With a static load model regardless of the voltage dependency the same PV curve is traced
 - But whether a point of maximum loadability exists depends on the assumed load model
 - If voltage exponent is > 1 then multiple solutions do not exist (see B.C. Lesieutre, P.W. Sauer and M.A. Pai “Sufficient conditions on static load models for network solvability,” NAPS 1992, pp. 262-271)



Change the load to constant impedance; hence it becomes a linear model

ZIP Model Coefficients



- One popular static load model is the ZIP; lots of papers on the “correct” amount of each type

TABLE VII

ACTIVE AND REACTIVE ZIP MODEL. FIRST HALF OF THE ZIPS WITH 100-V CUTOFF VOLTAGE. SECOND HALF REPORTS THE ZIPS WITH ACTUAL CUTOFF VOLTAGE

Equipment/ component	No. tested	V_{cut}	V_o	P_o	Q_o	Z_p	I_p	P_p	Z_q	I_q	P_q
Air compressor 1 Ph	1	100	120	1109.01	487.08	0.71	0.46	-0.17	-1.33	4.04	-1.71
Air compressor 3 Ph	1	174	208	1168.54	844.71	0.24	-0.23	0.99	4.79	-7.61	3.82
Air conditioner	2	100	120	496.33	125.94	1.17	-1.83	1.66	15.68	-27.15	12.47
CFL bulb	2	100	120	25.65	37.52	0.81	-1.03	1.22	0.86	-0.82	0.96
Coffeemaker	1	100	120	1413.04	13.32	0.13	1.62	-0.75	3.89	-6	3.11
Copier	1	100	120	944.23	84.57	0.87	-0.21	0.34	2.14	-3.67	2.53
Electronic ballast	3	100	120	59.02	5.06	0.22	-0.5	1.28	9.64	-21.59	12.95
Elevator	3	174	208	1381.17	1008.3	0.4	-0.72	1.32	3.76	-5.74	2.98
Fan	2	100	120	163.25	83.28	-0.47	1.71	-0.24	2.34	-3.12	1.78
Game consol	3	100	120	60.65	67.61	-0.63	1.23	0.4	0.76	-0.93	1.17
Halogen	3	100	120	97.36	0.84	0.46	0.64	-0.1	4.26	-6.62	3.36
High pressure sodium HID	4	100	120	276.09	52.65	0.09	0.7	0.21	16.6	-28.77	13.17
Incandescent light	2	100	120	87.16	0.85	0.47	0.63	-0.1	0.55	0.38	0.07
Induction light	1	100	120	44.5	4.8	2.96	-6.04	4.08	1.48	-1.29	0.81
Lanton charger	1	100	120	35.94	71.64	-0.28	0.5	0.78	-0.37	1.24	0.13

TABLE I

ZIP COEFFICIENTS FOR EACH CUSTOMER CLASS

Class	Z_p	I_p	P_p	Z_q	I_q	P_q
Large commercial	0.47	-0.53	1.06	5.30	-8.73	4.43
Small commercial	0.43	-0.06	0.63	4.06	-6.65	3.59
Residential	0.85	-1.12	1.27	10.96	-18.73	8.77
Industrial	0	0	1	0	0	1

Table 1 from M. Diaz-Aguilo, et. al., “Field-Validated Load Model for the Analysis of CVR in Distribution Secondary Networks: Energy Conservation,” IEEE Trans. Power Delivery, Oct. 2013

ZIP Model Coefficients

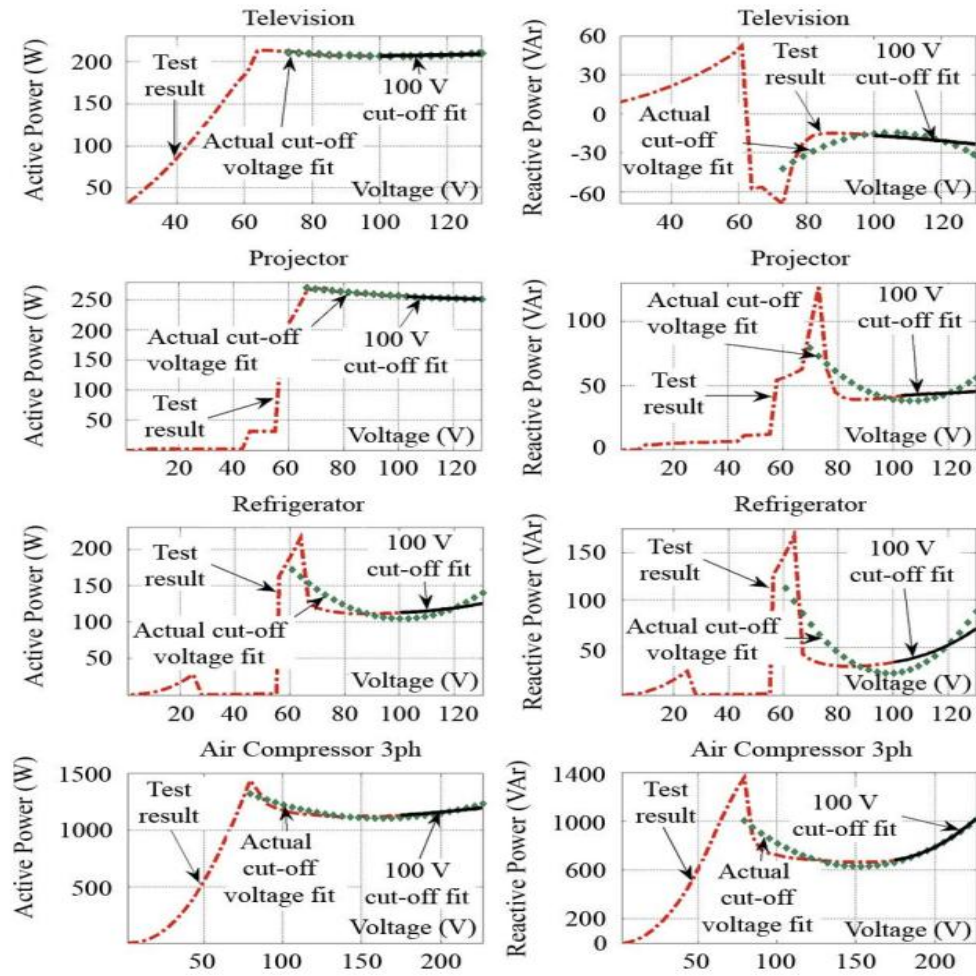


Fig. 3. Active and reactive test results with constrained curve fitting. The ZIP curve with the 100-V cutoff is shown in the solid line and ZIP with the actual cutoff voltage in the dashed line. The two sets of ZIPs are shown in Table VII.

Figure 3 from A, Bokhari, et. al.,
 “Experimental Determination of the
 ZIP Coefficients for Modern
 Residential, Commercial, and
 Industrial Loads,” IEEE Trans.
 Power Delivery, June. 2014

Application: Conservation Voltage Reduction (CVR)



- If the “steady-state” load has a true dependence on voltage, then a change (usually a reduction) in the voltage should result in a total decrease in energy consumption
- If an “optimal” voltage could be determined, then this could result in a net energy savings
- Some challenges are 1) the voltage profile across a feeder is not constant, 2) the load composition is constantly changing, 3) a decrease in power consumption might result in a decrease in useable output from the load, and 4) loads are dynamic and an initial decrease might be balanced by a later increase

CVR Issues

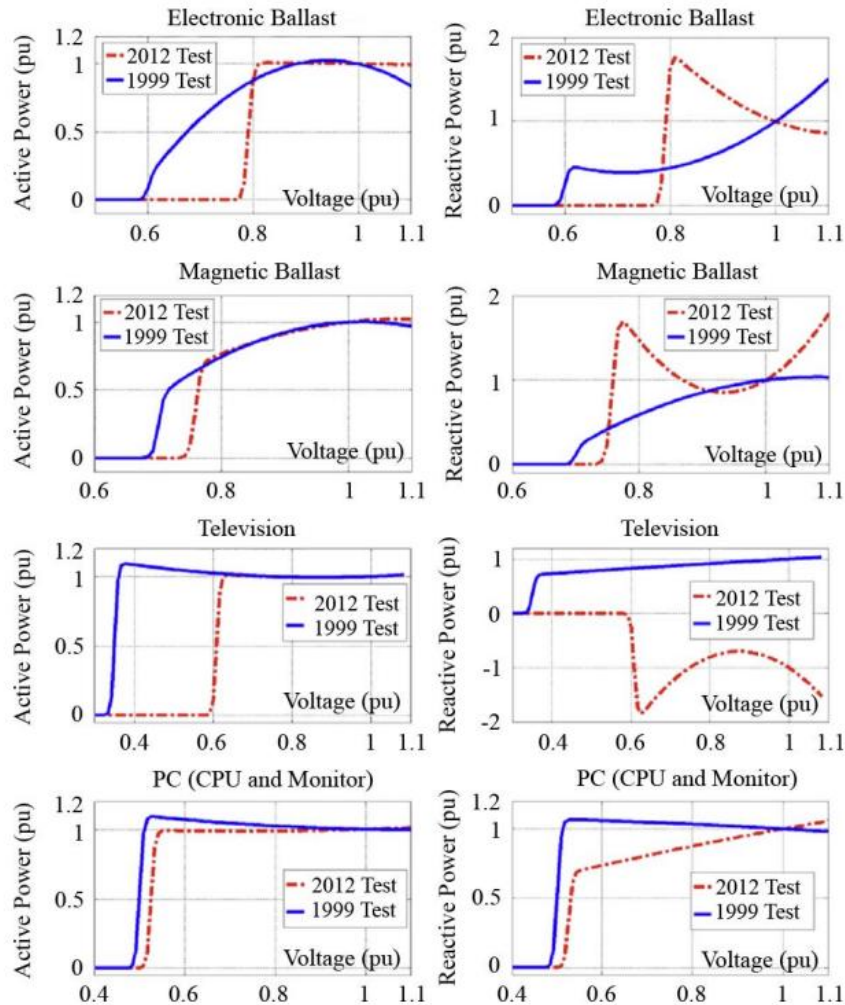


Fig. 4. Comparison of active and reactive powers between old and new appliances.

Conservation Voltage Reduction with Distributed Energy Resource Management System, Grid-Edge, and Legacy Devices

Preprint

Harsha Padullaparti, Murali Baggu, Jing Wang, Ismael Mendoza, Soumya Tiwari, Jiyu Wang, and Santosh Veda

National Renewable Energy Laboratory

Suggested Citation

Padullaparti, Harsha, Murali Baggu, Jing Wang, Ismael Mendoza, Soumya Tiwari, Jiyu Wang, and Santosh Veda. 2023. *Conservation Voltage Reduction with Distributed Energy Resource Management System, Grid-Edge, and Legacy Devices: Preprint*. Golden, CO: National Renewable Energy Laboratory, NREL/CP-5D00-84633. <https://www.nrel.gov/docs/fv23osti/84633.pdf>

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The gist of the 2023 NREL paper is distributed generation (like PV) can help with CVR through better feeder voltage regulation



Figure 4 from A, Bokhari, et. al., “Experimental Determination of the ZIP Coefficients for Modern Residential, Commercial, and Industrial Loads,” IEEE Trans. Power Delivery, June. 2014

Dynamic Load Response



- As first reported in the below paper, following a change in voltage there will be a dynamic load response
 - Residential supply voltage should be between 114 and 126 V
- If there is a heating load the response might be on the order of ten minutes
- Longer term issues can also come into play

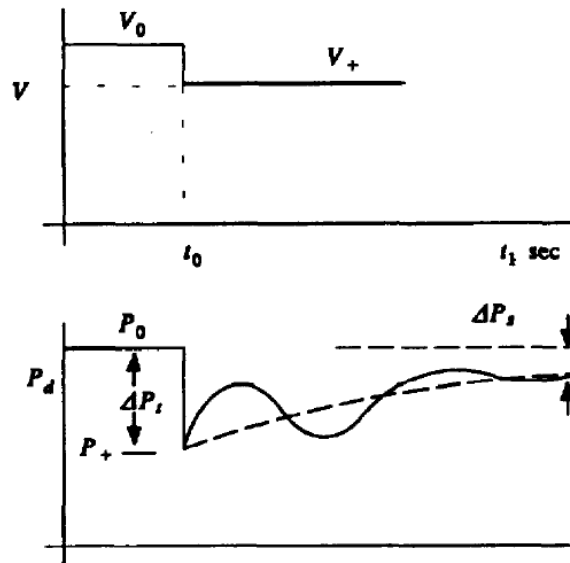


Figure 2.2 General Load Response

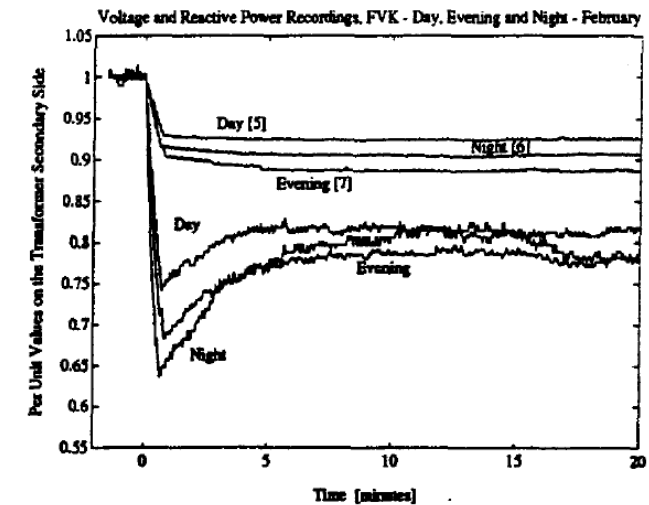


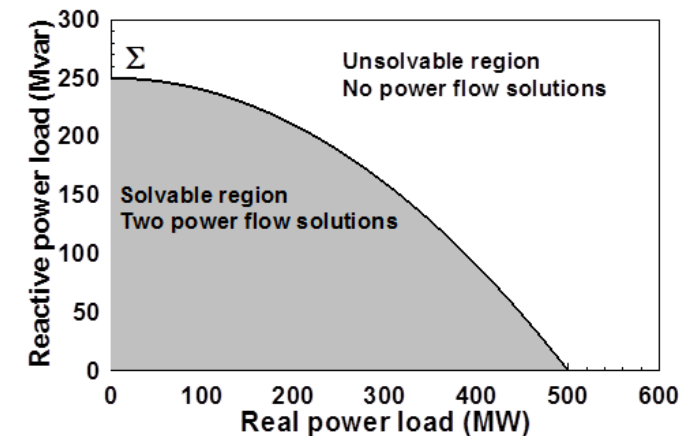
Figure 4.4b. Voltage and reactive power recordings after a voltage reduction in FVK; February; day, evening and night. The number of top-changer steps used for the ramp is shown in brackets.

Useful paper and figure reference: D. Karlsson, D.J. Hill, "Modeling and Identification of Nonlinear Dynamic Loads in Power Systems," IEEE. Trans. on Power Systems, Feb 1994, pp. 157-166

Determining a Metric to Voltage Collapse



- The goal of much of the voltage stability work was to determine an easy to calculate metric (or metrics) of the current operating point to voltage collapse
 - PV and QV curves (or some combination) can determine such a metric along a particular path
 - Goal was to have a path independent metric. The closest boundary point was considered, but this could be quite misleading if the system was not going to move in that direction
 - Any linearization about the current operating point (i.e., the Jacobian) does not consider important nonlinearities like generators hitting their reactive power limits



Assessing Voltage Margin Using PV and QV Curve Analysis



- A common method for assessing the distance in parameter space to voltage instability (or an undesirable voltage profile) is to trace how the voltage magnitudes vary as the system parameters (such as the loads) are changed in a specified direction
 - If the direction involves changing the real power (P) this is known as a PV curve; if the change is with the reactive power (Q) then this is a QV curve
- PV/QV curve analysis can be generalized to any parameter change, and can include the consideration of contingencies

PV and QV Analysis in PowerWorld



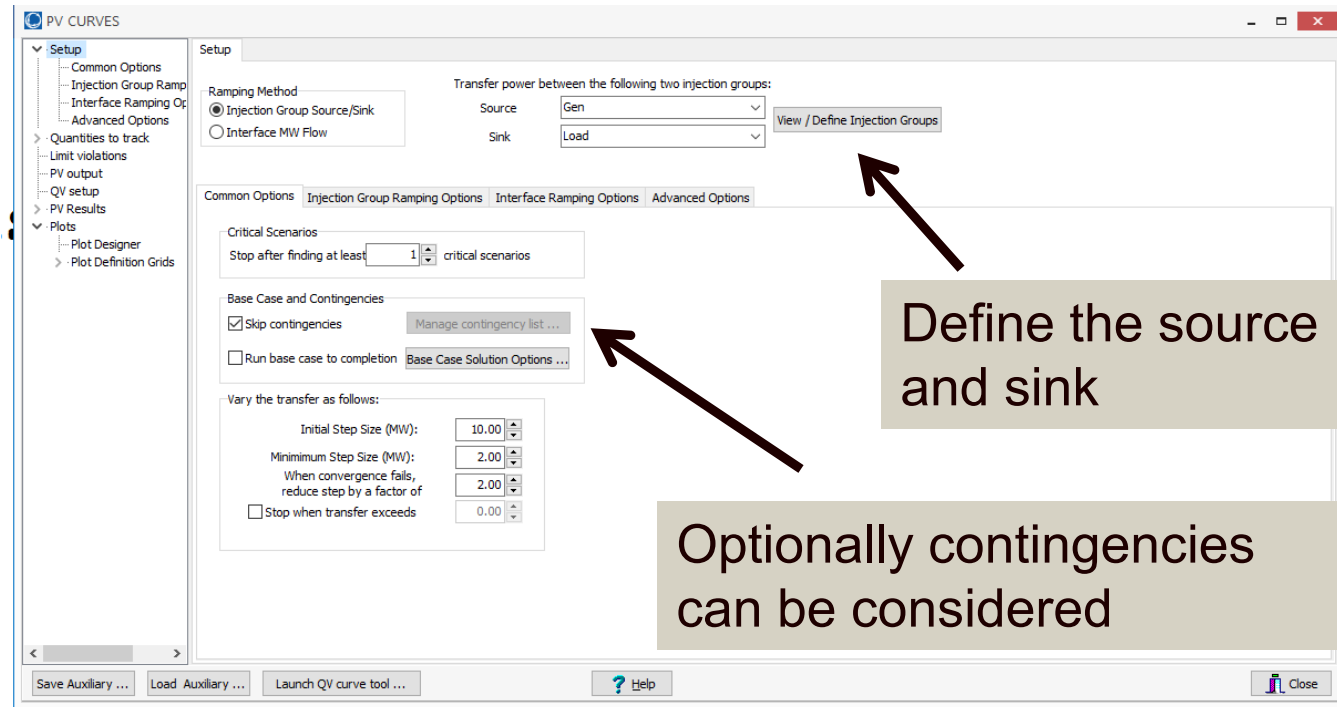
- Requires setting up what is known in PowerWorld as an injection group
 - An injection group specifies a set of objects, such as generators and loads, that can inject or absorb power
 - Injection groups can be defined by selecting **Case Information, Aggregation, Injection Groups**
- The PV and/or QV analysis then varies the injections in the injection group, tracing out the PV curve
- This allows optional consideration of contingencies
- The PV tool can be displayed by selecting **Add-Ons, PV**

This has already been done in the Bus2_PV case

PV and QV Analysis in PowerWorld: Two Bus Example



- Setup page defines the source and sink and step size



PV and QV Analysis in PowerWorld: Two Bus Example



- The PV Results Page does the actual solution
 - Plots can be defined to show the results
 - This should be done beforehand
 - **Other Actions, Restore initial state** restores the pre-study state

Base case could not be solved

Present nominal shift: 0.000

Present step size:

Found 1 limiting case.

Source	Gen MW	Load SMW	Load IMW	Load ZMW
Sink	150.00	0.00	0.00	0.00
	0.00	150.00	0.00	0.00

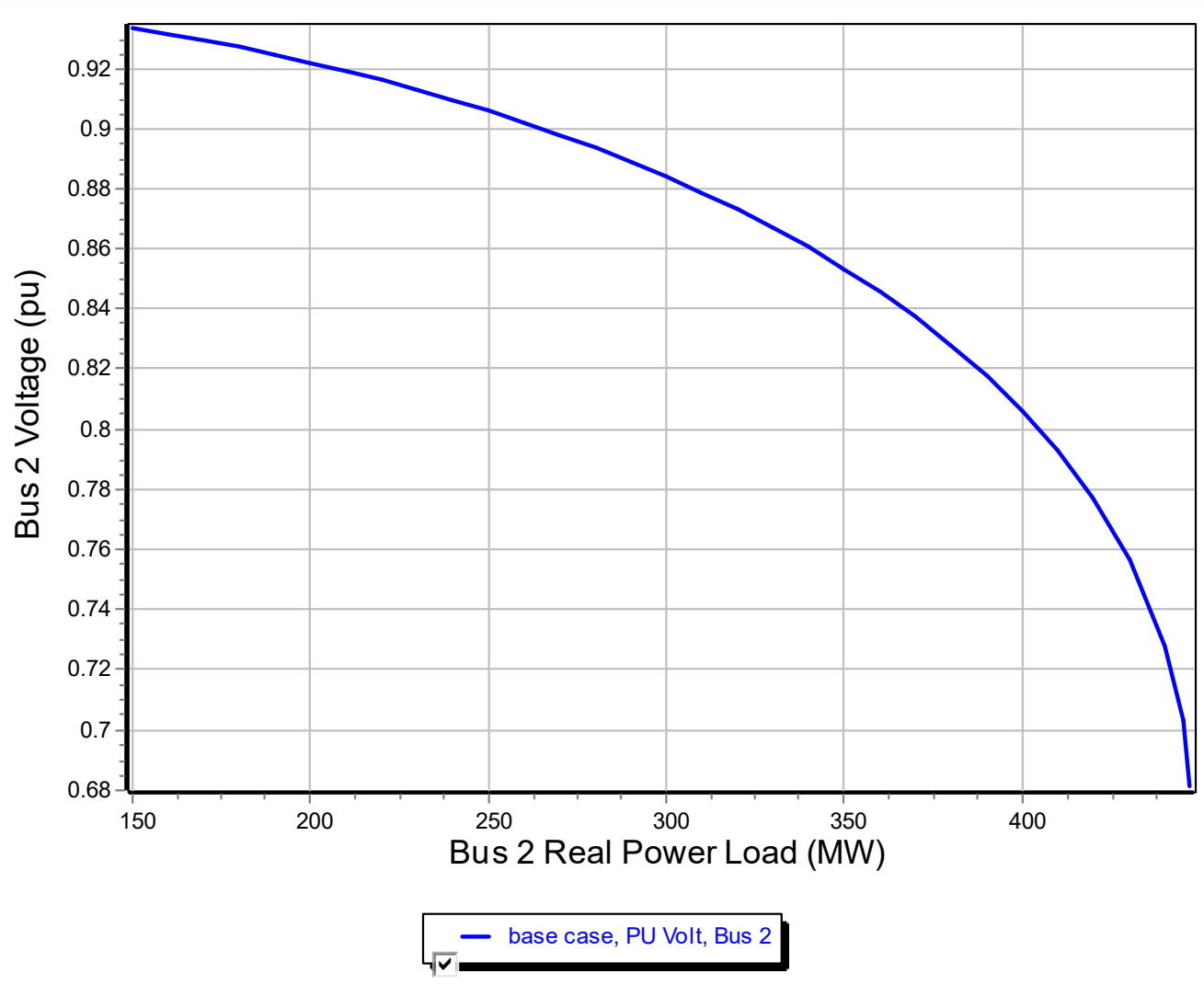
Overview Legacy Plots Track Limits

Scenario	Critical?	Critical Reason	Max Shift	Max Export	Max Import	# Viol	Worst V Viol	Worst V Bus
1 base case	YES	Reached Nose	297.00	297.04	-297.00	0		

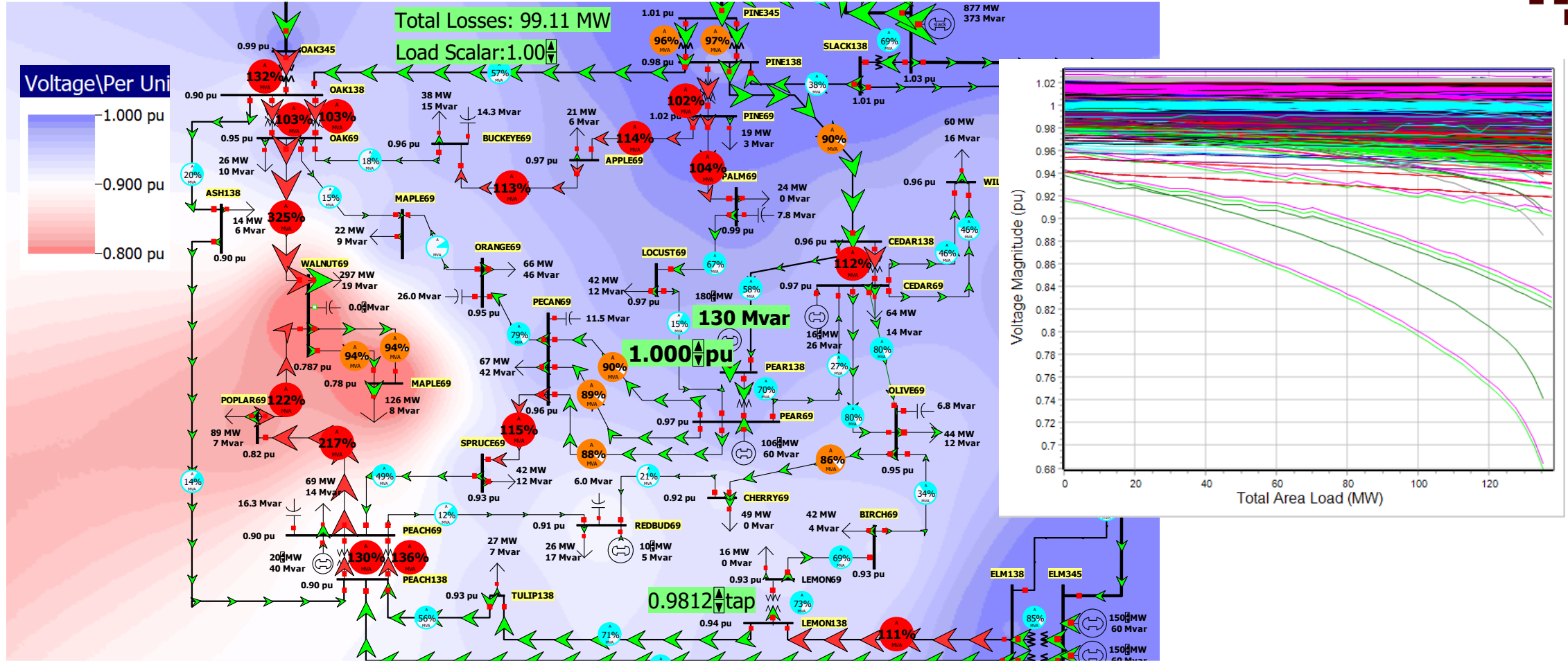
Click the Run button to run the PV analysis;

Check the Restore Initial State on Completion of Run to restore the pre-PV state (by default it is not restored)

PV and QV Analysis in PowerWorld: Two Bus Example



PV and QV Analysis in PowerWorld: 37 Bus Example



Usually other limits also need to be considered in doing a realistic PV analysis;
example case is Bus37_PV