ECEN 667
Power System Stability

Lecture 6: Synchronous Machine Modeling, Part 2

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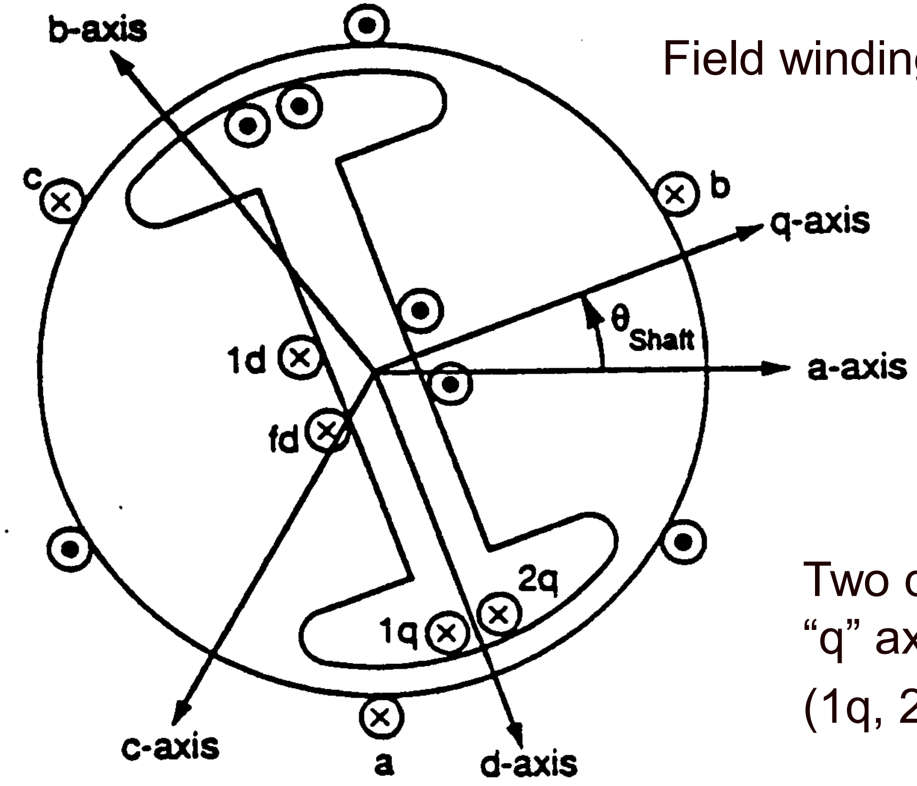
Announcements

- Homework Assignment #2 is due Thursday, Sept. 25th at 8 AM. Email me your solution as a single PDF.
- Read book chapters 3, 4, and 5
- Review the slides and PowerWorld examples

The Main Diagram

3ϕ bal. windings (a,b,c) – stator



Field winding (fd) on rotor

Damper in
"d" axis
(1d) on rotor

Two dampers in
"q" axis
(1q, 2q) on rotor

Full Per-Unit Model, Labeled

$$\frac{1}{\omega_s}\frac{d\psi_d}{dt} = V_d + I_d R_s + (\omega + 1)\psi_q$$

$$\frac{1}{\omega_s}\frac{d\psi_q}{dt} = V_q + I_q R_s - (\omega + 1)\psi_d$$

$$\frac{1}{\omega_s}\frac{d\psi_0}{dt} = V_0 + I_0 R_s$$

Stator Windings

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - (X_d - X'_d)\left[I_d - \frac{(X'_d - X''_d)}{(X'_d - X_{\ell s})^2}\left(\psi_{1d} + (X'_d - X_{\ell s})I_d - E'_q\right)\right] + E_{fd}$$

$$T''_{do}\frac{d\psi_{1d}}{dt} = -\psi_{1d} + E'_q - (X'_d - X_{\ell s})I_d$$

Rotor Windings

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + (X_q - X'_q)\left[I_q - \frac{X'_q - X''_q}{(X'_q - X_{\ell s})^2}\left(\psi_{2q} + (X'_q - X_{\ell s})I_q + E'_d\right)\right]$$

$$T''_{qo}\frac{d\psi_{2q}}{dt} = -\psi_{2q} - E'_d - (X'_q - X_{\ell s})I_q$$

$$\frac{d\delta}{dt} = \omega \cdot \omega_s$$

$$2H\frac{d\omega}{dt} = T_M + (\psi_q I_d - \psi_d I_q) - T_{FW}$$

Mechanical equations

Stator flux definitions

$$\psi_d = -X''_d I_d + \frac{X''_d - X_{\ell s}}{X'_d - X_{\ell s}}E'_q + \frac{X'_d - X''_d}{X'_d - X_{\ell s}}\psi_{1d}$$

$$\psi_q = -X''_q I_q - \frac{X''_q - X_{\ell s}}{X'_q - X_{\ell s}}E'_d + \frac{X'_q - X''_q}{X'_q - X_{\ell s}}\psi_{2q}$$

$$\psi_0 = -X_{\ell s} I_0$$

Balanced Operation

- Consider a balanced set of scaled sinusoidal voltages and currents

$$V_a = \sqrt{2}\, V_s \cos(\omega_s t + \theta_s) \qquad I_a = \sqrt{2}\, I_s \cos(\omega_s t + \phi_s)$$

$$V_b = \sqrt{2}\, V_s \cos\left(\omega_s t + \theta_s - \frac{2\pi}{3}\right) \qquad I_b = \sqrt{2}\, I_s \cos\left(\omega_s t + \phi_s - \frac{2\pi}{3}\right)$$

$$V_c = \sqrt{2}\, V_s \cos\left(\omega_s t + \theta_s + \frac{2\pi}{3}\right) \qquad I_c = \sqrt{2}\, I_s \cos\left(\omega_s t + \phi_s + \frac{2\pi}{3}\right)$$

- Applying Park's transformation and the definition of δ we get

$$V_d = V_s \sin(\delta - \theta_s) \qquad I_d = I_s \sin(\delta - \phi_s)$$

$$V_q = V_s \cos(\delta - \theta_s) \qquad I_q = I_s \cos(\delta - \phi_s)$$

- Which we can write compactly as two complex equations

$$\left. \begin{aligned} (V_d + jV_q)e^{j\left(\delta - \frac{\pi}{2}\right)} &= V_s \angle \theta_s \\ (I_d + jI_q)e^{j\left(\delta - \frac{\pi}{2}\right)} &= I_s \angle \phi_s \end{aligned} \right\}$$

We use this as our "reference frame transformation"
Instead of the full Park's transformation in stability studies

Steady-State Operation

$$0 = V_d + I_d R_s + (\omega + 1)\psi_q$$
$$0 = V_q + I_q R_s - (\omega + 1)\psi_d$$
$$0 = V_0 + I_0 R_s$$

Stator Windings

$$0 = -E'_q - (X_d - X'_d)\left[I_d - \cancel{\frac{(X'_d - X''_d)}{(X'_d - X_{\ell s})^2}\left(\psi_{1d} + (X'_d - X_{\ell s})I_d - E'_q\right)}\right] + E_{fd}$$
$$0 = -\psi_{1d} + E'_q - (X'_d - X_{\ell s})I_d$$
$$0 = -E'_d + (X_q - X'_q)\left[I_q - \cancel{\frac{X'_q - X''_q}{(X'_q - X_{\ell s})^2}\left(\psi_{2q} + (X'_q - X_{\ell s})I_q + E'_d\right)}\right]$$
$$0 = -\psi_{2q} - E'_d - (X'_q - X_{\ell s})I_q$$

Rotor Windings

$$0 = \omega \cdot \omega_s$$
$$0 = T_M + (\psi_q I_d - \psi_d I_q) - T_{FW}$$

Mechanical equations

$$\psi_d = -X''_d I_d + \frac{X''_d - X_{\ell s}}{X'_d - X_{\ell s}} E'_q + \frac{X'_d - X''_d}{X'_d - X_{\ell s}} \psi_{1d}$$
$$\psi_q = -X''_q I_q - \frac{X''_q - X_{\ell s}}{X'_q - X_{\ell s}} E'_d + \frac{X'_q - X''_q}{X'_q - X_{\ell s}} \psi_{2q}$$
$$\psi_0 = -X_{\ell s} I_0$$

Stator flux definitions

For steady-state operation,
Treat all derivatives as zero.

Then things begin to cancel…

Steady-State Operation, Results

- Eventually the dust settles and you end up with

$$0 = V_d + I_d R_s - X_q I_q$$
$$0 = V_q + I_q R_s - E_{fd} + X_d I_d$$

- Which we can combine into the following complex equation

$$jE - \left(V_d + jV_q\right) = (R_s + jX_q)(I_d + jI_q)$$

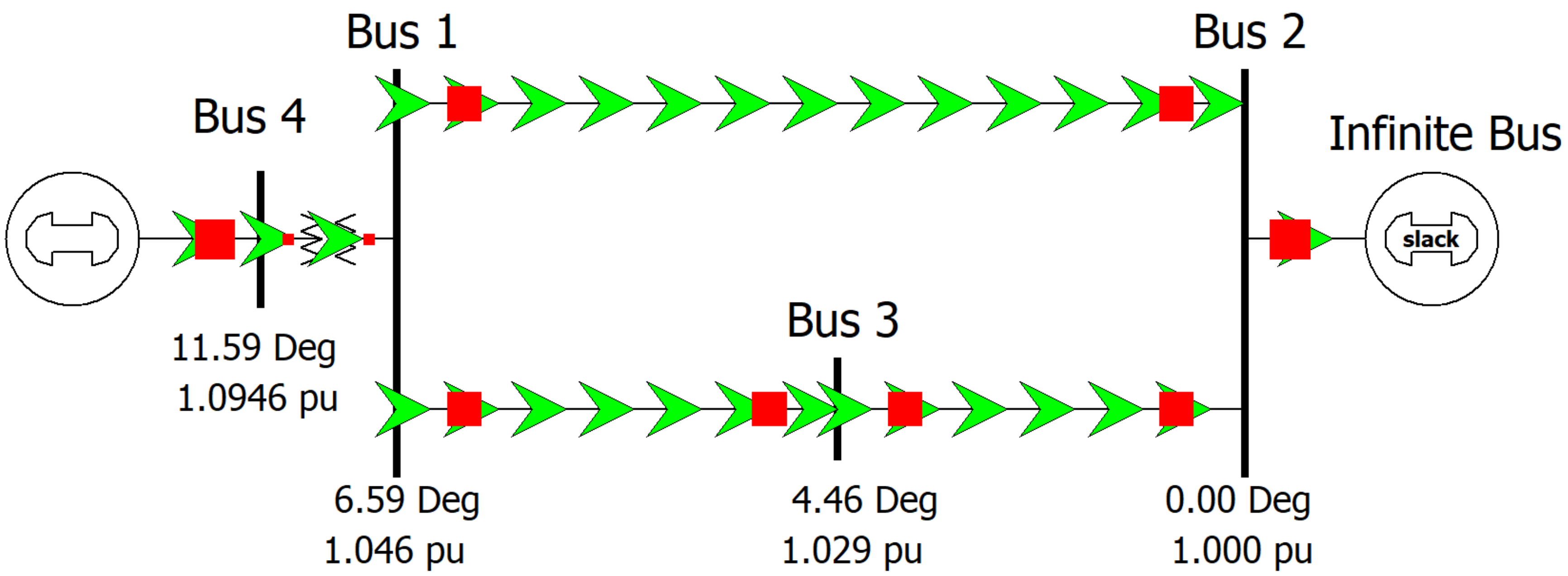
- Where $E = E_{fd} + \left(X_q - X_d\right) I_d = \frac{T_M - T_{FW}}{I_q}$
- If you then convert back to the “network reference frame” using the conversions equations a few slides back, you get

$$Ee^{j\delta} - V_s \angle \theta_s = \left(R_s + jX_q\right)(I_s \angle \phi_s)$$

- In other words, knowing terminal voltage and current you can get δ; then you can get I_d and I_q and from there E_{fd} and $(T_M - T_{FW})$
- This is all only **without** saturation! We will consider that soon.

Example, B4 Steady-State

- Assume a 100 MVA base, with the generator supplying 100 MW and 32.86 Mvar into an infinite bus at V=$1\angle 0$ with network impedance $j0.22$
- Generator per-unit values are $R_s = 0, X_d = 2.1, X_q = 2, X_d' = 0.3, X_q' = 0.5$
- Find the initial value of δ assuming no saturation



Example, B4 Steady-State, Solution

- From circuit, we can calculate $\bar{I}_s = 1.0526\angle - 18.2°$ and $\bar{V}_s = 1.0946\angle 11.59°$

$$Ee^{j\delta} = V_s\angle\theta_s + \left(R_s + jX_q\right)(I_s\angle\phi_s) = \bar{V}_s + j2\,\bar{I}_s = 2.814\angle 52.1°$$

- Hence $\delta = 52.1°$
- With "reference frame transformations"

$$V_d + jV_q = (V_s\angle\theta_s)e^{-j\left(\delta-\frac{\pi}{2}\right)} = 1.09\angle(11.59° - 52.1° + 90°) = 1.09\angle 49.5°$$

$$I_d + jI_q = (I_s\angle\phi_s)e^{-j\left(\delta-\frac{\pi}{2}\right)} = 1.0526\angle(-18.2° - 52.1° + 90°) = 1.0526\angle 19.7°$$

- We can also find other steady-state variables, for example

$$E'_q = V_q + R_sI_q + X'_dI_d = 0.8326 + 0.3 \cdot 0.9909 = 1.1299$$

Multiple Time Scale Analysis

- It is important to think about which states will change faster than others. Start by examining the coefficients of the derivatives
  - The slowest change will be in the swing equation, because $2H$ is bigger than the other constants
  - The "transient time constants" T'_{do} and T'_{qo} are larger values; these effects are generally considered slower
  - The "subtransient time constants" T''_{do} and T''_{qo} are smaller and happen faster
  - But the smallest coefficients are the $\frac{1}{\omega_s}$ terms in the stator transient equations
- Various simplifications of the synchronous machine equations exist which generally assume that the faster changing variables are algebraic, using time scale separation

Elimination of the Stator Transients

- First, it is convenient to define the following

$$\psi_d'' = \frac{X_d''-X_{\ell s}}{X_d'-X_{\ell s}} E_q' + \frac{X_d'-X_d''}{X_d'-X_{\ell s}} \psi_{1d} \quad \text{and} \quad \psi_q'' = -\frac{X_q''-X_{\ell s}}{X_q'-X_{\ell s}} E_d' + \frac{X_q'-X_q''}{X_q'-X_{\ell s}} \psi_{2q}$$

- So the algebraic equations become

$$\psi_d = -X_d'' I_d + \frac{X_d''-X_{\ell s}}{X_d'-X_{\ell s}} E_q' + \frac{X_d'-X_d''}{X_d'-X_{\ell s}} \psi_{1d} = -X_d'' I_d + (1+\omega)\psi_d''$$

$$\psi_q = -X_q'' I_q - \frac{X_q''-X_{\ell s}}{X_q'-X_{\ell s}} E_d' + \frac{X_q'-X_q''}{X_q'-X_{\ell s}} \psi_{2q} = -X_q'' I_q + (1+\omega)\psi_q''$$

- And so if you consider the stator transients to be so fast that $\frac{1}{\omega_s} = 0$

$$\frac{1}{\omega_s}\frac{d\psi_d}{dt} = V_d + I_d R_s + (\omega+1)\psi_q$$

$$\frac{1}{\omega_s}\frac{d\psi_q}{dt} = V_q + I_q R_s - (\omega+1)\psi_d$$

Stator Windings

$$0 = V_d + I_d R_s - X_q'' I_q + (1+\omega)\psi_q''$$

$$0 = V_q + I_q R_s + X_d'' I_d - (1+\omega)\psi_d''$$

New Network Interface Equations

Model Without Stator Transients, No Saturation

$$0 = V_d + I_d R_s - X_q'' I_q + (1+\omega)\psi_q''$$

$$0 = V_q + I_q R_s + X_d'' I_d - (1+\omega)\psi_d''$$

Network Interface

$$T_{do}' \frac{dE_q'}{dt} = -E_q' - (X_d - X_d')\left[I_d - \frac{(X_d' - X_d'')}{(X_d' - X_{\ell s})^2}\left(\psi_{1d} + (X_d' - X_{\ell s})I_d - E_q'\right)\right] + E_{fd}$$

$$T_{do}'' \frac{d\psi_{1d}}{dt} = -\psi_{1d} + E_q' - (X_d' - X_{\ell s})I_d$$

Rotor Windings

$$T_{qo}' \frac{dE_d'}{dt} = -E_d' + (X_q - X_q')\left[I_q - \frac{X_q' - X_q''}{(X_q' - X_{\ell s})^2}\left(\psi_{2q} + (X_q' - X_{\ell s})I_q + E_d'\right)\right]$$

$$T_{qo}'' \frac{d\psi_{2q}}{dt} = -\psi_{2q} - E_d' - (X_q' - X_{\ell s})I_q$$

$$\frac{d\delta}{dt} = \omega \cdot \omega_s$$

$$2H\frac{d\omega}{dt} = T_M + (\psi_q'' I_d - \psi_d'' I_q) - T_{FW}$$

Mechanical equations

$$\psi_d'' = \frac{X_d'' - X_{\ell s}}{X_d' - X_{\ell s}} E_q' + \frac{X_d' - X_d''}{X_d' - X_{\ell s}} \psi_{1d}$$

$$\psi_q'' = -\frac{X_q'' - X_{\ell s}}{X_q' - X_{\ell s}} E_d' + \frac{X_q' - X_q''}{X_q' - X_{\ell s}} \psi_{2q}$$

Flux definitions

(We're ignoring the zero axis here, which is common for balanced operation)

Elimination of Subtransients

- This is known as the two-axis model and comes from assuming that $T''_{do} = T''_{qo} = 0$ is an acceptable manifold for time separation

Network Interface

$$0 = V_d + I_d R_s - X'_q I_q - (1+\omega)E'_d$$
$$0 = V_q + I_q R_s + X'_d I_d - (1+\omega)E'_q$$

Rotor Windings

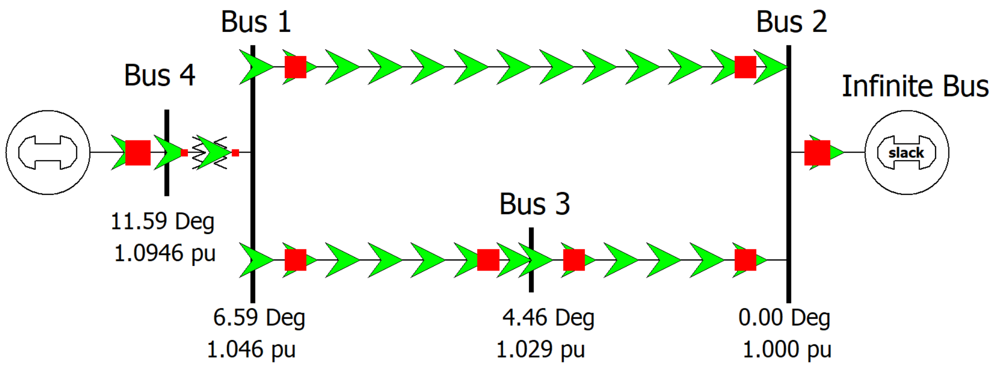
$$T'_{do}\frac{dE'_q}{dt} = -E'_q - (X_d - X'_d)I_d + E_{fd}$$
$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + \left(X_q - X'_q\right)I_q$$

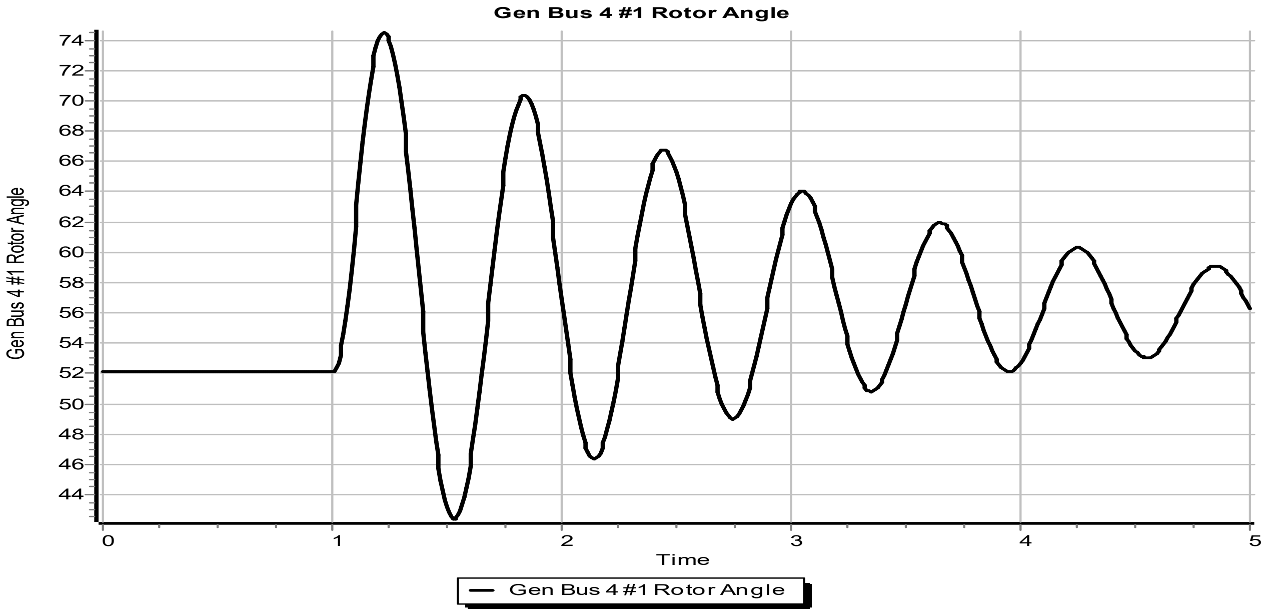
Mechanical equations

$$\frac{d\delta}{dt} = \omega \cdot \omega_s$$
$$2H\frac{d\omega}{dt} = T_M + \left(\left(-X'_q I_q - E'_d\right)I_d - (-X'_d I_d + E_q{}')I_q\right) - T_{FW}$$

Two-Axis Model Example

- Same example as before, with same initialization
- Assume a fault at bus 3 at time $t = 1.0$, cleared by opening both lines into bus 3 at time $t = 1.1$ seconds

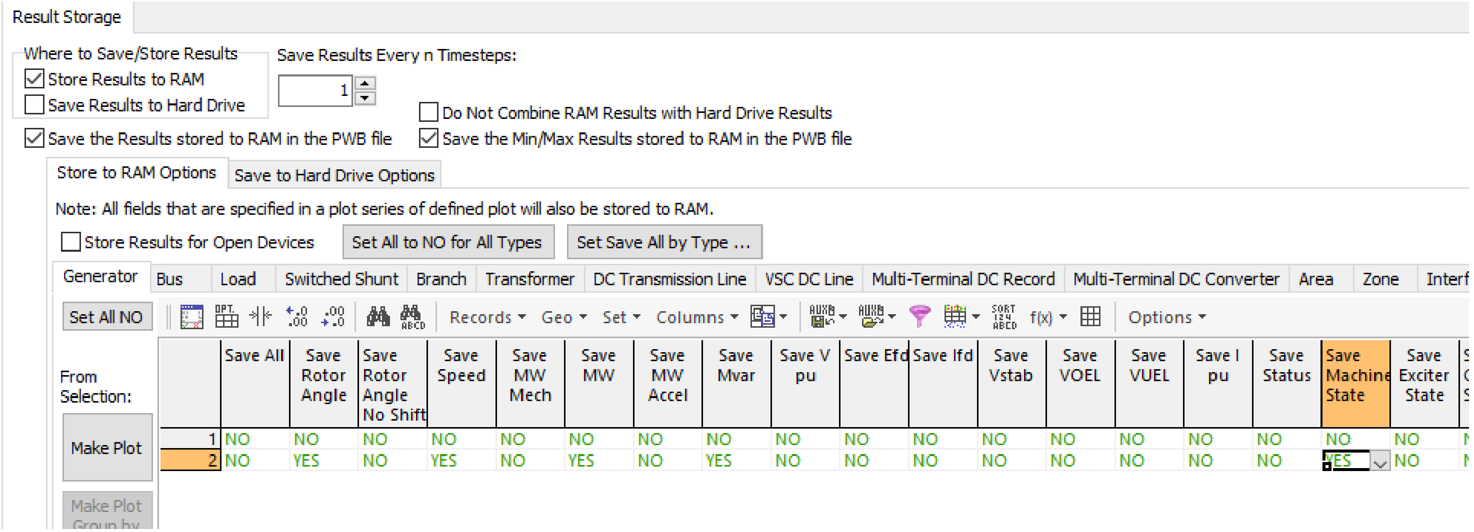


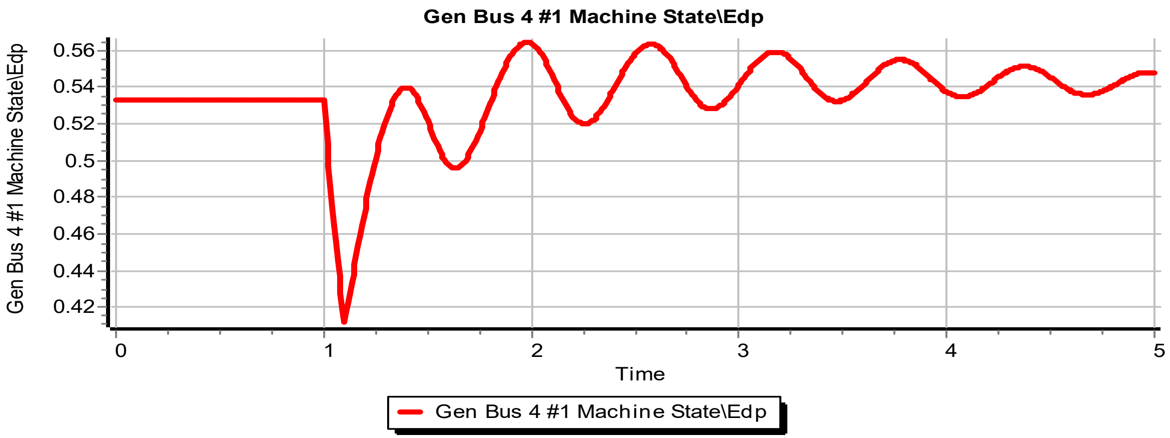


PowerWorld case B4_TwoAxis

Two-Axis Model Example, Viewing the States

- PowerWorld allows the generator states to be stored, such as E_d' below





Further Time Scale Elimination?

- The previous model reduces the size of the model down to six equations, with only four differential equations. This is sometimes called the fourth-order model or the two-axis model.
  - While not that common in industry, it does capture more details than the simpler models we will discuss next
- Next we can reduce the model to a third-order model often called the flux-decay model, by assuming the q-axis transient dynamics are slow ($T'_{qo} = 0$)
- Then finally we can make further simplifications to connect to the classical model we have previously introduced

Flux-Decay Model

- Assuming that $T'_{qo} = 0$ is an acceptable manifold for time separation

$$0 = V_d + I_d R_s - X_q I_q$$

$$0 = V_q + I_q R_s + X'_d I_d - (1+\omega)E'_q$$

Network Interface

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - (X_d - X'_d)I_d + E_{fd}$$

Rotor Winding

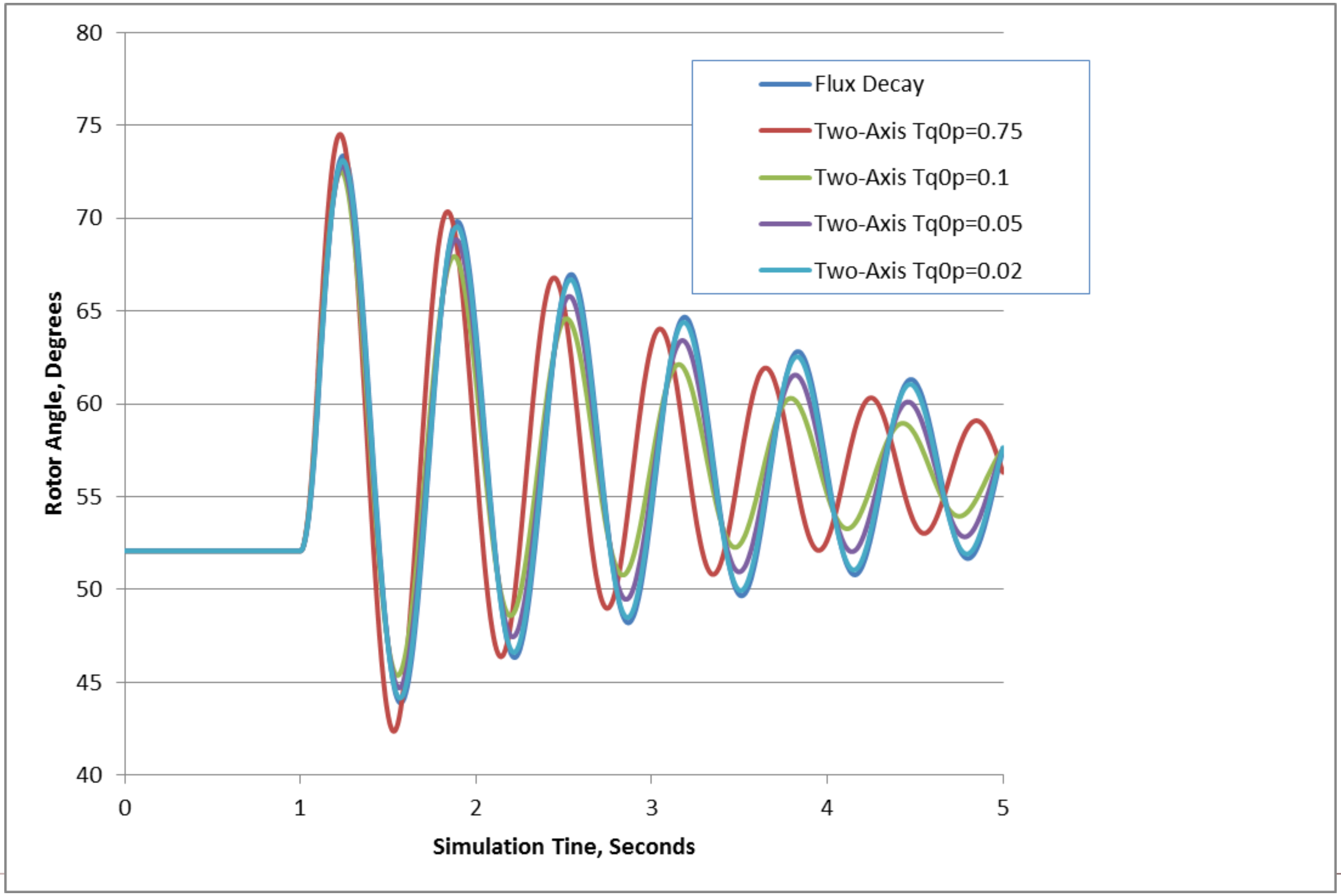
$$\frac{d\delta}{dt} = \omega \cdot \omega_s$$

$$2H\frac{d\omega}{dt} = T_M - E'_q I_q - T_{FW}$$

Mechanical equations

Flux-Decay Model Example

- This shows the sensitivity to changing T'_{qo} in the two-axis model
- Two-axis model with $T'_{qo} = 0$ is the same as the flux-decay model



Toward the Classical Model

- These last assumptions are the hardest to justify!

$$0 = V_d + I_d R_s - X_q I_q$$
$$0 = V_q + I_q R_s + X'_d I_d + E_{fd}$$

Network Interface

$$\frac{d\delta}{dt} = \omega \cdot \omega_s$$
$$2H \frac{d\omega}{dt} = T_M + E_{fd} I_q - T_{FW}$$

Mechanical equations

- If you apply the reference frame transformation you can get the equations we originally presented for the classical model (with $V_s \angle \theta_s = V_r + jV_i$)

$$\dot{\delta} = \omega \cdot \omega_s$$
$$\dot{\omega} = \frac{1}{2H}\left(\frac{P_m}{\omega+1} - \frac{E_p}{X'_d}\left(V_r \sin\delta - V_i \cos\delta\right)\right)$$
$$I_r = \frac{1}{X'_d}\left(-V_i + E_p \sin\delta\right)$$
$$I_i = \frac{1}{X'_d}\left(V_r - E_p \cos\delta\right)$$

Here, we use E_p instead of E_{fd} and make the common assumption for a classical model that $X_q = X_d = X'_d$

Saturation

- Relationship between current and flux linkage in magnetic materials is not linear
- Often generators are designed to be operated near saturation
- There is also hysteresis, meaning residual magnetism when current goes to zero (design goal to reduce)

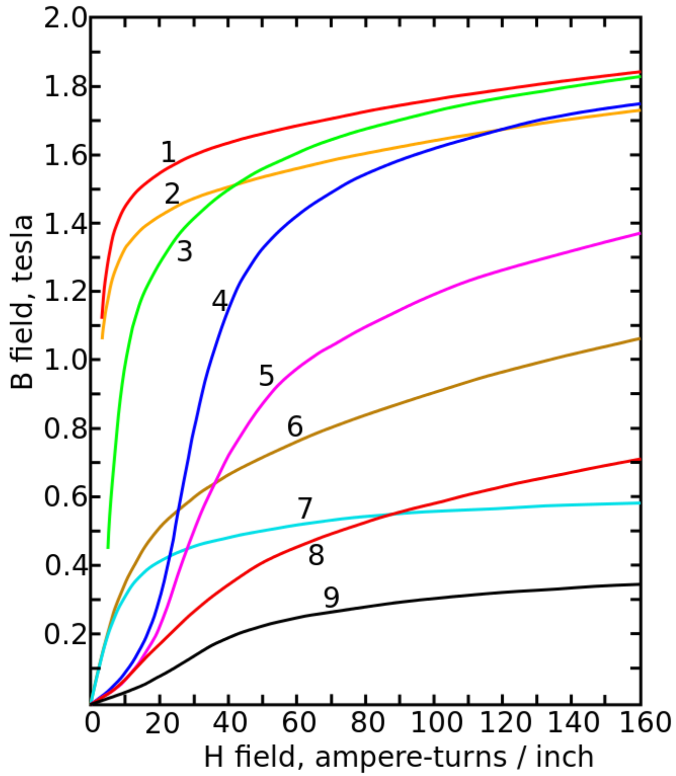


Image Source:
en.wikipedia.org/wiki/Saturation_(magnetic)

Saturation Models

- Tradeoff between accuracy and complexity
- One simple approach is to replace

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - (X_d - X'_d)I_d + E_{fd}$$

- With

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - (X_d - X'_d)I_d - Se(E'_q) + E_{fd}$$

- Where Se is a quadratic function

$$Se\left(E'_q\right) = B\left(E'_q - A\right)^2$$

- Usually specified at two points, such as $Se(1.0)$ and $Se(1.2)$

Saturation Example

- Given values $SE10 = 0.1$ and $SE12 = 0.5$, what are the values of A and B for the quadratic saturation model?
- Solution

$$0.1 = B(1.0 - A)^2 \quad \text{and} \quad 0.5 = B(1.2 - A)^2$$

- Solve these two equations simultaneously and get two possibilities

$$A = 0.838, B = 3.820 \quad \text{and} \quad A = 1.0618, B = 26.2$$

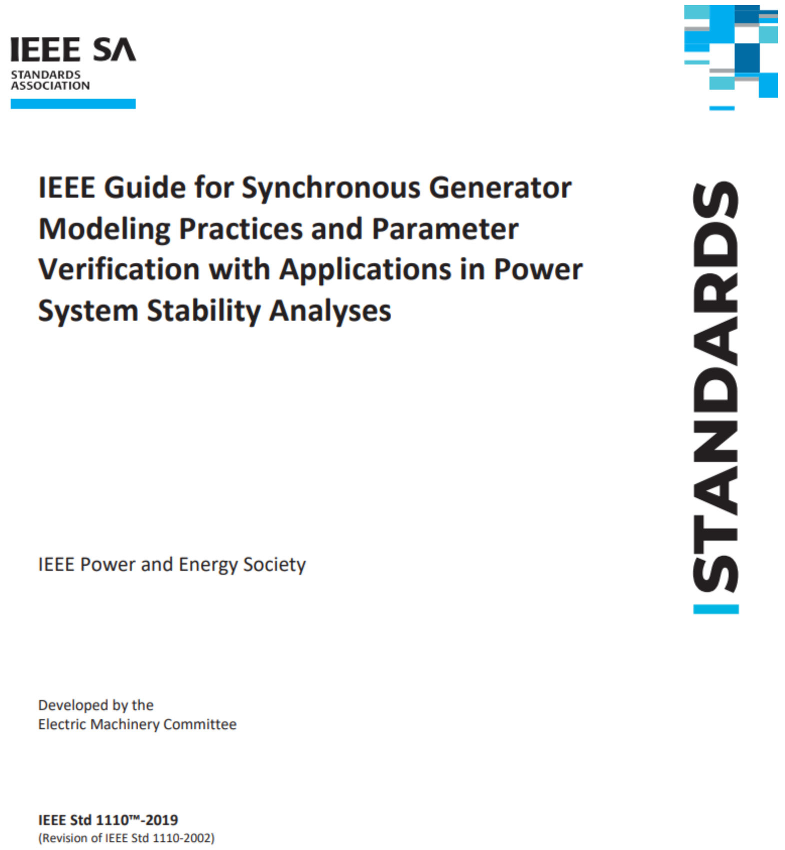
- The first one is physically meaningful, as A should be less than 1

Other Saturation Modeling Considerations

- Alternative models include scaled quadratic (same thing but divided by $E_q{}'$) and exponential $Se\left(E_q'\right) = Ae^{BE_q'}$
- In practice there are special cases, sometimes cause by user typos
  - What to do if Se(1.2) < Se(1.0)?
  - What to do if Se(1.0) = 0 and Se(1.2) ≠ 0
  - What to do if Se(1.0) = Se(1.2) ≠ 0

Introduction to GENSAL

- GENSAL is the first industry standard model we will study. It is a good introduction to subtransient models and models that include saturation
- It was designed going back to the ‘70s primarily for salient pole machines, such as hydro
- 5th order, saturation affects the d-axis only
- Very common in industry studies until about 2010-2015, in which it began to be replaced by newer models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IEEE Guide for Synchronous Generator Modeling Practices and Parameter Verification with Applications in Power System Stability Analyses

IEEE Power and Energy Society

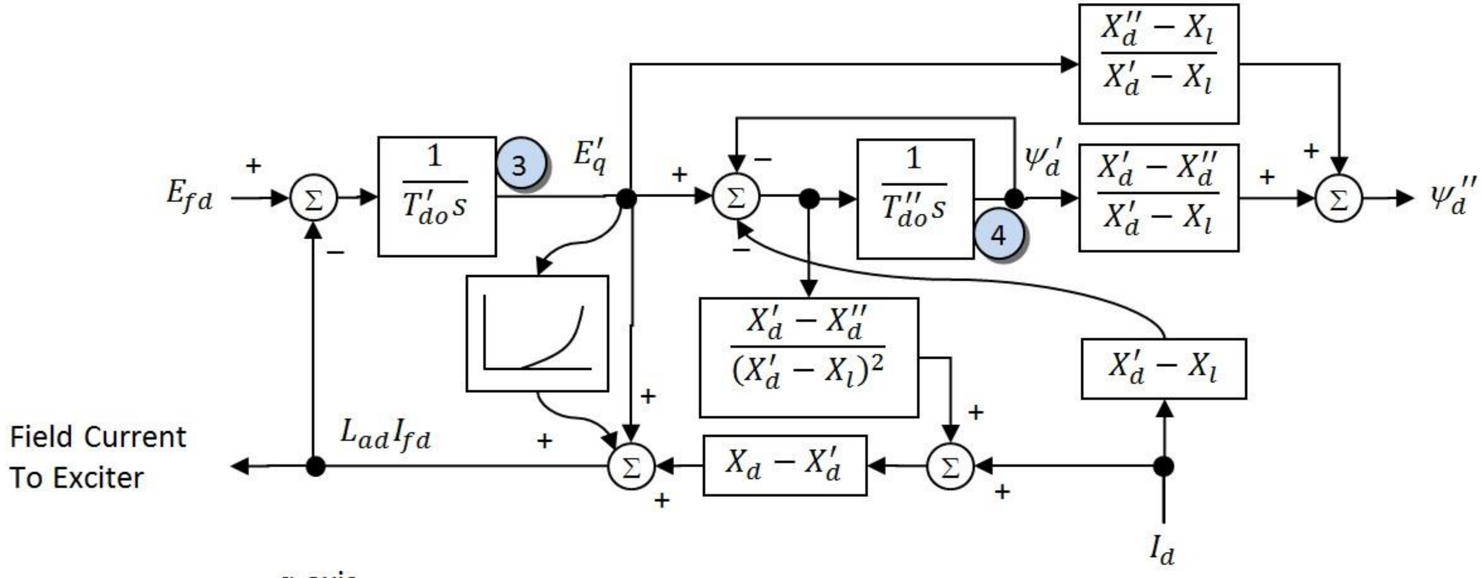
Developed by the
Electric Machinery Committee

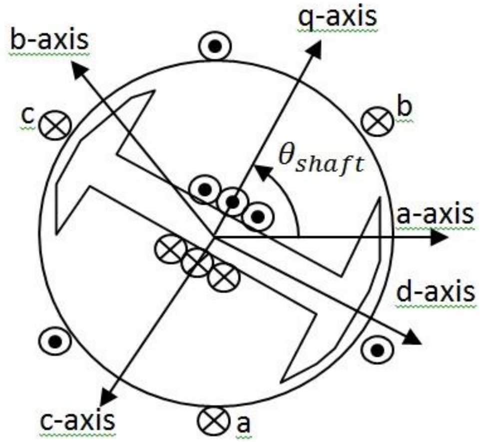
IEEE Std 1110™-2019
(Revision of IEEE Std 1110-2002)

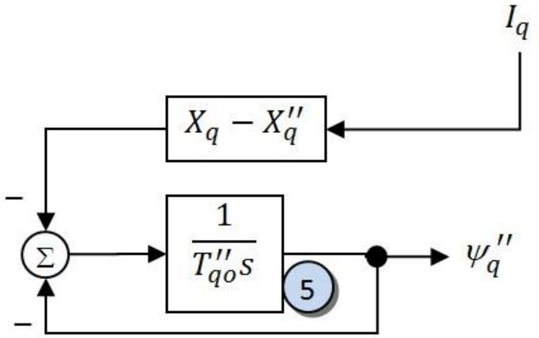
STANDARDS

GENSAL Definition

- Block diagram shows rotor equations
- Same machine interface equations and mechanical equations as the “model without stator transients” we developed above.
- For initialization, saturation only impacts calculation of E_{fd}







GENSAL Example

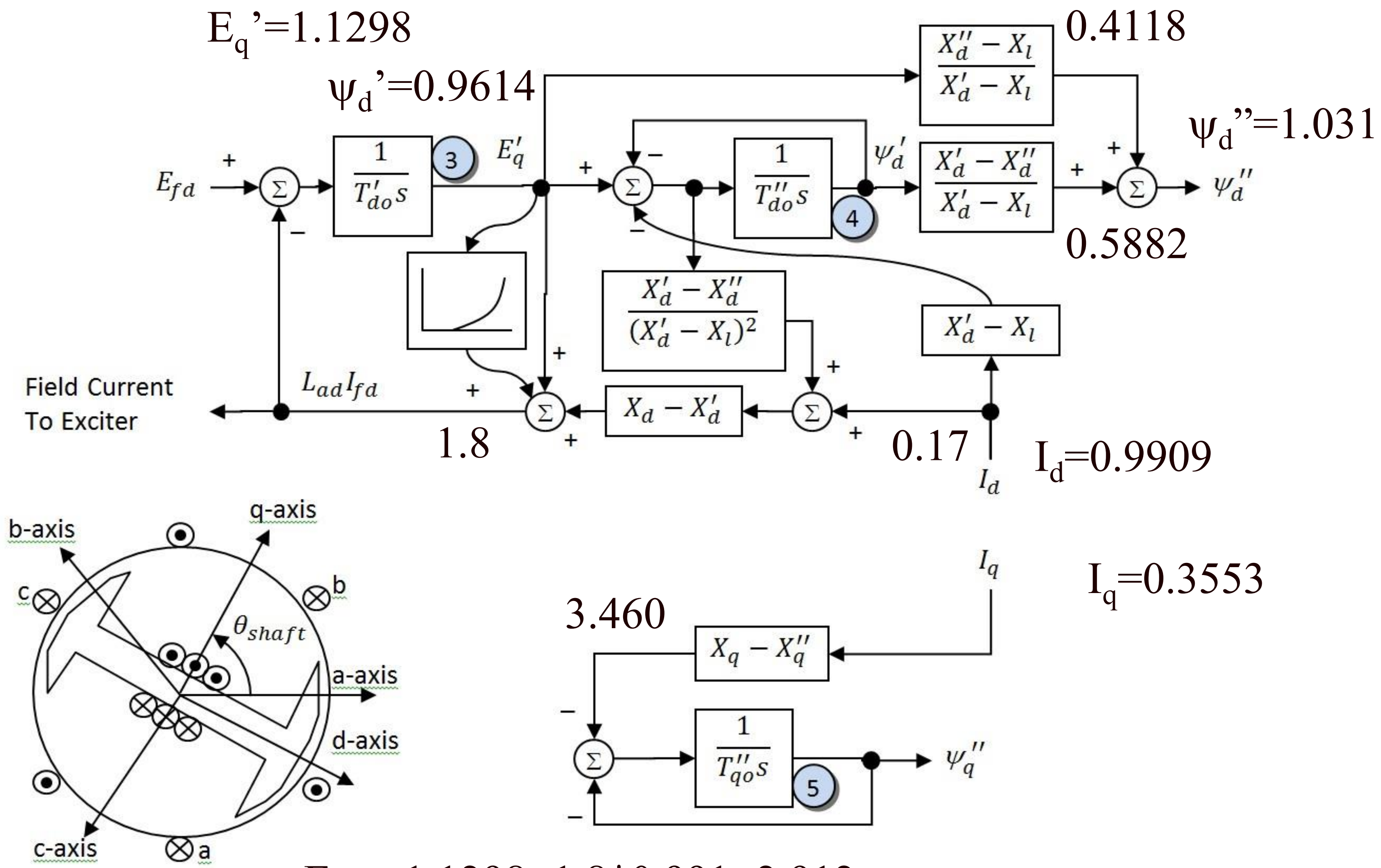
- Assume same system as before with same common generator parameters: H=3.0, D=0, R_a = 0, X_d = 2.1, X_q = 2.0, X'_d = 0.3, X''_d=X''_q=0.2, X_l = 0.13, T'_{do} = 7.0, T''_{do} = 0.07, T''_{qo} =0.07, S(1.0) =0, and S(1.2) = 0.
- Same terminal conditions as before, resulting in $\delta = 52.1°$
- Then we again get $V_d = 0.7107, V_q = 0.8326, I_d = 0.9909, I_q = 0.3553$
- Now, use the network interface equations to get ψ''_d, ψ''_q

$$0 = V_d + I_d R_s - X''_q I_q + (1+\omega)\psi''_q$$
$$0 = V_q + I_q R_s + X''_d I_d - (1+\omega)\psi''_d$$

- $\psi''_d = 1.031, \psi''_q = -0.6396$
- Then use the steady-state differential equations to find $\psi''_q = -0.6396,\ E'_q = 1.1298, \psi'_d = 0.9614$

On the d-axis you need to solve two linear equations for two unknowns

GENSAL Example, Solution



E_{fd} = 1.1298+1.8*0.991=2.912

GENSAL Example 2

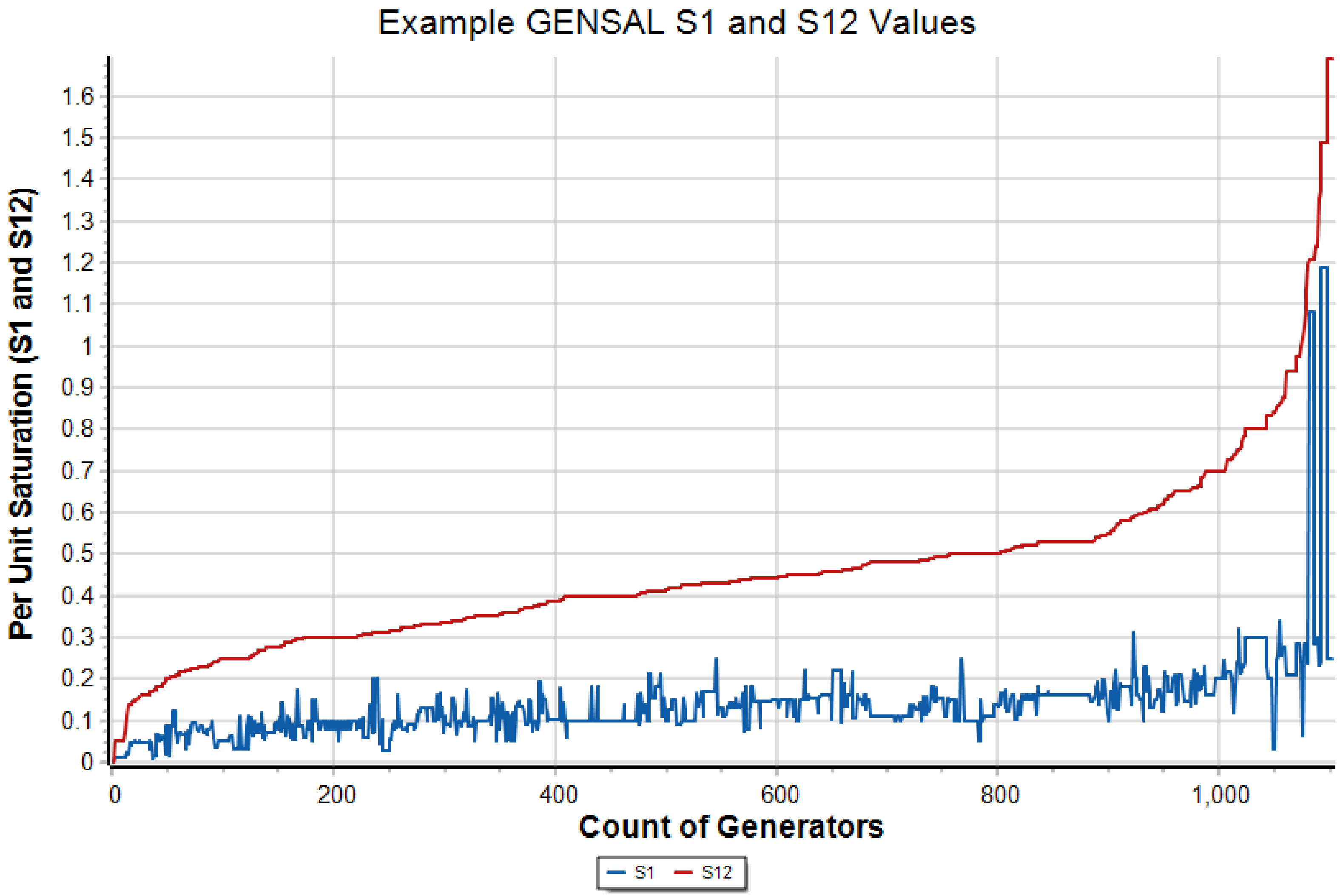
- Now repeat the initialization with the saturation values specified before, $SE10 = 0.1$ and $SE12 = 0.5$
- Recall, we found $A = 0.838$ and $B = 3.82$
- Now, use the block diagram and recognize that the derivative term before E_q' is zero

$$E_{fd} = E_q'\left(1 + Se\left(E_q'\right)\right) + (X_d - X_d')I_d$$
$$= 1.1298(1 + B(1.1298 - A)^2) + (2.1 - 0.3)(0.9909)$$
$$= 1.1298(1 + 3.82(1.1298 - 0.838)^2) + 1.784 = 3.28$$

- E_{fd} is the only initialized variable that is different because of saturation

GENSAL Saturation Values from Large Case

- Essentially all GENSAL generators have saturation values



Machine Models

- Common synchronous machine models
  - Classical – second order primarily for teaching purposes
  - Flux-decay – simplified third order model
  - Two-axis – simplified fourth order model
  - GENSAL – originally for salient pole generators, now becoming obsolete
  - GENROU – for round-rotor models
  - GENTPF – first replacement for GENSAL
  - GENTPJ – second replacement for GENSAL
  - GENQEC – third and current replacement for GENSAL, can also replace GENROU