ECEN 667
Power System Stability

Lecture 4: Time Domain Stability Analysis

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Announcements

- Homework Assignment #1 is due Thursday, Sept. 11th at 8 AM. Email me your solution as a single PDF.
- Homework Assignment #2 is due Thursday, Sept. 25th at 8 AM.
- Review the slides and PowerWorld examples

Recall, Stability

- Definition:

  Starting with a certain power system <u>operating condition</u>,
  and subjecting the system to some <u>disturbance</u>,
  will the system regain an <u>equilibrium point</u>
  with all <u>important variables within an acceptable range</u>?

- Notice that with this definition, instability can be affected in multiple ways
  - If you subject the system to a sufficiently large disturbance, you will eventually cause an unstable response
  - Alternatively, if you operate the system in sufficiently stressed conditions, even a very small disturbance could cause an unstable response
  - Design of various control systems are crucial to help increase stability
  - The definition of what variables are important and what is an acceptable range also matters

How Do We Analyze Stability?

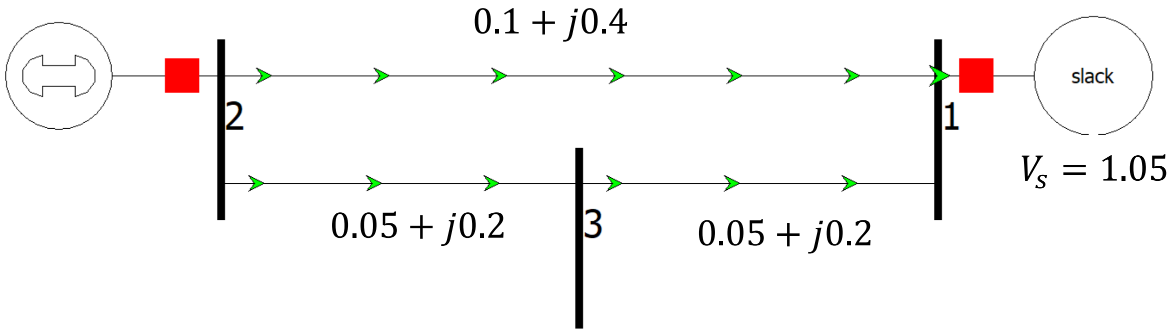
- Generally, the first step is some type of modeling, which usually results in a set of DAEs that describes the system
  - We also need to know the operating point, the disturbance(s) of interests, and desired system performance metrics
- From here there are two main categories of stability analysis techniques
  - **Analytical Stability Methods.** These look at the equations themselves, break them down, and attempt to make mathematical claims about their stability properties. For a linear system (or linearized non-linear system), this could involve eigenvalue analysis. For nonlinear systems, energy functions is an example of an approach. Sometimes a frequency sweep with phase-margin and gain-margin is used. We will talk about several of these approaches later in the semester.
  - **Time-Domain Numerical Methods.** We use numerical integration methods (as last class) to simulate disturbance responses in time. We then need to analyze the numerical results to determine the system's stability properties. This is the focus of today's lecture.

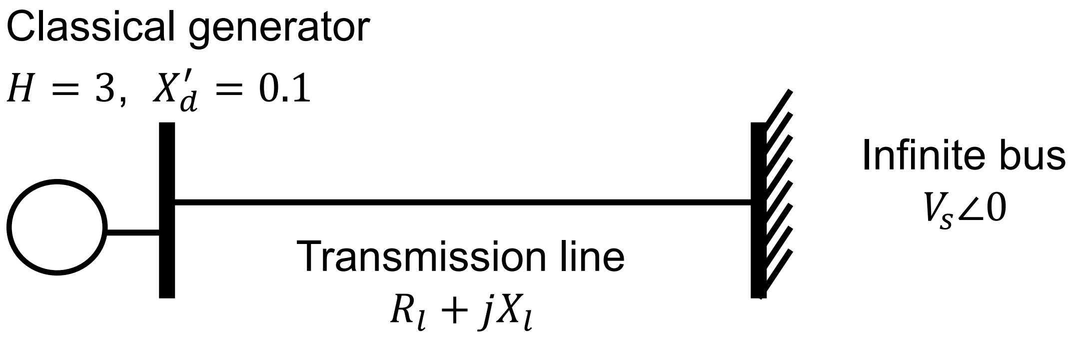
Time-Domain Stability Analysis Steps

1. Assemble a system model as a DAE set
2. Use Power Flow and equilibrium analysis to find initial conditions
3. Perform time integration using a numerical method
4. When a disturbance event occur, there will be discrete changes to the equations. Often there will be several events, such as fault apply/clear/open.
5. Assess the time-domain results to answer questions about system stability. For example: Did the system return to an equilibrium? How quickly did it reach equilibrium? Is the equilibrium within acceptable range? Did anything exceed acceptable bounds during the transient period? What frequencies of oscillation are present? What is the damping of oscillations?

Example 1, with Fault

- For Example 1, we want to find the results when there is a fault at Bus 3. After 6 cycles (0.1 seconds), the fault clears and also the 2-3 branch opens.
- The SMIB diagram (bottom) is actually valid for all 3 cases using Thevenin equivalents
  - Pre-fault, $\bar{Z} = 0.05 + j0.2, \;\; V_s = 1.05$
  - During fault we assume bus 3 is grounded. So $\bar{Z} = (0.05 + j0.2) || (0.1 + j0.4) = \frac{2}{3}(0.05 + j0.2)$ and $V_s = \frac{1}{3} \cdot 1.05 = 0.35$
  - After fault we remove line 2-3. So $\bar{Z} = 0.1 + j0.4$ and $V_s = 1.05$ again





Example 1, Model, Initial Conditions, Event

- Recall the equations from before

$$\dot{\delta} = \omega \cdot \omega_s$$

$$\dot{\omega} = \frac{1}{2H}\left(\frac{P_m}{\omega+1} - \frac{E_p}{X'_d}(V_r \sin\delta - V_i \cos\delta)\right)$$

$$E_p\angle\delta - \bar{V} = jX'_d\bar{I}$$

$$\bar{V} - V_s\angle 0 = (R_l + jX_l)\bar{I}$$

Constants are $H = 3$ and $X'_d = 0.1$ and $\omega_s = 2\pi 60$

Initial conditions are $\omega = 0, \bar{V} = 1.07984\angle 10.16°$, $\bar{I} = 0.9261\angle 10.16°$, $E_p = 1.0838$, $\delta = 15.06°$, $P_m = 1$

For the initial pre-fault conditions, $V_s = 1.05$, $R_l = 0.05$, $X_l = 0.2$

During the fault these constants change to $V_s = 0.35$, $R_l = 0.03333$, $X_l = 0.1333$

After the fault is cleared and the line opens, change $V_s = 1.05$, $R_l = 0.1$, $X_l = 0.4$

- On your own, verify that the pre-fault conditions are an equilibrium point

Solving Example 1 with RK2, First Time Step

- During fault equations (eliminating $\bar{I}$ for simplicity)

$$\dot{\delta} = \omega \cdot \omega_s \qquad \dot{\omega} = \frac{1}{2H}\left(\frac{P_m}{\omega+1} - \frac{E_p}{X_d'}(V_r \sin\delta - V_i \cos\delta)\right)$$

$$E_p\angle\delta - \bar{V} = \frac{jX_d'\bar{V}}{R_l+jX_l} - \frac{jX_d'V_s\angle 0}{R_l+jX_l} \rightarrow \bar{V} = \left(E_p\angle\delta + \frac{jX_d'V_s\angle 0}{R_l+jX_l}\right)\left(\frac{jX_d'}{R_l+jX_l}+1\right)^{-1}$$

- Setting $\Delta t = 0.01$ we find the variable values at $t = 0.01$ using RK2

First find $\bar{V}$ assuming $\delta = 15.06° = 0.26285$ and get $\bar{V}(0^+) = 0.77091 + j0.12154$

$$k_1 = \Delta t\, f\big(x(t)\big) = 0.01\, f\left(\begin{bmatrix}0.26285\\0\end{bmatrix}\right) = 0.01\begin{bmatrix}0\\ \frac{1}{2H}\left(P_m - \frac{E_p}{X_d'}(0.77091\sin 0.26285 - 0.012154\cos 0.26285)\right)\end{bmatrix} = \begin{bmatrix}0\\0.001684\end{bmatrix}$$

Note that $\bar{V}$ does not change for k_2 calculations since δ does not change

$$k_2 = \Delta t\, f(x(t) + k_1) = 0.01\, f\left(\begin{bmatrix}0.26285+0\\0+0.001684\end{bmatrix}\right) = \begin{bmatrix}0.006349\\0.001681\end{bmatrix}$$

So, the next values of the differential variables (x) are

$$\delta(0.01) = \delta(0) + \frac{1}{2}(0 + 0.006349) = 0.26317$$

$$\omega(0.01) = \omega(0) + \frac{1}{2}(0.001684 + 0.001681) = 0.0001683$$

Solving Example 1 with RK2, Second Time Step

- For the second time step, we follow the same procedure

First find $\bar{V}$ assuming $\delta = 0.26317$ and get $\bar{V}(0^+) = 0.77088 + j0.12174$

$$k_1 = \Delta t\ f\big(x(t)\big) = 0.01\ f\left(\begin{bmatrix} 0.26317 \\ 0.0001683 \end{bmatrix}\right) = \begin{bmatrix} 0.0063435 \\ 0.0016728 \end{bmatrix}$$

Now we need to recalculate $\bar{V}$ for k_2 calculations assuming $\delta = 0.26317 + 0.063435$

and we get $\bar{V} = 0.77081 + j0.12213$ and then for k_2

$$k_2 = \Delta t\ f(x(t) + k_1\ ) = 0.01\ f\left(\begin{bmatrix} 0.26285 + 0 \\ 0.0001683 + 0.001684 \end{bmatrix}\right) = \begin{bmatrix} 0.012650 \\ 0.0016531 \end{bmatrix}$$

So, the next values of the differential variables (x) are

$$\delta(0.02) = \delta(0.01) + \frac{1}{2}(0.063435 + 0.12650) = 0.26412$$

$$\omega(0.02) = \omega(0.01) + \frac{1}{2}(0.0016728 + 0.0016531) = 0.00033456$$

Solving Example 1 with RK2, Continuing

- At each time step we need to solve the differential equations (f) twice
  - But to solve them, we first need to solve the algebraic equations (g) to get $\bar{V}$
  - First we solve them based on the current values of δ and ω (x)
  - Then we solve them based on the k_1 values of δ and ω (x+k1)
- Continue step by step until we reach $t = 0.1$, when the fault clears
- Then do the same procedure, but with the equivalent circuit values $V_s = 1.05$ and $\bar{Z} = 0.1 + j0.4$ for future time steps starting with $t = 0.1$.
- Note: there are two values of $\bar{V}(0.1)$, before and after changing the equivalent circuit. We assume the voltage (which is an algebraic variable) changes instantaneously.
  - If you look carefully at the PowerWorld results, you will see two time steps inserted that correspond to $\bar{V}(0.1^-)$ and $\bar{V}(0.1^+)$.

Solving Example 1 with RK2, Full Results

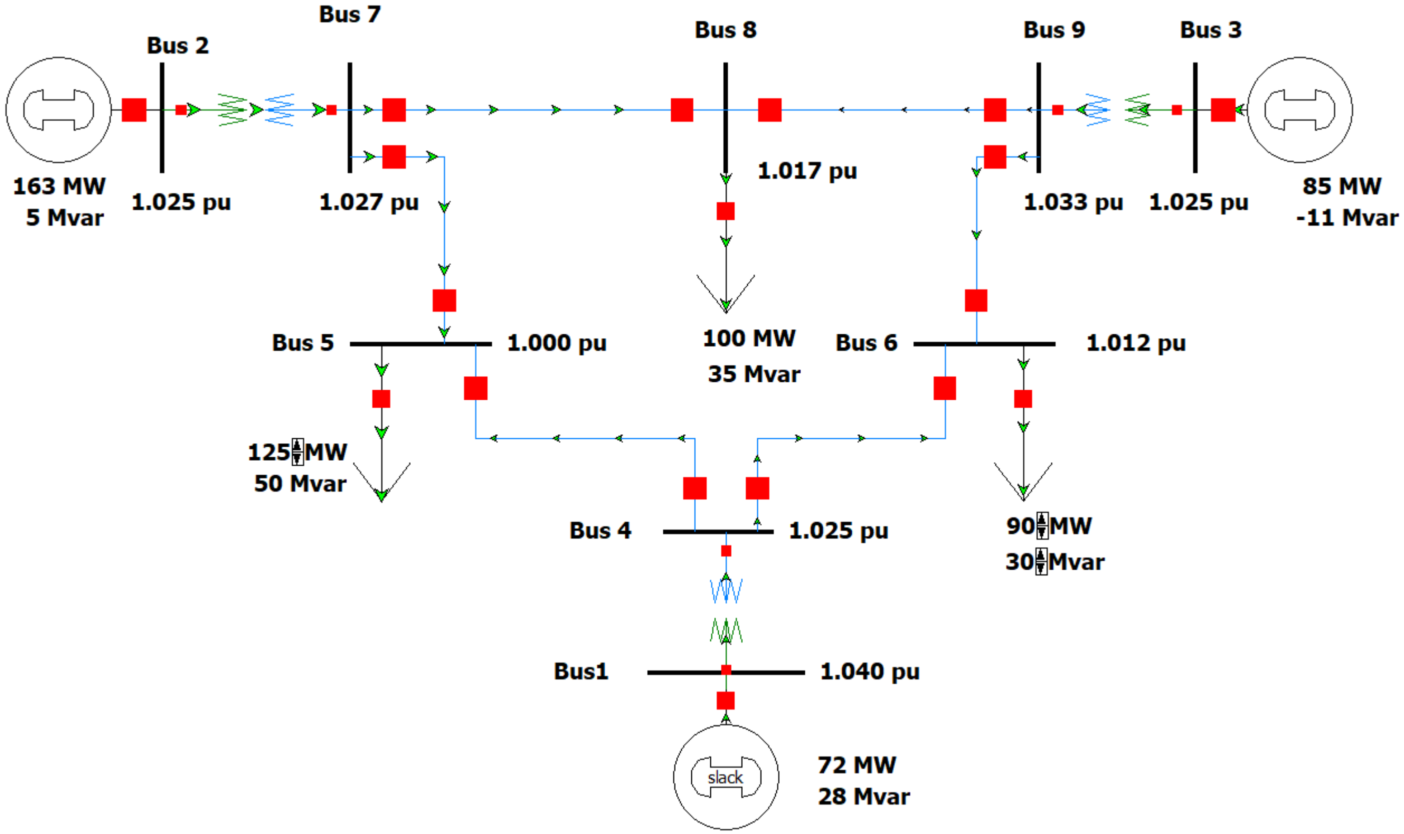
t	delta	omega	vr	vi	f1	f2	k1_delta	k1_omega	k1_vr	k2_vr	k1_f1	k2_f2
0	0.26285	0	0.770910041	0.121538005	0	0.016840646	0.26285	0.000168406	0.770910041	0.121538005	0.063487739	0.016812583
0.01	0.263167439	0.000168266	0.770878095	0.121736053	0.063434841	0.016728084	0.263801787	0.000335547	0.770814068	0.122131789	0.126498234	0.016531325
0.02	0.264117104	0.000334563	0.770782148	0.122328483	0.126127351	0.016447542	0.265378378	0.000499039	0.770653849	0.123115162	0.188133123	0.016084394
0.03	0.265688406	0.000497223	0.77062216	0.123308508	0.187448606	0.016002173	0.267562893	0.000657245	0.770429289	0.124477299	0.247775379	0.015476652
0.04	0.267864526	0.000654617	0.770398048	0.124665341	0.246784795	0.015396823	0.270332374	0.000808585	0.77014032	0.126203476	0.304829451	0.014714604
0.05	0.270622598	0.000805174	0.770109761	0.126384321	0.303543499	0.014637969	0.273658033	0.000951554	0.769787004	0.128275242	0.358727341	0.013806326
0.06	0.273933952	0.000947396	0.769757381	0.128447077	0.357159732	0.013733652	0.277505549	0.001084732	0.769369652	0.130670625	0.408934379	0.012761377
0.07	0.277764422	0.001079871	0.76934124	0.130831737	0.407101684	0.012693387	0.281835439	0.001206805	0.768888958	0.133364381	0.454954625	0.011590691
0.08	0.282074704	0.001201291	0.768862055	0.133513174	0.452876092	0.011528059	0.286603465	0.001316572	0.768346134	0.136328275	0.496335851	0.010306462
0.09	0.286820764	0.001310464	0.768321059	0.13646329	0.494033195	0.010249802	0.291761096	0.001412962	0.767743052	0.139531401	0.532674039	0.008922006
0.1	0.2919543	0.001406323	1.052254707	0.252418774	0.530171202	0.056016274	0.297256012	0.001966486	1.051118617	0.256923986	0.74134758	0.053960496
0.11	0.298311894	0.001956207	1.050889504	0.257820515	0.73747253	0.05357143	0.305686619	0.002491921	1.049262902	0.26407544	0.939432064	0.050754411
0.12	0.306696417	0.002477836	1.049036588	0.264930968	0.934122107	0.050383359	0.316037638	0.002981669	1.046902123	0.27283413	1.124062896	0.046847803
0.13	0.316987342	0.002963992	1.046680983	0.273636505	1.11739853	0.04649998	0.328161327	0.003428991	1.044021948	0.28306106	1.292699325	0.0422992
0.14	0.329037831	0.003407988	1.043808917	0.28379907	1.284781041	0.041979451	0.341885641	0.003827782	1.040612178	0.294595072	1.443039843	0.037176579
0.15	0.342676935	0.003803768	1.04041076	0.295258642	1.433986643	0.036889302	0.357016802	0.004172661	1.036669818	0.307255827	1.573056036	0.031556307
0.16	0.357712149	0.004145996	1.036484048	0.307836201	1.563003577	0.031305373	0.373342185	0.004459049	1.032202001	0.320847219	1.681022054	0.025521876
0.17	0.373932277	0.004430132	1.032036358	0.321337112	1.670120419	0.02531056	0.390633481	0.004683238	1.027228562	0.335161177	1.765538983	0.019162463
0.18	0.391110574	0.004652497	1.027087832	0.33555489	1.753950093	0.018993389	0.408650075	0.004842431	1.021784037	0.349981725	1.825553483	0.012571344
0.19	0.409008092	0.004810321	1.021673144	0.350275229	1.813448212	0.01244644	0.427142574	0.004934785	1.015918958	0.365089204	1.860370187	0.005844201
0.2	0.427377184	0.004901774	1.015842757	0.365280166	1.84792526	0.005764679	0.445856436	0.004959421	1.009700306	0.380264466	1.869657586	-0.00092261
0.21	0.445965098	0.004925984	1.009663369	0.380352239	1.857052345	-0.000956268	0.464535621	0.004916422	1.003211079	0.395292917	1.8534473	-0.007633785
0.22	0.464517596	0.004883034	1.003217476	0.395278473	1.840860477	-0.007621807	0.482926201	0.004806816	0.996548978	0.409968221	1.812126941	-0.014196706
0.23	0.482782533	0.004773942	0.996602067	0.409854055	1.799733548	-0.014140003	0.500779869	0.004632541	0.989824264	0.424095553	1.746426993	-0.020522938
0.24	0.500513336	0.004600627	0.989926508	0.423885538	1.734395445	-0.02042305	0.517857291	0.004396396	0.983156901	0.437494269	1.657402359	-0.026529517
0.25	0.517472325	0.004365864	0.983309717	0.437193491	1.645891941	-0.02638856	0.533931244	0.004101978	0.976673131	0.449999945	1.546409413	-0.032140021
0.26	0.533433832	0.004073221	0.976876783	0.449614523	1.535568165	-0.031960611	0.548789513	0.003753615	0.970501662	0.461465751	1.415079501	-0.037285373
0.27	0.54818707	0.003726991	0.9707552	0.46100265	1.40504256	-0.037070552	0.562237496	0.003356286	0.964769644	0.471763196	1.265289871	-0.0419044
0.28	0.561538732	0.003332116	0.965070889	0.471230041	1.256178283	-0.041657559	0.574100515	0.002915541	0.959598634	0.480782293	1.099132987	-0.04594414
0.29	0.573315289	0.002894108	0.959944212	0.480187211	1.091052971	-0.045668951	0.584225818	0.002437418	0.955100711	0.488431282	0.918885084	-0.049359967
0.3	0.583364979	0.002418963	0.955486129	0.487782751	0.911927682	-0.049060313	0.592484256	0.00192836	0.951374904	0.494636003	0.726974661	-0.052115534

Confirm that this matches PowerWorld, keeping in mind that you need the same fault duration and time, degrees vs radians, etc.

Hint: if you want to reproduce this you need to write some code that will make the complex calculations a bit easier

Example 2 (WSCC 9-Bus Case)

- For Example 2, we are going to use the commonly-known WSCC 9-Bus case, but with classical generators (and no machine controllers)
- The event is a solid fault at Bus 7 followed by opening line 7-5 to clear



This is the PowerWorld Oneline diagram with the power flow already solved

Example 2, Simplifying Algebraic Equations

- We need a more systematic way to find the network equations
- First, create the network Y-bus matrix. This is a 9x9 complex matrix that contains the branch conductances such that

$$\bar{\mathbf{Y}}\bar{V} = \bar{I} = \bar{I}_{load} + \bar{I}_{gen}$$

- Where $\bar{V}$ is the 9x1 vector of node complex voltages and $\bar{I}$ is the 9x1 vector of complex node current injections.
- Notice that if we are using constant impedance loads, $\bar{I}_{load}$ can be written as $\bar{\mathbf{Y}}_{\mathbf{load}}\bar{V}$, and if we are using classical generators, $\bar{I}_{gen} = \bar{\mathbf{Y}}_{\mathbf{gen}}\bar{V} + \bar{I}_{gnort}$, where $\bar{\mathbf{Y}}_{\mathbf{gen}}$ is just the inverse of jX'_d and $\bar{I}_{gnort}$ is $\frac{1}{jX'_d}E_p\angle\delta$. In other words, we convert the Thevenin source to Norton.
- Put this together and we get

$\left(\bar{\mathbf{Y}} - \bar{\mathbf{Y}}_{\mathbf{load}} - \bar{\mathbf{Y}}_{\mathbf{gen}}\right)\bar{V} = \bar{\mathbf{Y}}_{\mathbf{aug}}\bar{V} = \bar{I}_{gnort}$ where $\bar{I}_{gnort}$ is a function of δ only

Example 2, Variables, Equations, and Initialization

- With this reduced form, we have 24 total variables
  - 6 real differential variables (δ and ω for each of the 3 classical generators)
  - 9 complex algebraic variables ($\bar{V} = \mathrm{V_r} + \mathrm{jV_i}$)
- There are also 24 total equations
  - For each generator there are 2 differential equations of the form

$$\dot{\delta} = \omega \cdot \omega_s \qquad \dot{\omega} = \frac{1}{2H}\left(\frac{P_m}{\omega+1} - \frac{E_p}{X_d'}\left(V_r \sin\delta - V_i \cos\delta\right)\right)$$

  - And there are 9 complex network algebraic equations represented by

$$\bar{\mathbf{Y}}_{\mathbf{aug}}\bar{V} = \bar{I}_{gnort}$$

- Given a power flow solution we have $\bar{V}$ and hence we can get $\bar{I}_{gnort}$
  - $\bar{I}_{gnort}$ is directly related to $E_p\angle\delta$ for each generator
  - Then we use the swing equation at equilibrium to get P_m
  - Of course, $\omega = 0$ is an equilibrium point for all generators

Example 2, Solution Procedure

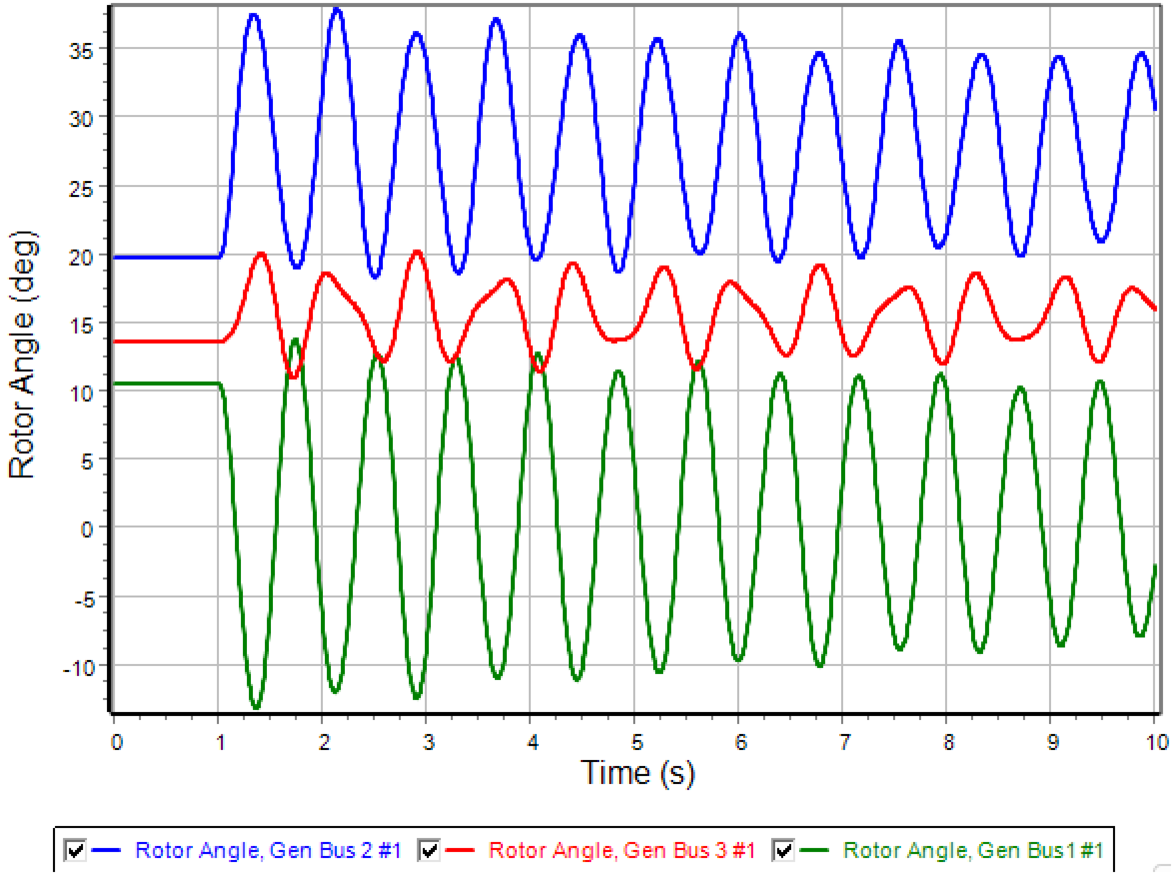
1. Solve the power flow to get $\bar{V}$, then do equilibrium point analysis to find initial values for $\omega, \delta, E_p,$ and T_m for each of the generators
2. At each time step do the following:
    1. Use δ values to get $\bar{I}_{gnort}$ and then solve $\bar{V} = \bar{\mathbf{Y}}_{\mathbf{aug}}^{-1}\, \bar{I}_{gnort}$
    2. Using $\bar{V}$, evaluate the differential equations $f(x)$ and get $k_1 = \Delta t\, f(x)$
    3. Use new values of δ from $x + k_1$ to follow the same procedure and get new values of $\bar{I}_{gnort}$ and $\bar{V}$
    4. Using the k_1 value of $\bar{V}$, evaluate the differential equations again for $f(x + k_1)$ and then get $k_2 = \Delta t\, f(x + k_1)$
    5. Find new values of δ and ω by using the equation $x(t + \Delta t) = x(t) + \frac{1}{2}(k_1 + k_2)$
3. When the fault occurs, change $\bar{\mathbf{Y}}_{\mathbf{aug}}$ to model the added fault. Then when the fault clears and the line opens, change $\bar{\mathbf{Y}}_{\mathbf{aug}}$ to reflect this

Example 2, Other Notes

- Generator parameters are specified on the machine's own per-unit base. To use for system studies, convert to the system base of 100 MVA
- Loads are given based on $P + jQ$. The admittance that becomes part of $\mathbf{Y_{aug}}$ for the stability analysis is operating point dependent and given by $(P - jQ)/|V|^2$
- Rotor angle results are often given by shifting them relative to the average rotor angle.

Results in PowerWorld, Rotor Angle

- This is for a 6-cycle fault with time step of ½ cycle (0.083333 s)

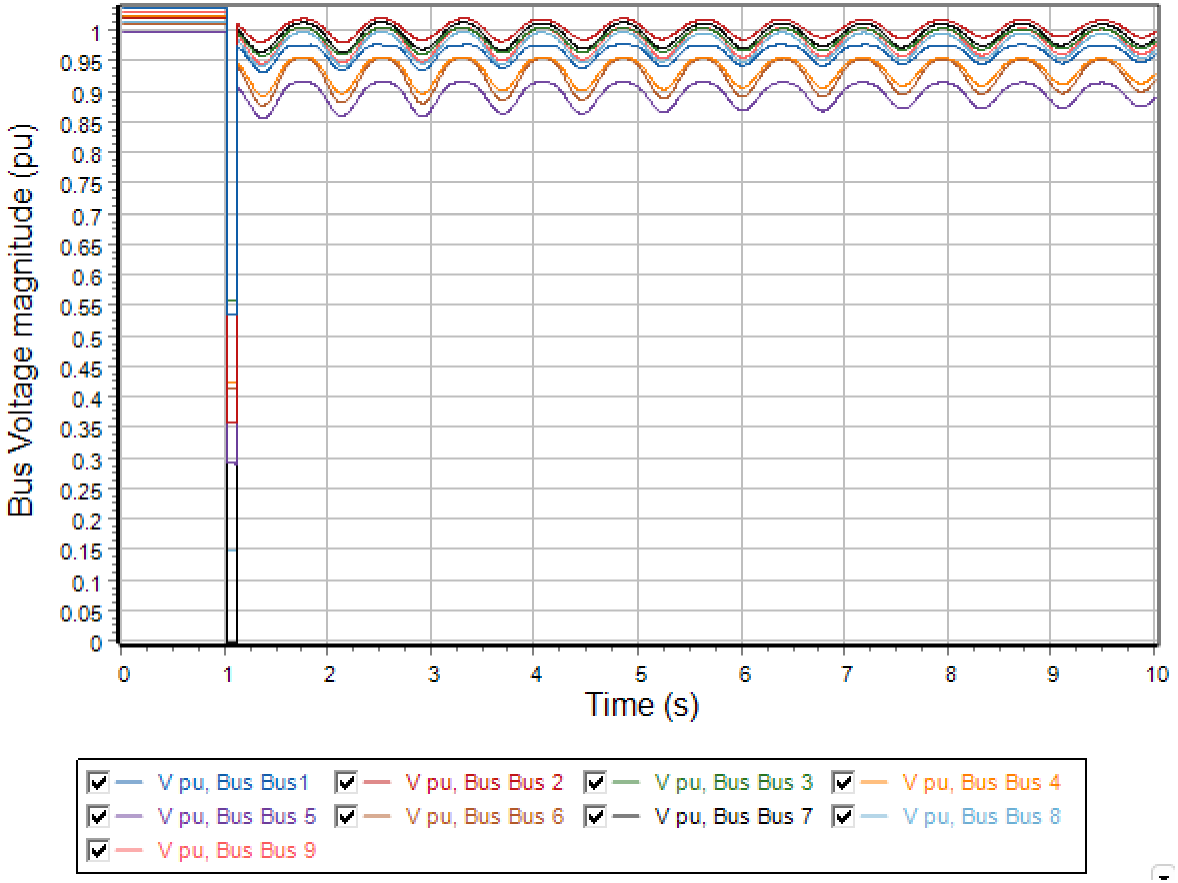


Note that the rotor angles stay together
But have relatively low damping of their oscillations

Generators 1 and 2 oscillate against each other,
With Generator 3 caught in the middle

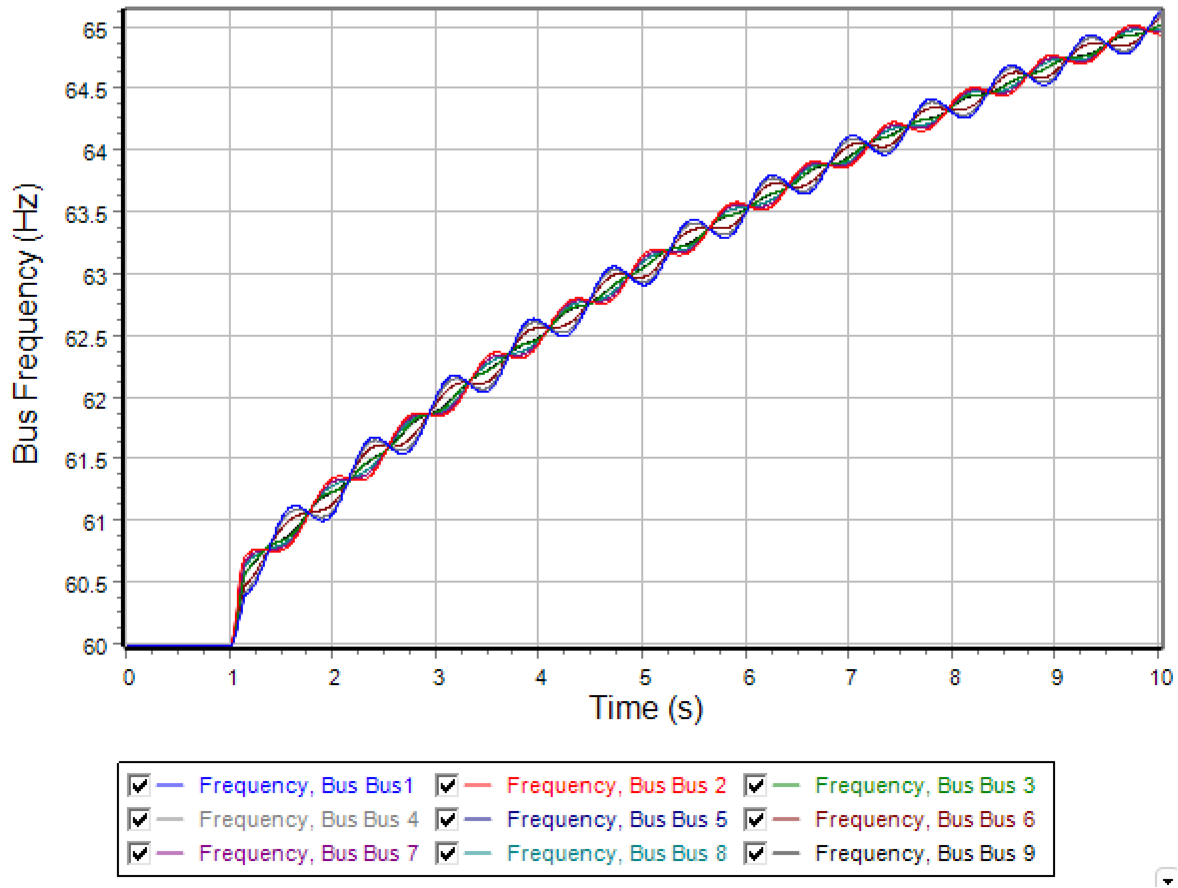
Results in PowerWorld, Voltage

- Similar to before, voltages drop during fault and then recover and oscillate. In this case there is essentially no damping



Results in PowerWorld, Frequency

- We haven’t yet discussed how to calculate frequency, but it is related to machine speed ω.
- This is not a good result! But it should not be surprising
- Even though the machines stay together, the overall frequency increases since there is more energy entering the system than leaving



Critical Clearing Time (CCT)

- Change the fault duration and find the time at which the generators lose synchronism with one another (this would lead to a blackout for sure!)