ECEN 667
Power System Stability

Lecture 15: Small Signal Stability Analysis of Systems Driven by Inverter-Based Resources

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Announcements

- HW #4 is on the website, due Oct 30th at 8 AM.
- Read book chapters 9 and 8
- Review the slides and PowerWorld examples

Recall, Dynamic System Linearization

- For a general system of the form

$$\dot{x} = f(x,y)$$
$$0 = g(x,y)$$

- We can *linearize* the system by finding various partial derivative matrices about a given operating point x_0, y_0

$$A = \frac{\partial f}{\partial x};\ \ B = \frac{\partial f}{\partial y};\ \ C = \frac{\partial g}{\partial x};\ \ D = \frac{\partial g}{\partial y}$$

- Then we get the linearized system as follows

$$\dot{x} = Ax + By$$
$$0 = Cx + Dy$$

- If D is a non-singular matrix we can also create an equivalent ODE system

$$\dot{x} = (A - BD^{-1}C)x = A_{sys}x$$

Recall, Small Signal Stability Analysis

- Small signal stability is the ability of the power system to maintain synchronism following a small disturbance
  - System is continually subject to small disturbances, such as changes in the load
- The operating equilibrium point (EP) must be stable
- Small system stability analysis (SSA) is studied to get a feel for how close the system is to losing stability and to get additional insight into the system response
  - There must be positive damping
- This is primarily done by looking at the properties of the A_{sys} matrix
  - Eigenvalues and eigenvectors
  - Participation factors
  - Mode shape
- Goal is to determine how various parameters affect system response

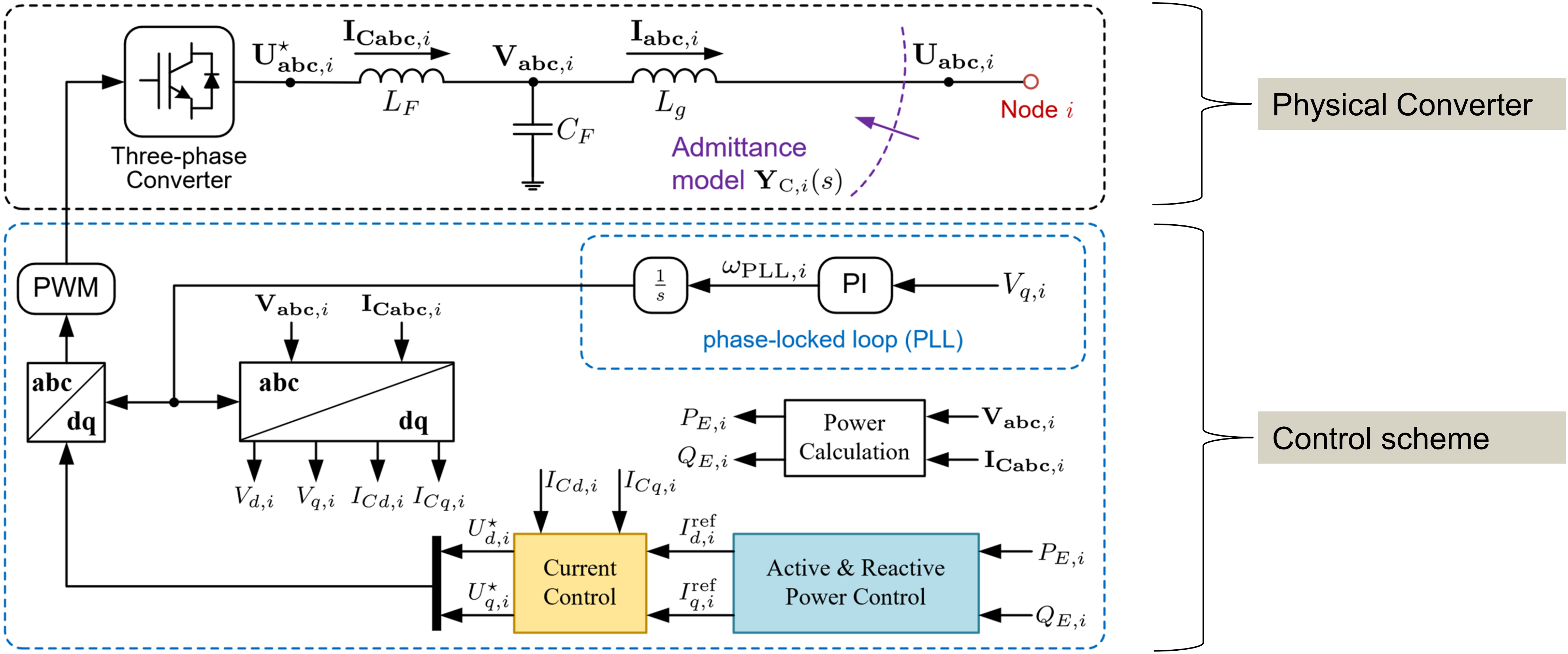
Stability Analysis of Converter-Driven Systems

- With growth in solar, wind, and batteries, there is much interest in understanding power system stability in the context of dynamics driven by inverter-based resources
- This is still very much an active area of research
- We previously discussed industry models for time-domain stability analysis with wind and solar generators
- These slides outline some analytical stability approaches from a linearized, small signal stability point of view, from recent research
- An interesting recent paper in this area is the following:

[1] L. Huang *et al*., "Gain and Phase: Decentralized Stability Conditions for Power Electronics-Dominated Power Systems," in *IEEE Transactions on Power Systems*, vol. 39, no. 6, pp. 7240-7256, Nov. 2024, doi: 10.1109/TPWRS.2024.3380528

A Linearized GFL Model

- A grid following inverter (GFL) has the following basic control structure (Fig. 2 from the referenced paper)



Small-Signal IBR Model

- A small-signal IBR model could be generically given as

$$-\begin{bmatrix} I_{d,i} \\ I_{q,i} \end{bmatrix} = S_i e^{J\theta_i} \mathbf{Y}_{C,i}(s) e^{-J\theta_i} \begin{bmatrix} V_{d,i} \\ V_{q,i} \end{bmatrix}$$

- Here, V and I are given in system-wide d-q coordinates.
- Constants S_i and θ_i represent the device’s steady-state per-unit capacity ratio and steady-state angle with respect to the converter’s local d-q coordinates.
- The matrix $\mathbf{Y}_{C,i}(s)$ gives the frequency-domain representation of the converter’s controllers and physical structure, including phase-locked loop, current control, modulation dynamics, and L-C-L filtering

Network Dynamics

- Network dynamics are formulated slightly differently in this analysis, because IBR dynamics are so fast that there is a need to consider the transients associated with the L and C of the network

$$\mathbf{I} = \mathbf{Y}(\mathbf{s})\,\mathbf{V}$$

- Where $\mathbf{Y}$ is a matrix that includes the s-domain dynamics. For example, an R-L branch would be modeled with

$$Y_{ij}(s) = B_{ij}\begin{bmatrix} \frac{s}{\omega_o} + \epsilon_{ij} & -1 \\ 1 & \frac{s}{\omega_o} + \epsilon_{ij} \end{bmatrix}^{-1}$$

- By Kron reduction, you can eliminate all but the converter nodes and get

$$\mathbf{Y_{grid}}(s) = \left[\mathbf{Y_1}(s) - \mathbf{Y_2}(s)\mathbf{Y_4^{-1}}(s)\mathbf{Y_3}(s)\right]$$

Multi-Converter System Representation

- Consider a matrix with all the converter dynamics as block-diagonal

$$\mathbf{Y}_{\mathbf{C}}^{\mathrm{N}}(s) = \begin{bmatrix} \mathbf{Y}_{\mathbf{C},\mathbf{1}}(s) & & & \\ & \mathbf{Y}_{\mathbf{C},\mathbf{2}}(s) & & \\ & & \cdots & \\ & & & \mathbf{Y}_{\mathbf{C},\mathbf{N}}(s) \end{bmatrix}$$

- Now the combined system closed-loop model is

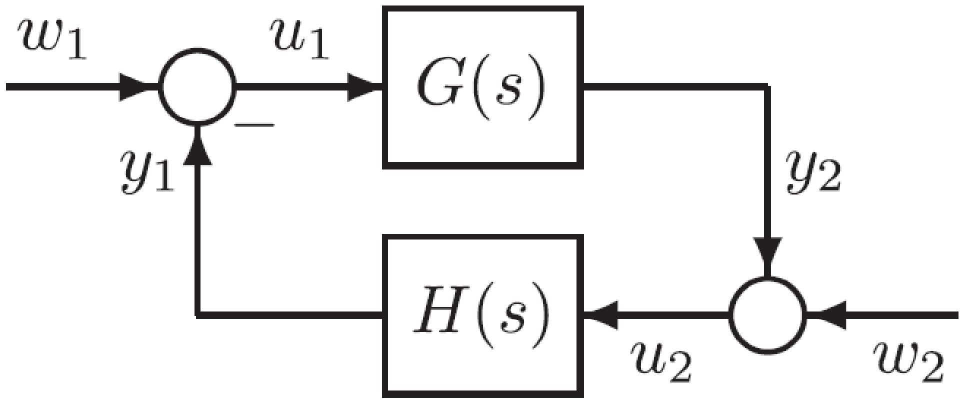
$$-\mathbf{I} = \mathbf{S}e^{J\theta}\mathbf{Y}_{\mathbf{C}}^{\mathrm{N}}(s)e^{-J\theta}\mathbf{V}$$
$$\mathbf{I} = \mathbf{Y}_{\mathbf{grid}}(s)\mathbf{V}$$

- How can we determine if such a closed-loop system is stable?

Mixed Small Gain-Phase Theorem

- According to [1], one way to approach this problem is with the mixed small gain-phase theorem:
  - Let he open-loop systems G and H be real, rational, stable, and proper transfer function matrices. Then, the closed-loop system (see Fig) is stable if, for every frequency ω, one of the following two conditions are met:
  - (1) $\bar{\sigma}\big(G(j\omega)\big)\bar{\sigma}\big(H(j\omega)\big) < 1$ ($\bar{\sigma}$ is the gain margin, i.e. the largest singular value)
  - (2) Three conditions need to be met:
    - (2A) $G(j\omega)$ and $H(j\omega)$ are sectorial, meaning for all $||x|| = 1,\ x^T G x \neq 0$
    - (2B) $\bar{\phi}\big(G(j\omega)\big) + \bar{\phi}\big(H(j\omega)\big) < \pi$
    - (2C) $\underline{\phi}\big(G(j\omega)\big) + \underline{\phi}\big(H(j\omega)\big) > -\pi$

    ($\bar{\phi}$ and $\underline{\phi}$ are the largest and smallest angles in the "sectorial decomposition" of G or H)



Decentralized Stability Conditions

- Applying this to the power system, recent research has shown that there can be decentralized stability conditions [1] for converter-dominated power systems, as follows --

1. Need to ensure that both the open-loop grid model $\mathbf{Y_{grid}}(s)$ and the open-loop of each individual converter model $\mathbf{Y_{C,i}}(s)$ is stable
2. Go through every frequency (conceptually) and determine if one of the following two conditions holds
   1. $\max_{\mathrm{i}} \overline{\sigma}\left(\mathbf{Y_{C,i}}(\mathrm{j}\omega)\right) < \underline{\sigma}\left(\mathbf{Y_{grid}}(\mathrm{j}\omega)\right)$
   2. All three of the following hold
      - A. $\max_{i} \bar{\phi}\left(Y_{C,i}(j\omega)\right) < \pi - \bar{\phi}\left(Y_{grid}^{-1}(j\omega)\right)$
      - B. $\min_{i} \underline{\phi}\left(Y_{C,i}(j\omega)\right) > -\pi - \underline{\phi}\left(Y_{grid}^{-1}(j\omega)\right)$
      - C. $\max_{i} \bar{\phi}\left(Y_{C,i}(j\omega)\right) - \min_{i} \underline{\phi}\left(Y_{C,i}(j\omega)\right) < \pi$

Features of a Decentralized Stability Analysis Method

- Do not need to form the full system A matrix or do large system eigenvalue analysis
- Valid over full range of frequencies
- Can consider quite complex converter control properties
- Full paper [1] explains method in more detail, including how to consider GFM controls and mixed converter and synchronous machine systems
- Potential challenges / ongoing research
  - If an actual system does not meet these stability criteria, it may still be stable. These are only sufficient conditions
  - Only valid for a single linearized operating point with small-signal disturbances