

ECEN 667

Power System Stability

Lecture 14: Small Signal Stability Analysis

Prof. Adam Birchfield

Dept. of Electrical and Computer Engineering

Texas A&M University

abirchfield@tamu.edu



TEXAS A&M
UNIVERSITY

Announcements



- HW #4 is on the website, due Oct 30th at 8 AM.
- Read book chapters 9 and 8
- Review the slides and PowerWorld examples

Dynamic System Linearization



- For a general system of the form

$$\begin{aligned}\dot{x} &= f(x, y) \\ 0 &= g(x, y)\end{aligned}$$

- We can *linearize* the system by finding various partial derivative matrices about a given operating point x_0, y_0

$$A = \frac{\partial f}{\partial x}; \quad B = \frac{\partial f}{\partial y}; \quad C = \frac{\partial g}{\partial x}; \quad D = \frac{\partial g}{\partial y}$$

- Then we get the linearized system as follows

$$\begin{aligned}\dot{x} &= Ax + By \\ 0 &= Cx + Dy\end{aligned}$$

- If D is a non-singular matrix we can also create an equivalent ODE system

$$\dot{x} = (A - BD^{-1}C)x = A_{sys}x$$

Small Signal Stability Analysis



- Small signal stability is the ability of the power system to maintain synchronism following a small disturbance
 - System is continually subject to small disturbances, such as changes in the load
- The operating equilibrium point (EP) must be stable
- Small system stability analysis (SSA) is studied to get a feel for how close the system is to losing stability and to get additional insight into the system response
 - There must be positive damping
- This is primarily done by looking at the properties of the A_{sys} matrix
 - Eigenvalues and eigenvectors
 - Participation factors
 - Mode shape
- Goal is to determine how various parameters affect system response

Eigenvalues, Right Eigenvectors



- For an n -by- n matrix \mathbf{A} the eigenvalues of \mathbf{A} are the roots of the characteristic equation:

$$\det[\mathbf{A} - \lambda\mathbf{I}] = |\mathbf{A} - \lambda\mathbf{I}| = 0$$

- Assume $\lambda_1 \dots \lambda_n$ as distinct (no repeated eigenvalues).
- For each eigenvalue λ_i there exists an eigenvector \mathbf{v}_i such that:

$$\mathbf{A}\mathbf{v}_i = \lambda_i\mathbf{v}_i$$

- \mathbf{v}_i is called a right eigenvector
- If λ_i is complex, then \mathbf{v}_i has complex entries

Left Eigenvectors, Eigenvector Properties



- For each eigenvalue λ_i there exists a left eigenvector \mathbf{w}_i such that:

$$\mathbf{w}_i^t \mathbf{A} = \mathbf{w}_i^t \lambda_i$$

- Equivalently, the left eigenvector is the right eigenvector of \mathbf{A}^T ; that is,

$$\mathbf{w}_i^t \mathbf{A} = \mathbf{w}_i^t \lambda_i$$

- The right and left eigenvectors are orthogonal i.e.

$$\mathbf{w}_i^t \mathbf{v}_i \neq 0, \mathbf{w}_i^t \mathbf{v}_j = 0 (i \neq j)$$

- We can normalize the eigenvectors so that:

$$\mathbf{w}_i^t \mathbf{v}_i = 1, \mathbf{w}_i^t \mathbf{v}_j = 0 (i \neq j)$$

Eigenvector Example, Part 2



$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}, |\mathbf{A} - \lambda \mathbf{I}| = 0 \Rightarrow \begin{vmatrix} 1 - \lambda & 4 \\ 3 & 2 - \lambda \end{vmatrix} = 0$$

$$\lambda^2 - 3\lambda - 10 = 0$$

$$\Rightarrow \lambda_{1,2} = \frac{3 \pm \sqrt{(3)^2 + 4(10)}}{2} = \frac{3 \pm \sqrt{49}}{2} = 5, -2$$

$$\lambda_1 = 5$$

$$\mathbf{A}\mathbf{v}_1 = 5\mathbf{v}_1 \Rightarrow \mathbf{v}_1 = \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} \Rightarrow \begin{cases} v_{11} + 4v_{21} = 5v_{11} \\ 3v_{11} + 2v_{21} = 5v_{21} \end{cases}$$

Similarly,

$$\lambda_2 = -2 \Rightarrow \mathbf{v}_2 = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$

Choose

$$\mathbf{v}_{21} = 1 \Rightarrow \mathbf{v}_{11} = 1$$

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Eigenvector Example, Part 3



- Left eigenvectors

$$\lambda_1 = 5 \quad \mathbf{w}_1^t \mathbf{A} = \mathbf{w}_1^t 5 \Rightarrow [w_{11} \ w_{21}] \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} = 5[w_{11} \ w_{21}]$$

$$\begin{aligned} w_{11} + 3w_{21} &= 5w_{11} \\ 4w_{11} + 2w_{21} &= 5w_{21} \end{aligned} \Rightarrow \text{Let } w_{21} = 4, \text{ then } w_{11} = 3$$

$$\mathbf{w}_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \quad \lambda_2 = -2 \Rightarrow \mathbf{w}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 4 \\ -3 \end{bmatrix} \quad \mathbf{w}_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \quad \mathbf{w}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\text{Verify } \mathbf{w}_1^t \mathbf{v}_1 = 7, \quad \mathbf{w}_2^t \mathbf{v}_2 = 7, \quad \mathbf{w}_2^t \mathbf{v}_1 = 0, \quad \mathbf{w}_1^t \mathbf{v}_2 = 0$$

- We would like to make $\mathbf{w}_i^t \mathbf{v}_i = 1$.
- This can be done in many ways.

Eigenvector Example, Part 4



- It can be verified that $\mathbf{W}^T = \mathbf{V}^{-1}$.
- The left and right eigenvectors are used in computing the participation factor matrix.

$$\text{Let } \mathbf{W} = \frac{1}{7} \begin{bmatrix} 3 & 1 \\ 4 & -1 \end{bmatrix}$$

$$\text{Then } \mathbf{W}^T \mathbf{V} = \mathbf{I}$$

$$\text{Verify } \frac{1}{7} \begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Modal Matrices



- The deviation away from an equilibrium point can be defined as

$$\Delta \dot{\mathbf{x}} = \mathbf{A} \Delta \mathbf{x}$$

- From this equation it is difficult to determine how parameters in \mathbf{A} affect a particular \mathbf{x} because of the variable coupling
- To decouple the problem first define the matrices of the right and left eigenvectors (the modal matrices)

$$\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n] \quad \& \quad \mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_n]$$
$$\mathbf{A}\mathbf{V} = \mathbf{V}\mathbf{\Lambda} \quad \text{when} \quad \mathbf{\Lambda} = \text{Diag}(\lambda_i)$$

Modal Matrices, Part 2



- It follows that

$$\mathbf{V}^{-1}\mathbf{A}\mathbf{V} = \mathbf{\Lambda}$$

- To decouple the variables define \mathbf{z} so

$$\Delta\mathbf{x} = \mathbf{V}\mathbf{z} \rightarrow \Delta\dot{\mathbf{x}} = \mathbf{V}\dot{\mathbf{z}} = \mathbf{A}\Delta\mathbf{x} = \mathbf{A}\mathbf{V}\mathbf{z}$$

- Then

$$\dot{\mathbf{z}} = \mathbf{V}^{-1}\mathbf{A}\mathbf{V}\mathbf{z} = \mathbf{W}\mathbf{A}\mathbf{V}\mathbf{z} = \mathbf{\Lambda}\mathbf{z}$$

- Since $\mathbf{\Lambda}$ is diagonal, the equations are now uncoupled with

$$\dot{z}_i = \lambda_i z_i$$

- So

$$\Delta\mathbf{x}(t) = \mathbf{V}\mathbf{z}(t)$$

Example

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$$

$$\mathbf{V} = \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix}$$

$$\mathbf{V}^{-1}\mathbf{A}\mathbf{V} = \begin{bmatrix} 5 & 0 \\ 0 & -2 \end{bmatrix}$$

Modal Matrices, Part 3



- Thus, the response can be written in terms of the individual eigenvalues and right eigenvectors as

$$\Delta \mathbf{x}(t) = \sum_{i=1}^n \mathbf{v}_i z_i(0) e^{\lambda_i t}$$

Note, we are requiring that the eigenvalues be distinct!

- Furthermore with

$$\Delta \mathbf{x} = \mathbf{VZ} \Rightarrow \mathbf{z} = \mathbf{V}^{-1} \mathbf{x} = \mathbf{W}^T \mathbf{x}$$

- So, $\mathbf{z}(t)$ can be written as using the left eigenvectors as

$$\mathbf{z}(t) = \mathbf{W}^t \mathbf{x}(t) = [\mathbf{w}_1 \mathbf{w}_2 \dots \mathbf{w}_n]^t \begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix}$$

Modal Matrices, Part 4



- We can then write the response $\mathbf{x}(t)$ in terms of the modes of the system

$$z_i(t) = \mathbf{w}_i^t \mathbf{x}(t)$$

$$z_i(0) = \mathbf{w}_i^t \mathbf{x}(0) \underline{\Delta} c_i$$

$$\text{so } \mathbf{x}(t) = \sum_{i=1}^n \mathbf{v}_i c_i e^{\lambda_i t}$$

$$\text{Expanding } \Delta x_i(t) = v_{i1} c_1 e^{\lambda_1 t} + v_{i2} c_2 e^{\lambda_2 t} + \dots + v_{in} c_n e^{\lambda_n t}$$

- So c_i is a scalar that represents the magnitude of excitation of the i^{th} mode from the initial conditions

Numerical Example



$$\begin{bmatrix} \Delta \dot{x}_1 \\ \Delta \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 8 & -2 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}, \Delta \mathbf{x}(0) = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

Eigenvalues are $\lambda_1 = -4, \lambda_2 = 2$

Eigenvectors are $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Modal matrix $\mathbf{V} = \begin{bmatrix} 1 & 1 \\ -4 & 2 \end{bmatrix}$

Normalize so $\mathbf{V} = \begin{bmatrix} 0.2425 & 0.4472 \\ -0.9701 & 0.8944 \end{bmatrix}$

Left eigenvector matrix is:

$$\mathbf{W}^T = \mathbf{V}^{-1} = \begin{bmatrix} 1.3745 & -0.6872 \\ 1.4908 & 0.3727 \end{bmatrix}$$

$$\dot{\mathbf{z}} = \mathbf{W}^T \mathbf{A} \mathbf{V} \mathbf{z}$$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

Numerical Example (cont'd)



$$\dot{z}_1 = -4z_1, \mathbf{z}(0) = V^{-1}\mathbf{x}(0)$$

$$\dot{z}_2 = 2z_2, \begin{bmatrix} z_1(0) \\ z_2(0) \end{bmatrix} = \begin{bmatrix} 4.123 \\ 0 \end{bmatrix}$$

$$z_1(t) = z_1(0)e^{-4t}; z_2(t) = z_2(0)e^{2t}, \mathbf{c} = \mathbf{W}^T \mathbf{x}(0) = \begin{bmatrix} 4.123 \\ 0 \end{bmatrix}$$

$$\mathbf{x} = \mathbf{V}\mathbf{z}$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix}$$

$$= c_1 \begin{bmatrix} 0.2425 \\ -0.9701 \end{bmatrix} z_1(t) + c_2 \begin{bmatrix} 0.4472 \\ 0.8944 \end{bmatrix} z_2(t) = \sum_{i=1}^2 c_i \mathbf{v}_i z_i(0) e^{\lambda_i t}$$

Because of the initial condition, the 2nd mode does not get excited

Mode Shape, Sensitivity and Participation Factors



- So we have

$$\mathbf{x}(t) = \mathbf{V}\mathbf{z}(t), \quad \mathbf{z}(t) = \mathbf{W}^t \mathbf{x}(t)$$

- $\mathbf{x}(t)$ are the original state variables, $\mathbf{z}(t)$ are the transformed variables so that each variable is associated with only one mode.
- From the first equation the right eigenvector gives the “mode shape” i.e. relative activity of state variables when a particular mode is excited.
- For example, the degree of activity of the state variable x_k in the \mathbf{v}_i mode is given by the element V_{ki} of the right eigenvector matrix \mathbf{V}

Mode Shape, Sensitivity and Participation Factors



- The magnitude of elements of \mathbf{v}_i give the extent of activities of n state variables in the i^{th} mode and angles of elements (if complex) give phase displacements of the state variables with regard to the mode.
- The left eigenvector \mathbf{w}_i identifies which combination of original state variables display only the i^{th} mode.

Eigenvalue Parameter Sensitivity



- To derive the sensitivity of the eigenvalues to the parameters recall $\mathbf{A}\mathbf{v}_i = \lambda_i\mathbf{v}_i$; take the partial derivative with respect to A_{kj} by using the chain rule

$$\frac{\partial \mathbf{A}}{\partial A_{kj}} \mathbf{v}_i + \mathbf{A} \frac{\partial \mathbf{v}_i}{\partial A_{kj}} = \frac{\partial \lambda_i}{\partial A_{kj}} \mathbf{v}_i + \lambda_i \frac{\partial \mathbf{v}_i}{\partial A_{kj}}$$

Multiply by \mathbf{w}_i^t

$$\mathbf{w}_i^t \frac{\partial \mathbf{A}}{\partial A_{kj}} \mathbf{v}_i + \mathbf{w}_i^t \mathbf{A} \frac{\partial \mathbf{v}_i}{\partial A_{kj}} = \mathbf{w}_i^t \frac{\partial \lambda_i}{\partial A_{kj}} \mathbf{v}_i + \mathbf{w}_i^t \lambda_i \frac{\partial \mathbf{v}_i}{\partial A_{kj}}$$

$$\mathbf{w}_i^t \frac{\partial \mathbf{A}}{\partial A_{kj}} \mathbf{v}_i + \mathbf{w}_i^t [\mathbf{A} - \lambda_i \mathbf{I}] \frac{\partial \mathbf{v}_i}{\partial A_{kj}} = \mathbf{w}_i^t \frac{\partial \lambda_i}{\partial A_{kj}} \mathbf{v}_i$$

Eigenvalue Parameter Sensitivity, Part 2



- This is simplified by noting that $\mathbf{w}_i^t (\mathbf{A} - \lambda_i \mathbf{I}) = 0$ by the definition of \mathbf{w}_i being a left eigenvector
- Therefore

$$\mathbf{w}_i^t \frac{\partial \mathbf{A}}{\partial A_{kj}} \mathbf{v}_i = \frac{\partial \lambda_i}{\partial A_{kj}}$$

- Since all elements of $\frac{\partial \mathbf{A}}{\partial A_{kj}}$ are zero, except the k^{th} row, j^{th} column is 1
- Thus $\frac{\partial \lambda_i}{\partial A_{kj}} = W_{ki} V_{ji}$

Sensitivity Example



- In the previous example we had

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}, \quad \lambda_{1,2} = 5, -2, \quad \mathbf{V} = \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix}, \quad \mathbf{W} = \frac{1}{7} \begin{bmatrix} 3 & 1 \\ 4 & -1 \end{bmatrix}$$

- Then the sensitivity of λ_1 and λ_2 to changes in \mathbf{A} are

$$\frac{\partial \lambda_i}{\partial A_{kj}} = W_{ki} V_{ji} \rightarrow \frac{\partial \lambda_1}{\partial \mathbf{A}} = \frac{1}{7} \begin{bmatrix} 3 & 3 \\ 4 & 4 \end{bmatrix}, \quad \frac{\partial \lambda_2}{\partial \mathbf{A}} = \frac{1}{7} \begin{bmatrix} 4 & -3 \\ -4 & 3 \end{bmatrix}$$

- For example with $\hat{\mathbf{A}} = \begin{bmatrix} 1 & 4 \\ 3 & 2.1 \end{bmatrix}$, $\hat{\lambda}_{1,2} = 5.057, -1.957$
- Or if $\hat{\mathbf{A}} = \begin{bmatrix} 1 & 4 \\ 3 & 3 \end{bmatrix}$, $\hat{\lambda}_{1,2} = 5.61, -1.61$,

Participation Factors



- The participation factors, P_{ki} , are used to determine how much the k^{th} state variable participates in the i^{th} mode

$$P_{ki} = V_{ki}W_{ki}$$

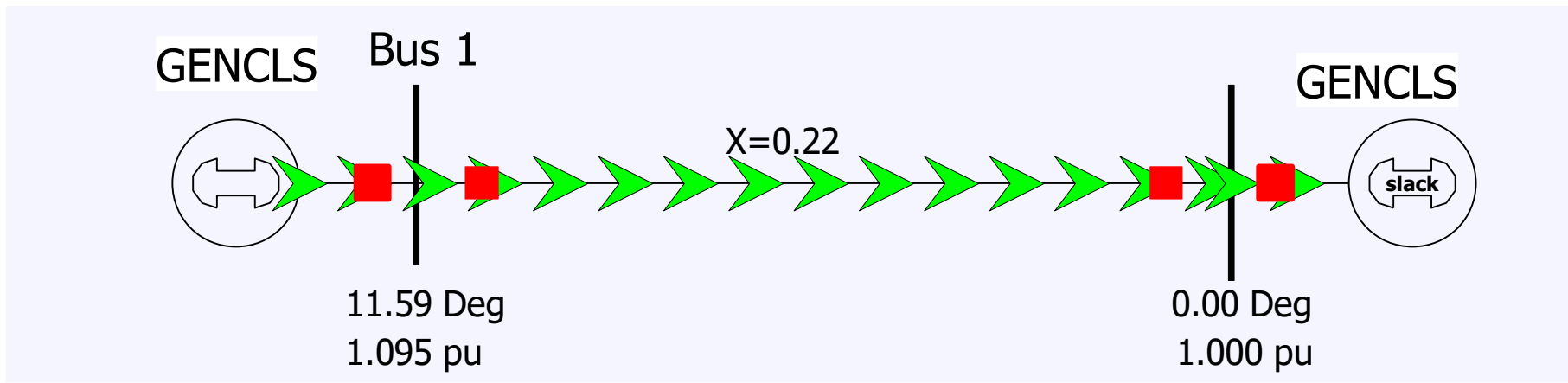
- The sum of the participation factors for any mode or any variable sum to 1
- The participation factors are quite useful in relating the eigenvalues to portions of a model
- For the previous example with $P_{ki} = V_{ki}W_{ik}$ and

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix}, \quad \mathbf{W} = \frac{1}{7} \begin{bmatrix} 3 & 1 \\ 4 & -1 \end{bmatrix} \rightarrow \mathbf{P} = \frac{1}{7} \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}$$

SSA Two Generator Example



- Consider the two bus, two classical generator system from before with $X_{d1}'=0.3$, $H_1=3.0$, $X_{d2}'=0.2$, $H_2=6.0$



- Essentially everything needed to calculate the **A**, **B**, **C** and **D** matrices was covered previously

SSA Two Generator Example, Part 2



- The **A** matrix is calculated differentiating **f(x,y)** with respect to **x** (where **x** is $\delta_1, \Delta\omega_1, \delta_2, \Delta\omega_2$)

$$\frac{d\delta_1}{dt} = \Delta\omega_{1.pu}\omega_s$$

$$\frac{d\Delta\omega_{1.pu}}{dt} = \frac{1}{2H_1} (P_{M1} - P_{E1} - D_1\Delta\omega_{1.pu})$$

$$\frac{d\delta_2}{dt} = \Delta\omega_{2.pu}\omega_s$$

$$\frac{d\Delta\omega_{2.pu}}{dt} = \frac{1}{2H_2} (P_{M2} - P_{E2} - D_2\Delta\omega_{2.pu})$$

$$P_{Ei} = (E_{Di}^2 - E_{Di}V_{Di})G_i + (E_{Qi}^2 - E_{Qi}V_{Qi})G_i + (E_{Di}V_{Qi} - E_{Qi}V_{Di})B_i$$

$$E_{Di} + jE_{Qi} = E_i'(\cos \delta_i + j \sin \delta_i)$$

SSA Two Generator Example, Part 3



- Giving

$$\mathbf{A} = \begin{bmatrix} 0 & 376.99 & 0 & 0 \\ -0.761 & 0 & 0 & 0 \\ 0 & 0 & 0 & 376.99 \\ 0 & 0 & -0.389 & 0 \end{bmatrix}$$

- **B**, **C** and **D** are as calculated previously for the implicit integration, except the elements in **B** are not multiplied by $\Delta t/2$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -0.2889 & 0.6505 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0833 & 0.3893 \end{bmatrix}$$

SSA Two Generator Example, Part 4



- The **C** and **D** matrices are

$$\mathbf{C} = \begin{bmatrix} -3.903 & 0 & 0 & 0 \\ -1.733 & 0 & 0 & 0 \\ 0 & 0 & -4.671 & 0 \\ 0 & 0 & 1.0 & 0 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 0 & 7.88 & 0 & -4.54 \\ -7.88 & 0 & 4.54 & 0 \\ 0 & -4.54 & 0 & 9.54 \\ 4.54 & 0 & -9.54 & 0 \end{bmatrix}$$

- Giving

$$\mathbf{A}_{sys} = \mathbf{A} - \mathbf{BD}^{-1}\mathbf{C} = \begin{bmatrix} 0 & 376.99 & 0 & 0 \\ -0.229 & 0 & 0.229 & 0 \\ 0 & 0 & 0 & 376.99 \\ 0.114 & 0 & -0.114 & 0 \end{bmatrix}$$

SSA Two Generator

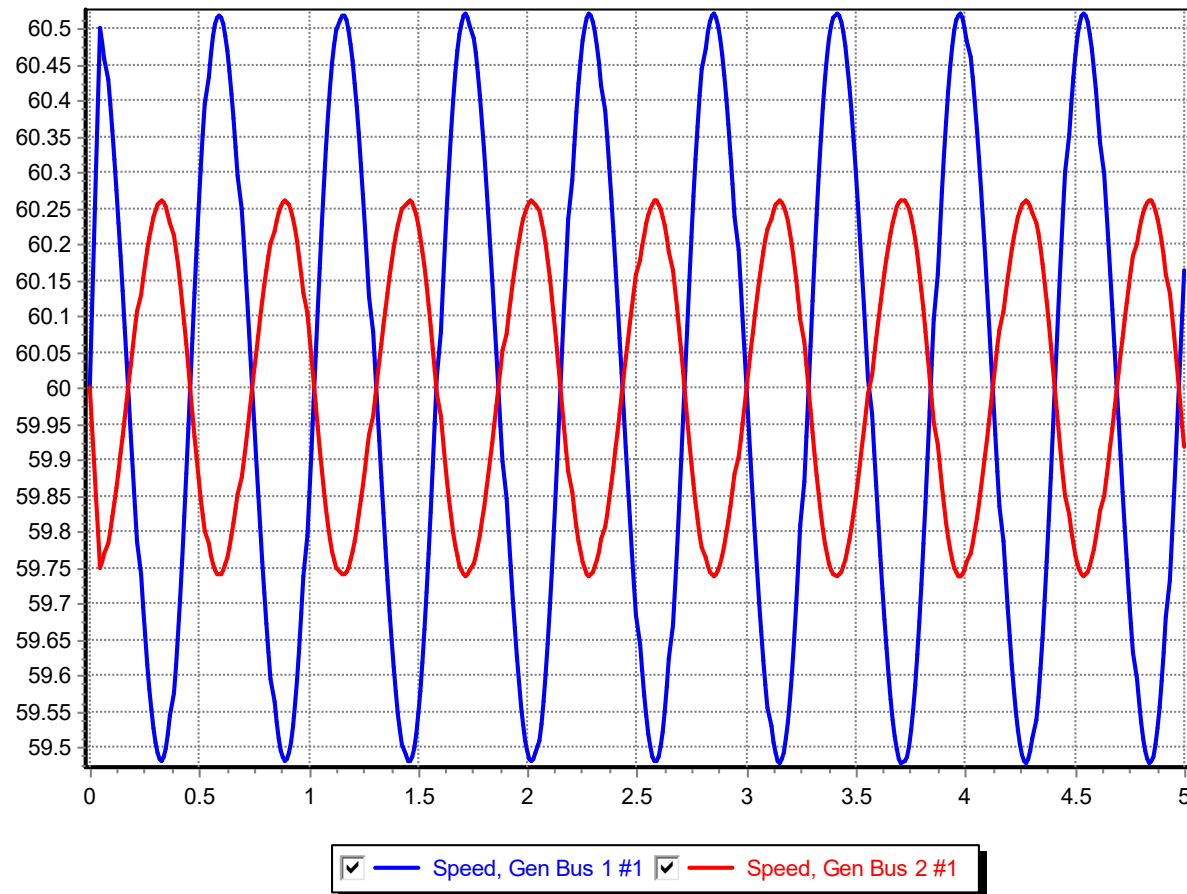


- Calculating the eigenvalues gives a complex pair and two zero eigenvalues
- The complex pair, with values of $\pm j11.39$ corresponds to the generators oscillating against each other at 1.81 Hz
- One of the zero eigenvalues corresponds to the lack of an angle reference
 - Could be rectified by redefining angles to be with respect to a reference angle (see book 226) or we just live with the zero
- Other zero is associated with lack of speed dependence in the generator torques

SSA Two Generator Speeds



- The two generator system response is shown below for a small disturbance

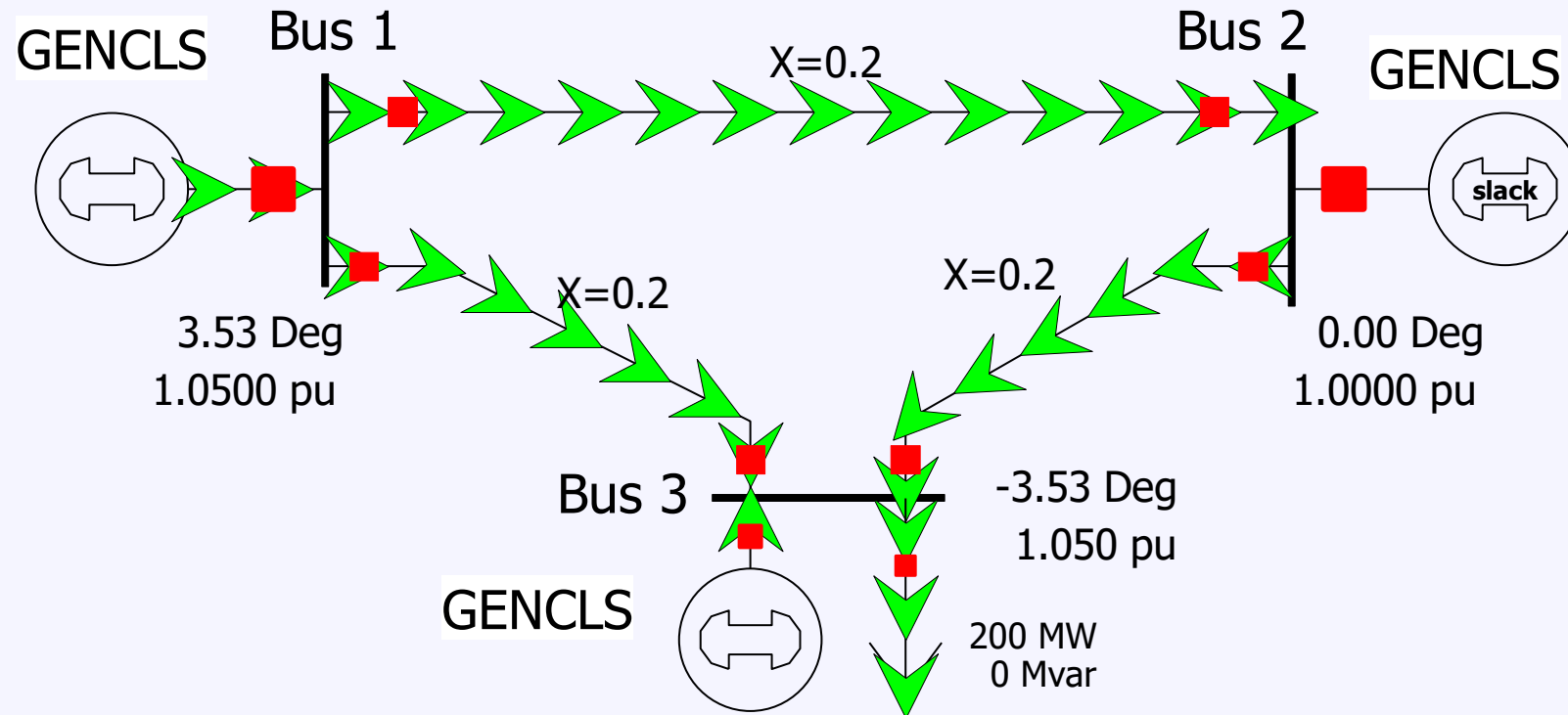


Notice the numerical simulated response closely matches the calculated frequency

SSA Three Generator Example



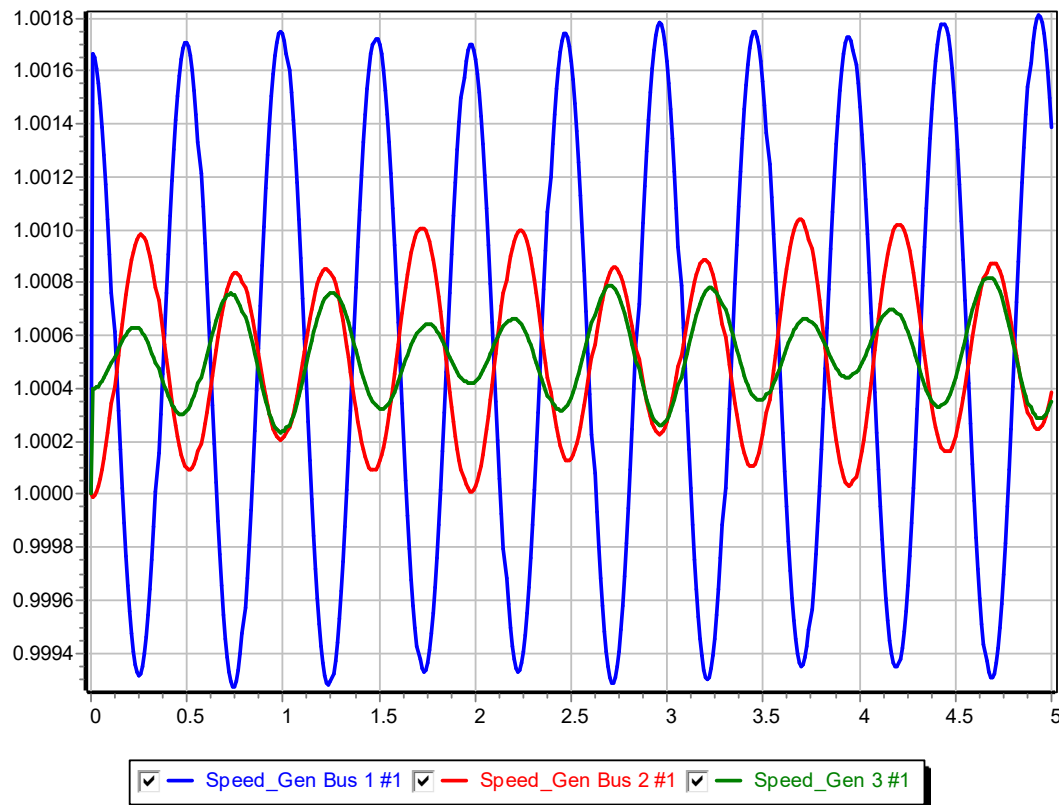
- The two generator system is extended to three generators with the third generator having H_3 of 8 and $X_{d3}'=0.3$



SSA Three Generator Example, Part 2



- Using SSA, two frequencies are identified: one at 2.02 Hz and one at 1.51 Hz



The oscillation is started with a short, self-clearing fault

Shortly we'll discuss modal analysis to determine the contribution of each mode to each signal

PowerWorld case
B2_CLS_3Gen_SSA

Large System Studies



- The challenge with large systems, which could have more than 100,000 states, is the sheer size
 - Most eigenvalues are associated with the local plants
 - Computing all the eigenvalues is computationally challenging, order n^3
- Specialized approaches can be used to calculate particular eigenvalues of large matrices
 - See Kundur, Section 12.8 and associated references

Single Machine Infinite Bus

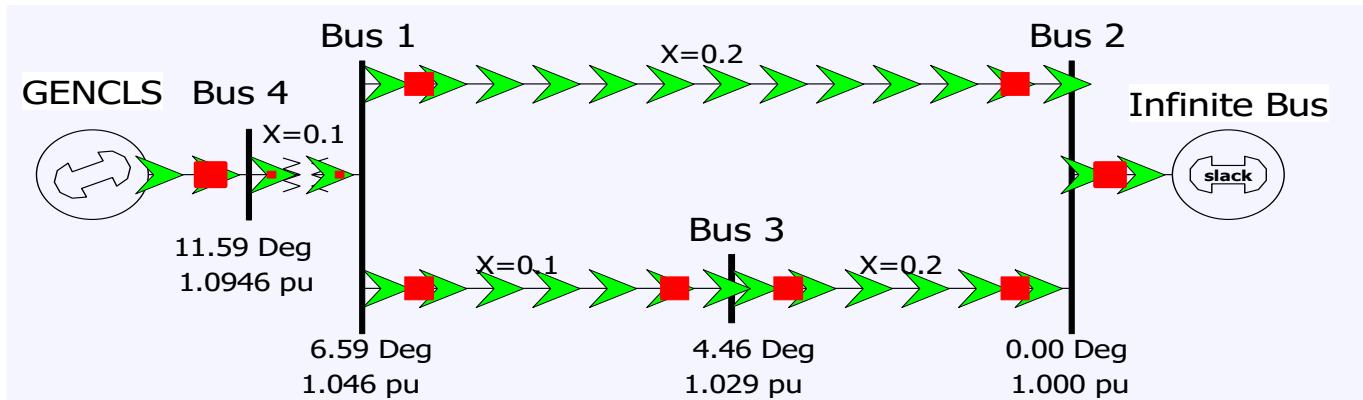


- A quite useful analysis technique is to consider the small signal stability associated with a single generator connected to the rest of the system through an equivalent transmission line
- Driving point impedance looking into the system is used to calculate the equivalent line's impedance
 - The Z_{ij} value can be calculated quite quickly using sparse vector methods
- Rest of the system is assumed to be an infinite bus with its voltage set to match the generator's real and reactive power injection and voltage

Small SMIB Example



- As a small example, consider the 4 bus system shown below, in which bus 2 really is an infinite bus



Z_{44} is Z_{th} in parallel with $jX'_{d,4}$ (which is $j0.3$) so Z_{th} is $j0.22$

- To get the SMIB for bus 4, first calculate Z_{44}

$$Y_{bus} = j \begin{bmatrix} -25 & 0 & 10 & 10 \\ 0 & 1 & 0 & 0 \\ 10 & 0 & -15 & 0 \\ 10 & 0 & 0 & -13.33 \end{bmatrix} \rightarrow Z_{44} = j0.1269$$

Small SMIB Example, Part 2



- The infinite bus voltage is then calculated so as to match the bus i terminal voltage and current

$$\bar{V}_{\text{inf}} = \bar{V}_i - Z_i \bar{I}_i$$

$$\text{where } \left(\frac{P_i + jQ_i}{\bar{V}_i} \right)^* = \bar{I}_i$$

While this was demonstrated on an extremely small system for clarity, the approach works the same for any size system

- In the example we have

$$\left(\frac{P_4 + jQ_4}{\bar{V}_4} \right)^* = \left(\frac{1 + j0.572}{1.072 + j0.220} \right)^* = 1 - j0.328$$

$$\bar{V}_{\text{inf}} = (1.072 + j0.220) - (j0.22)(1 - j0.328)$$

$$\bar{V}_{\text{inf}} = 1.0$$

Calculating the A Matrix



- The SMIB model **A** matrix can then be calculated either analytically or numerically
 - The equivalent line's impedance can be embedded in the generator model so the infinite bus looks like the "terminal"
- This matrix is calculated in PowerWorld by selecting Transient Stability, SMIB Eigenvalues
 - Select Run SMIB to perform an SMIB analysis for all the generators in a case
 - Right click on a generator on the SMIB form and select Show SMIB to see the Generator SMIB Eigenvalue Dialog
 - These two bus equivalent networks can also be saved, which can be quite useful for understanding the behavior of individual generators

Example: Bus 4 SMIB Dialog



- On the SMIB dialog, the General Information tab shows information about the two bus equivalent

Generator SMIB Eigenvalue Information

Bus Number: 4
Bus Name: Bus 4
ID: 1
Area Name: Home (1)

Status: Open Closed

Find By Number
Find By Name
Find ...

Generator Information (on Generator MVA Base)

General Info | A Matrix | Eigenvalues

Generator MVA Base: 100.000

Infinite Bus Voltage Magnitude (pu): 1.0000
Infinite Bus Angle (deg): -0.0000

Terminal Current Magnitude (pu): 1.0526
Terminal Current Angle (deg): -18.193

Terminal Voltage Magnitude (pu): 1.0946
Terminal Voltage Angle (deg): 11.5942

Network Impedance on Generator MVA Base
Network R (Gen Base): 0.00000
Network X (Gen Base): 0.22000

Network Impedance on System MVA Base
Network R (System Base): 0.00000
Network X (System Base): 0.22000

OK Save Cancel Help Print

PowerWorld
case **B4_SMIB**

Example: Bus 4 SMIB Dialog, Part 2



- On the SMIB dialog, the **A** Matrix tab shows the \mathbf{A}_{sys} matrix for the SMIB generator

Row Name	Machine Angle	Machine Speed w
1 Machine Angle	0.0000	376.9911
2 Machine Speed w	-0.3753	0.0000

- In this example A_{21} is showing

$$\begin{aligned} \frac{\partial \Delta \omega_{4,pu}}{\partial \delta_4} &= \frac{1}{2H_4} \left(\frac{-\partial P_{E,4}}{\partial \delta_4} \right) = - \left(\frac{1}{6} \right) \left(\left(\frac{-1}{0.3 + 0.22} \right) (-1.2812 \cos(23.94^\circ)) \right) \\ &= -0.3753 \end{aligned}$$

Example: Bus 4 with GENROU



- The eigenvalues can be calculated for any set of generator models
- This example replaces the bus 4 generator classical machine with a GENROU model
 - There are now six eigenvalues, with the dominate response coming from the electro-mechanical mode with a frequency of 1.84 Hz, and damping of 6.9%

Generator Information (on Generator MVA Base)

General Info A Matrix Eigenvalues

Records Set Columns Options

	Real Part	Imag Part	Magnitude	Damping Ratio	Damped Freq (Hz)	Damped Period (Sec)	Undamped Freq (Hz)
1	-21.2472	0.0000	21.2472	1.0000	0.0000		3.3816
2	-0.8040	11.5563	11.5842	0.0694	1.8392	0.5437	1.8437
3	-0.8040	-11.5563	11.5842	0.0694	-1.8392	-0.5437	1.8437
4	-14.2256	0.0000	14.2256	1.0000	0.0000		2.2641
5	-3.7087	0.0000	3.7087	1.0000	0.0000		0.5903
6	-0.4248	0.0000	0.4248	1.0000	0.0000		0.0676

PowerWorld case **B4_GENROU_Sat_SMIB**

Example: Bus 4 with GENROU Model and Exciter



- Adding a relatively slow EXST1 exciter adds additional states (with $K_A=200$, $T_A=0.2$)
 - As the initial reactive power output of the generator is decreased, the system becomes unstable (below example is with a generator reactive power output of 0 Mvar)

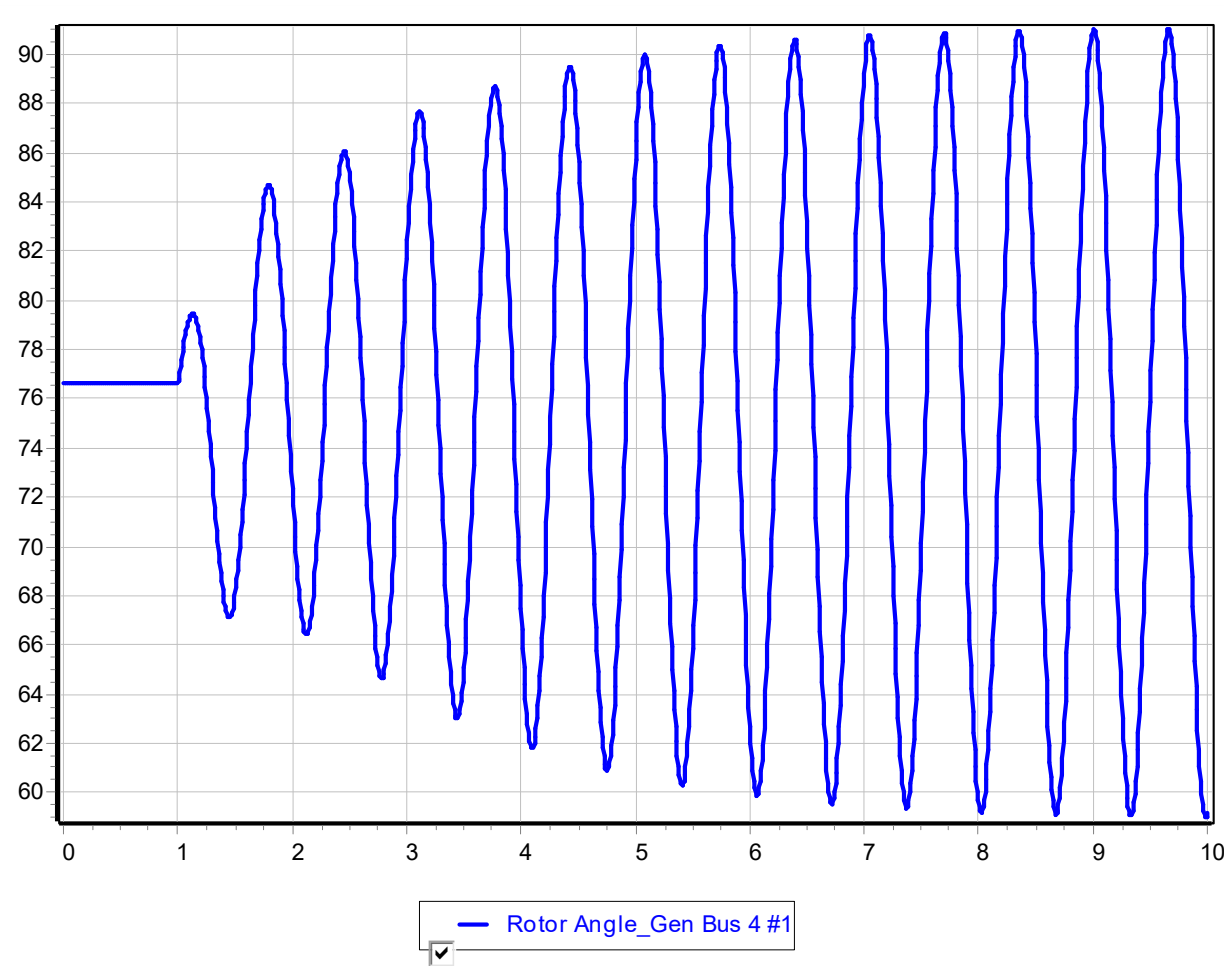
	Real Part	Imag Part	Magnitude	Damping Ratio	Damped Freq (Hz)	Damped Period (Sec)	Undamped Freq (Hz)	Ma
1	0.2704	-9.5336	9.5374	-0.0283	-1.5173	-0.6591	1.5179	
2	0.2704	9.5336	9.5374	-0.0283	1.5173	0.6591	1.5179	
3	-1.0000	0.0000	1.0000	1.0000	0.0000		0.1592	
4	-3.0137	0.0000	3.0137	1.0000	0.0000		0.4796	
5	-3.6849	-6.4281	7.4094	0.4973	-1.0231	-0.9775	1.1792	
6	-3.6849	6.4281	7.4094	0.4973	1.0231	0.9775	1.1792	
7	-14.4234	0.0000	14.4234	1.0000	0.0000		2.2956	
8	-21.6978	0.0000	21.6978	1.0000	0.0000		3.4533	

PowerWorld case **B4_GENROU_Sat_SMIB_QZero**

Example: Bus 4 with GENROU Model and Exciter



- The below image shows the system response to a brief bus 4 self-clearing fault



Example: Bus 4 with GENROU Model and Exciter



- The remainder of the Eigenvalues page shows the participation factors for the various states in the modes

Generator SMIB Eigenvalue Information

Bus Number: 4 Find By Number Status: Open Closed

Bus Name: Bus 4 Find By Name

ID: 1 Find ... Area Name: Home (1)

Generator Information (on Generator MVA Base)

General Info A Matrix **Eigenvalues**

	Real Part	Imag Part	Magnitude	Damping Ratio	Damped Freq (Hz)	Damped Period (Sec)	Undamped Freq (Hz)	Machine Angle	Machine Speed w	Machine Eqp	Machine PsiDp	Machine PsiQpp	Machine Edp	Exciter EField before limit	Exciter VF
1	0.2704	-9.5336	9.5374	-0.0283	-1.5173	-0.6591	1.5179	0.6920	0.6810	0.1642	0.0250	0.0137	0.0139	0.1714	0.0000
2	0.2704	9.5336	9.5374	-0.0283	1.5173	0.6591	1.5179	0.6920	0.6810	0.1642	0.0250	0.0137	0.0139	0.1714	0.0000
3	-1.0000	0.0000	1.0000	1.0000	0.0000		0.1592	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
4	-3.0137	0.0000	3.0137	1.0000	0.0000		0.4796	0.0071	0.0098	0.0573	0.0011	0.1263	0.9865	0.0865	0.0000
5	-3.6849	-6.4281	7.4094	0.4973	-1.0231	-0.9775	1.1792	0.1643	0.1764	0.6494	0.0534	0.0350	0.0964	0.7120	0.0000
6	-3.6849	6.4281	7.4094	0.4973	1.0231	0.9775	1.1792	0.1643	0.1764	0.6494	0.0534	0.0350	0.0964	0.7120	0.0000
7	-14.4234	0.0000	14.4234	1.0000	0.0000		2.2956	0.0054	0.0049	0.0219	0.9995	0.0013	0.0028	0.0226	0.0000
8	-21.6978	0.0000	21.6978	1.0000	0.0000		3.4533	0.0030	0.0037	0.0009	0.0006	0.9971	0.0762	0.0011	0.0000

OK Save Cancel Help Print

SMIB Eigenvalues for TSGC_2000 Case



- All the SMIB eigenvalues can be calculated quickly even for relatively large grids

Transient Stability Analysis

Simulation Status: **Initialized**

Run Transient Stability | Pause | Abort | Restore Reference | For Contingency: Find | Tornado

Select Step: Simulation, Options, Result Storage, Plots, Result Analyzer - Damping, Results from RAM, Transient Limit Monitors, States/Manual Control, Validation, **SMIB Eigenvalues**, Modal Analysis, Dynamic Simulator Options

Run SMIB Eigen Analysis | Re-Initialize | Eigenvalue Analysis Last Run: 11/8/2021 4:58:07 PM

	Number of Bus	Name of Bus	ID	MVA Base	Area Name of Gen	Machine	Exciter	Governor	Stabilizer	Calculate	Number of Eigenvalues	Number of Zero Eigenvalues	Min Eigenvalue	Max Eigenvalue	Swing Equation Freq. (Hz)	Swing Equation Damping	Swing Equation D Equivalent
1	1004	O DONNELL 1 1	1	253.2	Far West	WT4G	WT4E			YES	9	0	-0.2258	-49.9323	0.0000	0.0000	0.0000
2	1006	BIG SPRING 5 1	1	41.2	Far West	WT4G	WT4E			YES	9	0	-0.1842	-49.9543	0.0000	0.0000	0.0000
3	1009	IRAAN 2 1	1	99.0	Far West	WT4G	WT4E			YES	9	0	-0.2436	-49.9203	0.0000	0.0000	0.0000
4	1011	PRESIDIO 1 1	1	12.0	Far West					YES	0	0	0.0000	0.0000	0.0000	0.0000	0.0000
5	1021	BIG SPRING 1 1	1	239.4	Far West	WT4G	WT4E			YES	9	0	-0.2693	-49.8987	0.0000	0.0000	0.0000
6	1021	O DONNELL 2 1	1	216.0	Far West	WT4G	WT4E			YES	9	0	-0.2793	-49.8853	0.0000	0.0000	0.0000
7	1026	BIG SPRINGS 5 1	1	149.0	Far West	WT4G	WT4E			YES	9	0	-0.2736	-49.8928	0.0000	0.0000	0.0000
8	1033	MCCAMEY 1 1	1	333.6	Far West	WT4G	WT4E			YES	9	0	-0.2110	-49.9407	0.0000	0.0000	0.0000
9	1035	BIG SPRING 4 1	1	108.0	Far West	WT4G	WT4E			YES	9	0	-0.2462	-49.9185	0.0000	0.0000	0.0000
10	1039	FORT STOCKTOI 1	1	177.0	Far West	WT4G	WT4E			YES	9	0	-0.2692	-49.8992	0.0000	0.0000	0.0000
11	1042	FORSAN 1	1	146.3	Far West	WT4G	WT4E			NO	0	0	0.0000	0.0000	0.0000	0.0000	0.0000
12	1043	FORSAN 2	1	70.6	Far West	WT4G	WT4E			YES	9	0	-0.2235	-49.9335	0.0000	0.0000	0.0000
13	1048	MONAHANS 1 1	1	390.2	Far West	GENROU	ESST4B	GGOV1	IEEEST	NO	0	0	0.0000	0.0000	0.0000	0.0000	0.0000
14	1049	MONAHANS 1 2	1	390.2	Far West	GENROU	EXAC2	GGOV1	IEEEST	NO	0	0	0.0000	0.0000	0.0000	0.0000	0.0000
15	1050	MONAHANS 1 3	1	107.3	Far West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.0015	-77.1569	1.8102	0.0643	8.4396 C
16	1051	MONAHANS 1 4	1	107.3	Far West	GENROU	EXPIC1	GGOV1	IEEEST	YES	21	1	-0.0818	-76.9490	0.9537	0.0733	13.6859 C
17	1052	MONAHANS 1 5	1	107.3	Far West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.0830	-76.9891	1.1103	0.0620	13.8306 C
18	1053	MONAHANS 1 6	1	214.6	Far West	GENROU	ESST4B	GGOV1	IEEEST	NO	0	0	0.0000	0.0000	0.0000	0.0000	0.0000
19	1057	LENORAH 1	1	144.0	Far West	WT4G	WT4E			NO	0	0	0.0000	0.0000	0.0000	0.0000	0.0000
20	1059	IRAAN 3 1	1	6.7	Far West	GENROU	EXAC2	GGOV1	IEEEST	NO	0	0	0.0000	0.0000	0.0000	0.0000	0.0000
21	1060	IRAAN 3 2	1	2.4	Far West	GENROU	ESST4B	GGOV1	IEEEST	NO	0	0	0.0000	0.0000	0.0000	0.0000	0.0000
22	1062	BIG SPRING 6 1	1	1.8	Far West					NO	0	0	0.0000	0.0000	0.0000	0.0000	0.0000
23	1063	BIG SPRING 6 2	1	276.0	Far West					NO	0	0	0.0000	0.0000	0.0000	0.0000	0.0000
24	1066	BIG SPRING 3 1	1	171.0	Far West	WT4G	WT4E			YES	9	0	-0.2728	-49.8939	0.0000	0.0000	0.0000
25	1070	IRAAN 1 1	1	192.6	Far West	WT4G	WT4E			YES	9	0	-0.2670	-49.8965	0.0000	0.0000	0.0000
26	1072	ODESSA 1 1	1	230.6	Far West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.1053	-64.1893	0.6171	0.6281	69.8414 C
27	1073	ODESSA 1 2	1	230.6	Far West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.1350	-63.4621	8.5123	0.6088	799.2446 C
28	1074	ODESSA 1 3	1	230.6	Far West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.0812	-48.4635	0.5445	0.7105	66.8468 C
29	1075	ODESSA 1 4	1	230.6	Far West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.0479	-73.9551	0.5988	0.7503	83.5653 C
30	1076	ODESSA 1 5	1	230.6	Far West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.0315	-54.6018	13.1315	0.4923	942.2911 C
31	1077	ODESSA 1 6	1	230.6	Far West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.1454	-56.7124	0.9590	0.5193	61.6237 C
32	1078	ODESSA 1 7	1	114.6	Far West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.0932	-53.9516	0.6988	0.6303	99.7896 C
33	1079	ODESSA 1 8	1	114.6	Far West	GENROU	EXAC2	GGOV1	IEEEST	YES	23	1	-0.0081	-72.9229	1.9024	0.3274	87.7561 C
34	1080	ODESSA 1 9	1	114.6	Far West	GENROU	ESST4B	GGOV1	IEEEST	NO	0	0	0.0000	0.0000	0.0000	0.0000	0.0000
35	1081	ODESSA 1 10	1	343.8	Far West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.2000	-77.2607	9.2208	0.5061	815.9991 C
36	1082	FORT STOCKTOI 1	1	180.0	Far West	WT4G	WT4E			YES	9	0	-0.2453	-49.9189	0.0000	0.0000	0.0000
37	1084	BIG SPRING 2 0	1	138.6	Far West	WT4G	WT4E			YES	9	0	-0.1892	-49.9505	0.0000	0.0000	0.0000
38	1088	MCCAMEY 2 0	1	90.0	Far West	WT4G	WT4E			YES	9	0	-0.2307	-49.9289	0.0000	0.0000	0.0000
39	1090	COLEMAN 0	1	183.0	Far West	WT4G	WT4E			YES	9	0	-0.2263	-49.9300	0.0000	0.0000	0.0000

Process Contingencies: One Contingency at a time, Multiple Contingencies

Save All Settings To | Load All Settings From | Show Transient Contour Toolbar | Auto Insert... | Critical Clearing Time Calculator... | Help | Close

Saving a Two Bus Equivalent



- PowerWorld makes it easy to save a two bus equivalent from the **SMIB Eigenvalues** page
 - Right-click and select **Save Two Bus Equivalent**
- As the name implies, the two bus equivalent is the generator connected to an infinite bus through its driving point impedance
- Two bus equivalents provide a convenient way to track down at least some causes of instability issues