ECEN 667
Power System Stability

Lecture 13: Energy Methods for Stability Analysis

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Announcements

- HW #4 is on the website, due Oct 30^{th} at 8 AM.
- Read book chapters 9 and 8
- Review the slides and PowerWorld examples

Recall, Stability

- Definition:

  Starting with a certain power system operating condition,
  and subjecting the system to some disturbance,
  will the system regain an equilibrium point
  with all important variables within an acceptable range?

- Notice that with this definition, instability can be affected in multiple ways
  - If you subject the system to a sufficiently large disturbance, you will eventually cause an unstable response
  - Alternatively, if you operate the system in sufficiently stressed conditions, even a very small disturbance could cause an unstable response
  - Design of various control systems are crucial to help increase stability
  - The definition of what variables are important and what is an acceptable range also matters

Recall, How Do We Analyze Stability?

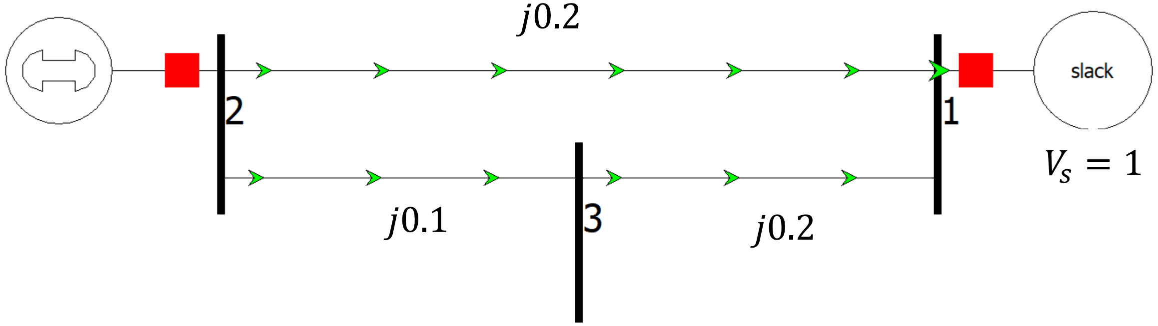
- Generally, the first step is some type of modeling, which usually results in a set of DAEs that describes the system
  - We also need to know the operating point, the disturbance(s) of interests, and desired system performance metrics
- From here there are two main categories of stability analysis techniques
  - **Analytical Stability Methods.** These look at the equations themselves, break them down, and attempt to make mathematical claims about their stability properties. For a linear system (or linearized non-linear system), this could involve eigenvalue analysis. For nonlinear systems, energy functions is an example of an approach. Sometimes a frequency sweep with phase-margin and gain-margin is used. We will talk about several of these approaches later in the semester.
  - **Time-Domain Numerical Methods.** We use numerical integration methods (as last class) to simulate disturbance responses in time. We then need to analyze the numerical results to determine the system's stability properties. This is the focus of today's lecture.

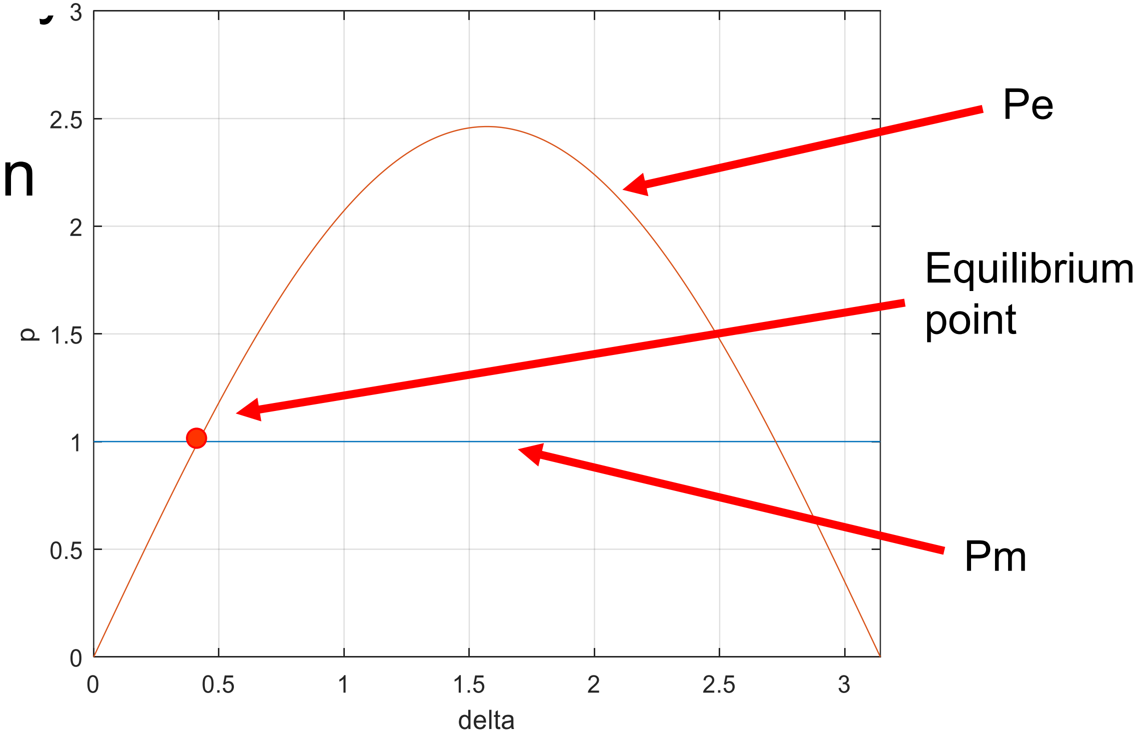
SMIB Example

- Consider a single-machine, infinite bus system as shown, with $X'_d = 0.4$ and $H = 3$
- Let's assume, with some simplifications, that $\delta = 23.95°$ and $p_e = 2.4628 \sin \delta$ initi
- Hence the initial $p_m = 1.0$
- Use the following simplified swing equation

$$\frac{2H}{\omega_s}\ddot{\delta} = p_m - p_e = p_m - 2.4628 \sin \delta$$

- We can draw an angle-power curve



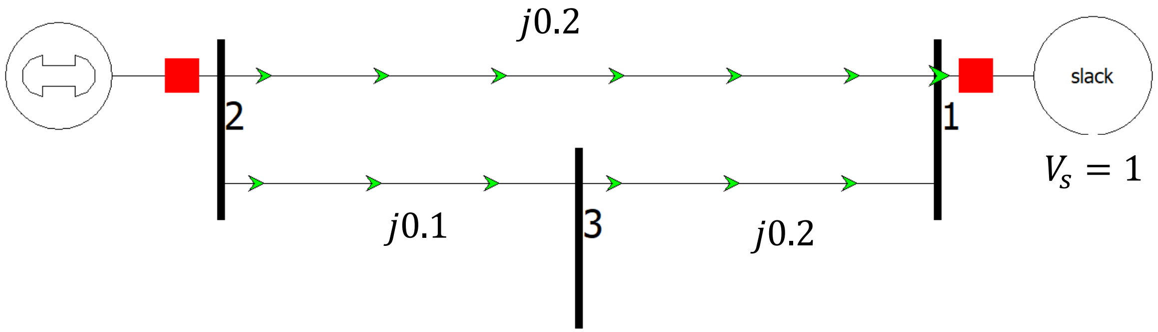


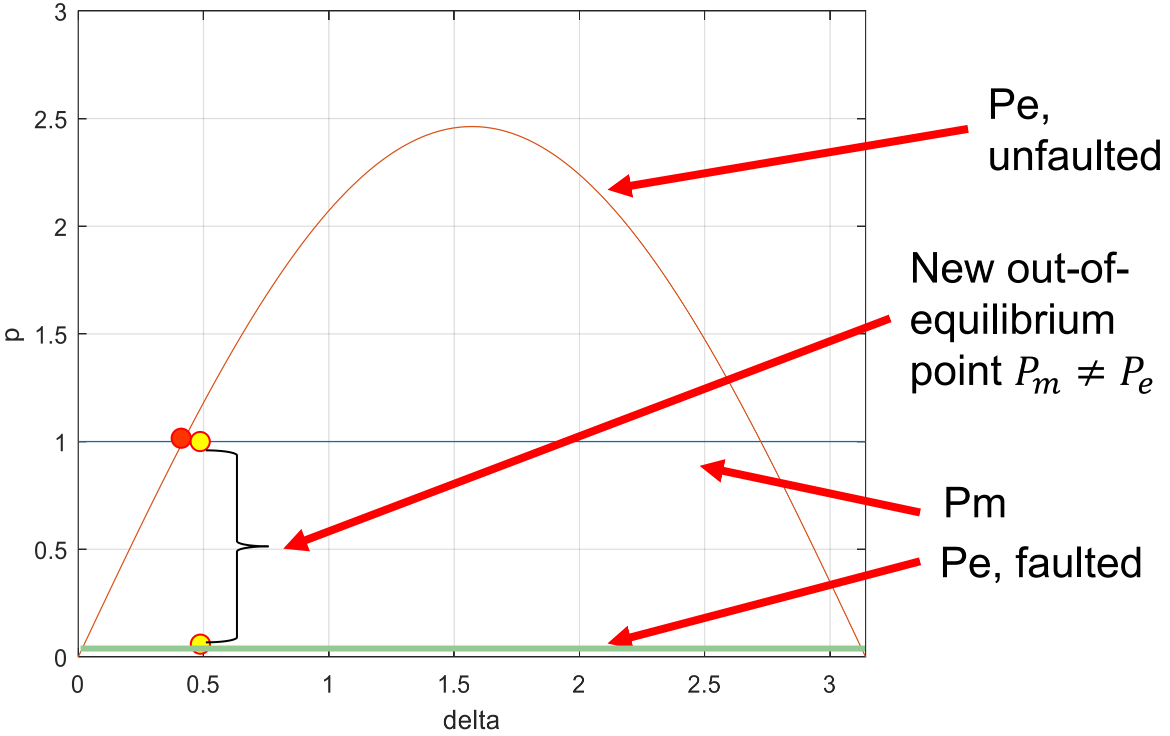
SMIB Example, Step 2

- Say a three-phase fault occurs at bus 2, with a duration of three seconds
- P_e drops to zero, what happens to delta?
- Constant acceleration of $\frac{\omega_s}{2H} p_m$

$$\delta_1 = \delta_0 + \frac{\omega_s}{2H} p_m \left(t_{fault}\right)^2 = 28.44°$$

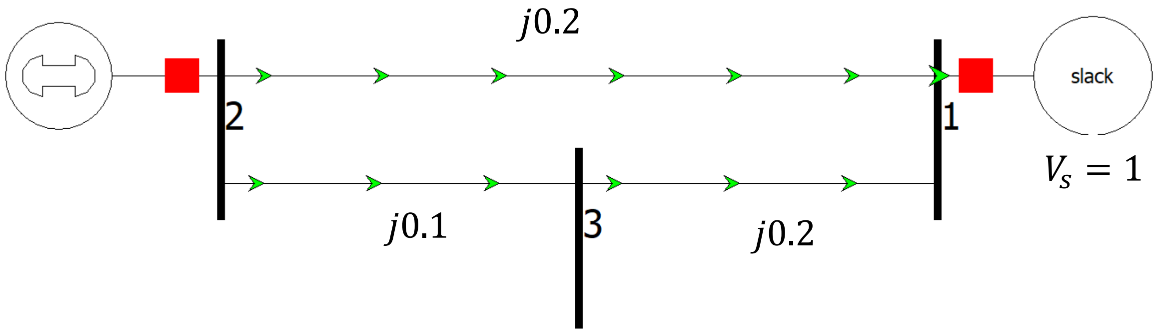
- When the fault then clears, acceleration will be negative, but $\dot{\delta}$ will be positive
- Notice on the curve, if Pe < Pm we have positive acceleration, and vice versa
- The total *energy* is the integration of the (Pm-Pe) difference over δ.

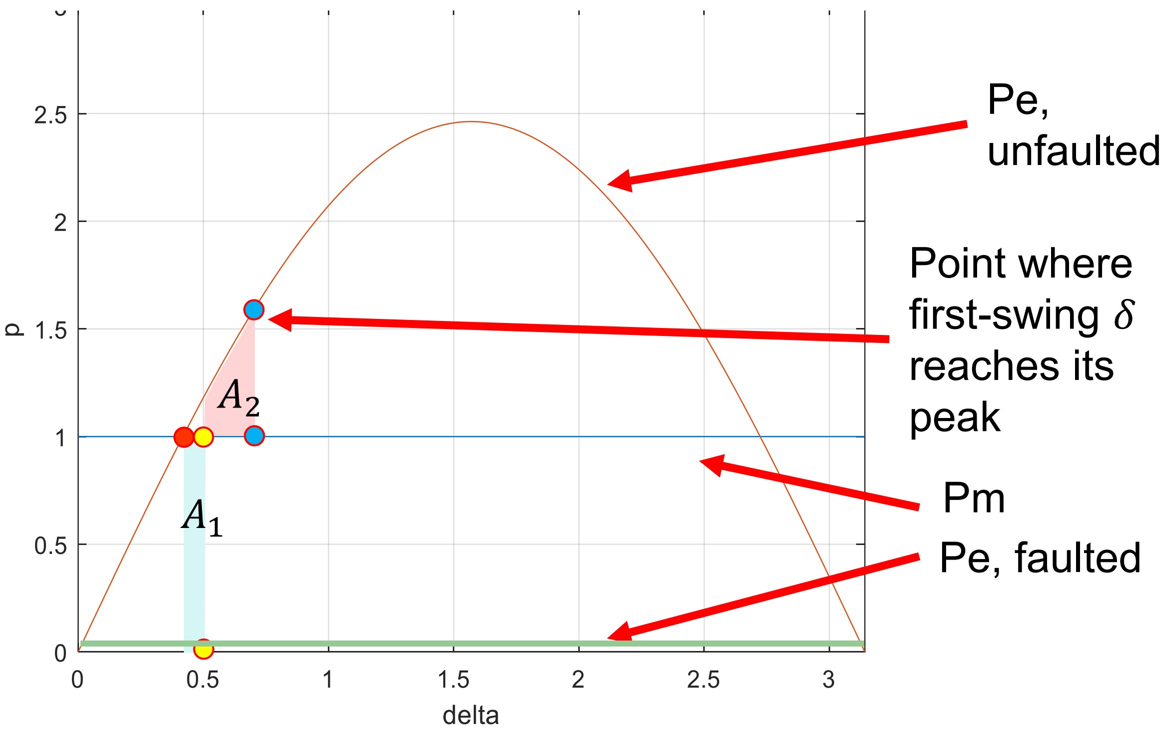




SMIB Example, Step 3

- After fault clears, delta will continue to increase until decelerating power pulls it back down
- As long as this occurs before $\delta = \pi - \delta_0$, system will remain stable
- The **Equal Area Criterion** specifies that $A_1 = A_2$
- This is a simplified "energy method"
- Energy built up during accelerating period must equal energy reversed during the decelerating period
- Calculate $\delta_{\max}$ using integration
- Confirm with time-domain analysis





Equal Area Criterion, Methodology

For a single machine, infinite bus system with a classical generator model

1. Plot both P_m and P_e as a function of δ
2. Determine the conditions directly following the event
3. The total net area between Pm and Pe will be zero at the end of the first angle swing. This criterion can be used to determine the maximum value of δ and show whether system stability will be maintained.

Analytical Methods

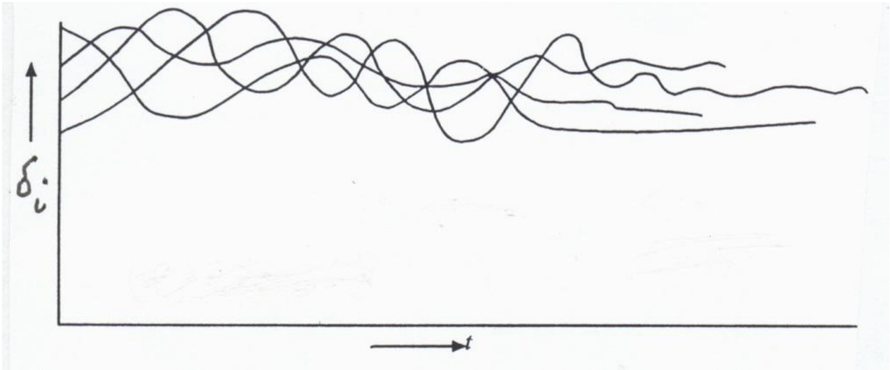
- Equal area criteria
- Lyapunov energy methods
- Introduction to small signal stability
- Phase and gain margin approaches to small-signal stability
- Eigenvalue analysis

Transient Energy Function (TEF) Techniques

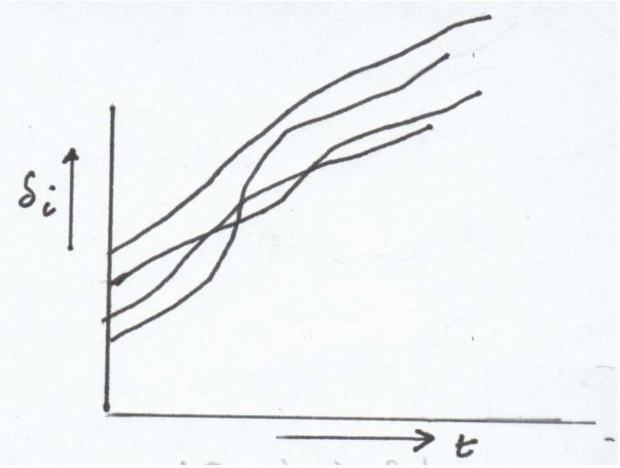
- No repeated simulations are involved.
- Limited somewhat by modeling complexity.
- Energy of the system used as Lyapunov function.
- Computing energy at the "controlling" unstable equilibrium point (CUEP) (critical energy).
- CUEP defines the mode of instability for a particular fault.
- Computing critical energy is <u>not</u> easy.

Judging Stability and Instability

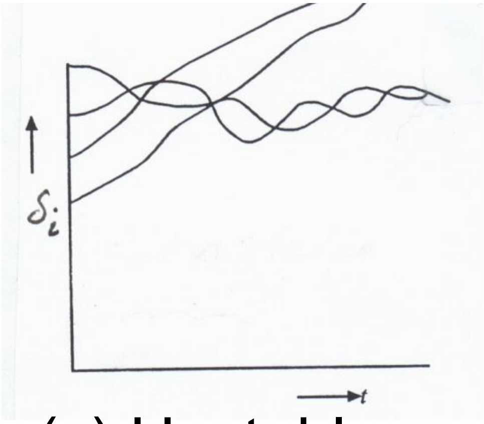
Monitor Rotor Angles



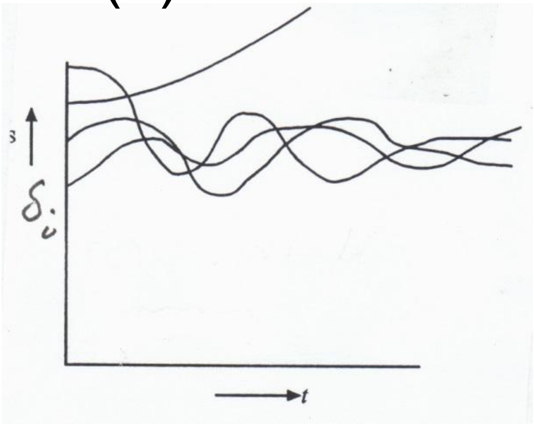
(a) Stable



(b) Stable



(c) Unstable



(d) Unstable

Stability is judged by Relative Rotor Angles.

Mathematical Formulation

- A power system undergoing a disturbance (fault, etc.), followed by clearing of the fault, has the following model

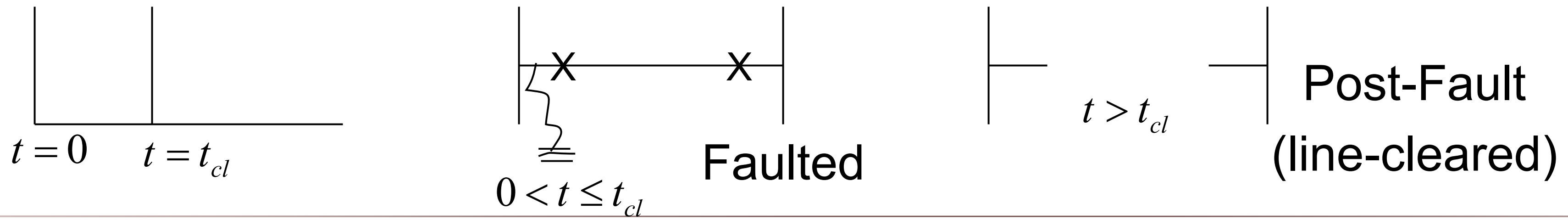
$$\dot{\mathbf{x}}(t) = \mathbf{f}^{\mathbf{I}}(\mathbf{x}(t)) \quad -\infty < t \leq 0 \qquad (1)$$

$$\dot{\mathbf{x}}(t) = \mathbf{f}^{\mathbf{F}}(\mathbf{x}(t)) \quad 0 < t \leq t_{cl} \qquad (2)$$

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t)) \quad t_{cl} < t \leq \infty \qquad (3)$$

  - (1) Prior to fault (Pre-fault)
  - (2) During fault (Fault-on or faulted)
  - (3) After the fault (Post-fault)

T_{cl} is the clearing time

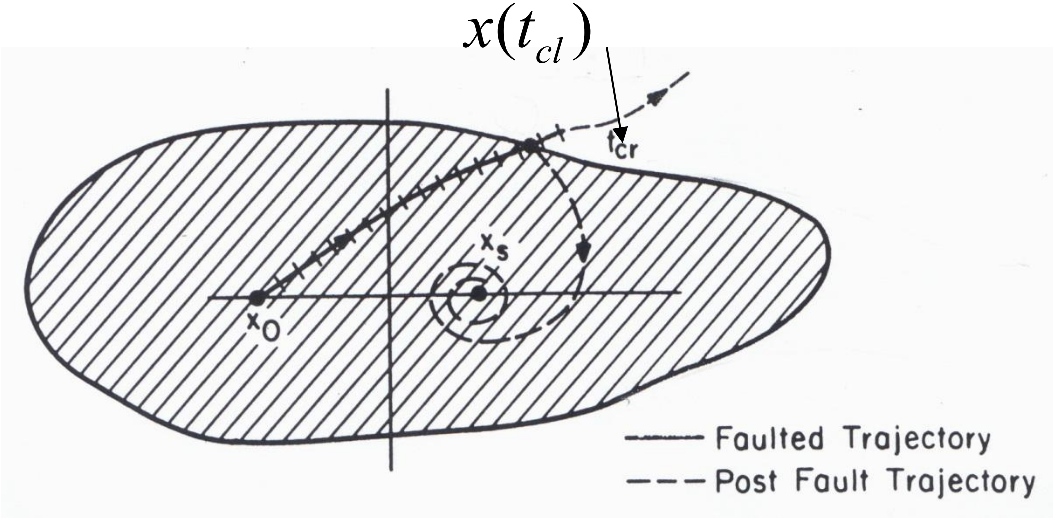


Critical Clearing Time

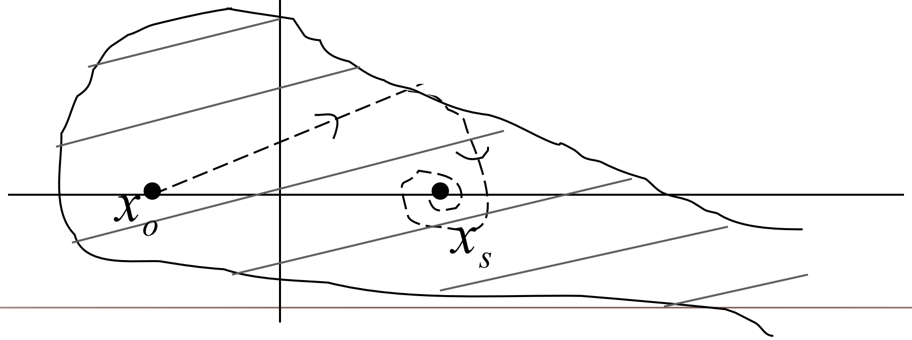
- Assume the post-fault system has a stable equilibrium point $\mathbf{x}_s$
- All possible values of $\mathbf{x}(t_{cl})$ for different clearing times provide the initial conditions for the post-fault system
  - Question is then will the trajectory of the post fault system, starting at $\mathbf{x}(t_{cl})$, converge to $\mathbf{x}_s$ as $t \to \infty$
- Largest value of t_{cl} for which this is true is called the critical clearing time, t_{cr}
- The value of t_{cr} is different for different faults

Region of Attraction (ROA)

All faulted trajectories cleared before they reach the boundary of the ROA will tend to $\mathbf{x}_s$ as $t \to \infty$ (stable)



The region need not be closed; it can be open:



Methods to Compute RoA

- Had been a topic of research in power system literature since early 1960's.
- The stable equilibrium point (SEP) of the post-fault system, $\mathbf{x}_s$, is generally close to the pre-fault EP, $\mathbf{x}_0$
- Surrounding $\mathbf{x}_s$ there are a number of unstable equilibrium points (UEPs).
- The boundary of ROA is characterized via these UEPs

$$\mathbf{x}_{u,i}\ , i = 1, 2\ldots$$

$$\mathbf{f}(\mathbf{x}) = 0 \quad i.e. \quad \mathbf{f}(\mathbf{x}_{u,i}) = 0 \quad i = 1, 2\ldots$$

Characterization of RoA

- Define a scalar energy function $V(\mathbf{x})$ = sum of the kinetic and potential energy of the post-fault system.
- Compute $V(\mathbf{x}_{u,i})$ at each UEP, i=1,2,…
- Defined V_{cr} as

$$V_{cr} = Min\, V(\mathbf{x}) \Big|\, \mathbf{x}_{u,i}$$

  - RoA is defined by $V(\mathbf{x}) < V_{cr}$
  - But this can be an extremely conservative result.
- Alternative method: Depending on the fault, identify the critical UEP, $\mathbf{x}_{u,cr}$, towards which the faulted trajectory is headed; then $V(\mathbf{x}) < V(\mathbf{x}_{u,cr})$ is a good estimate of the ROA.

Lyapunov's Method

- Defining the function V(x) is a key challenge
- Consider the system defined by
- $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}), \quad \mathbf{f}(\mathbf{x}_s) = \mathbf{0}$
- Lyapunov's method: If there exists a scalar function V(x) such that
- 1) $V(\mathbf{x}_s) = 0$

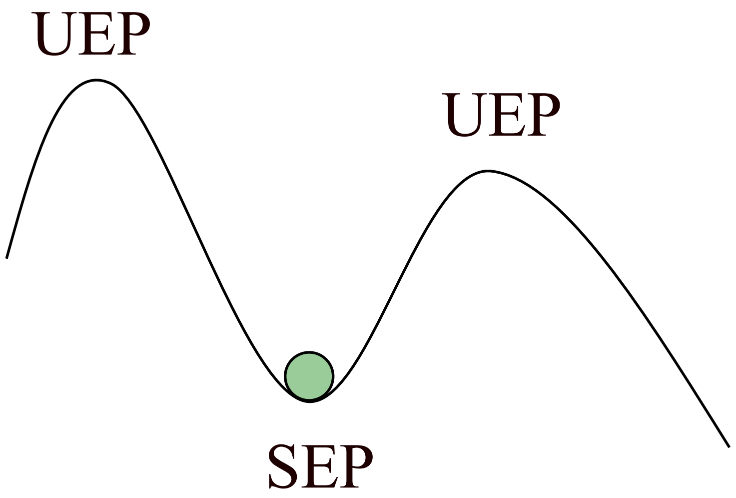
  2) $V(\mathbf{x}) > 0$ for all $\mathbf{x}$ around $\mathbf{x}_s$

  3) $\dot{V}(\mathbf{x}) \leq 0$ for all $\mathbf{x}$ around $\mathbf{x}_s$
- Then $\mathbf{x}_s$ is stable in the sense of Lyapunov

  EP $\mathbf{x}_s$ is asympotically stable if $\dot{V}(\mathbf{x}) < 0$ for $\mathbf{x} \neq \mathbf{x}_s$ around $\mathbf{x}_s$

Ball in Well Analogy

- The classic Lyapunov example is the stability of a ball in a well (valley) in which the Lyapunov function is the ball's total energy (kinetic and potential)



- For power systems, defining a true Lyapunov function often requires using restrictive models

Power System Example

- Consider the classical generator model using an internal node representation (load buses have been equivalenced)

$$M_i \frac{d^2\delta_i}{dt^2} + D_i \frac{d\delta_i}{dt} = P_i - \sum_{\substack{j=1 \\ j\neq i}}^{m} (C_{ij} \sin\delta_{ij} + D_{ij}\cos\delta_{ij})$$

$$P_i = T_{Mi} - E_i^2 G_{ii}$$

C_{ij} are the susceptance terms, D_{ij} the conductance terms

- Functionally

$$M_i \frac{d^2\delta_i}{dt^2} + D_i \frac{d\delta_i}{dt} = P_i - P_{ei}(\delta_1, \ldots, \delta_m) i = 1, \ldots, m$$

$$\dot{\delta}_i = \omega_i - \omega_s$$

$$\dot{\omega}_i = \frac{1}{M_i}(P_i - P_{ei}(\delta_i, \ldots, \delta_m) - D_i(\omega_i - \omega_s))$$

Constructing the Transient Energy Function (TEF)

- The reference frame matters. Either relative rotor angle formulation, or COI reference frame.
  - COI is preferable since we measure angles with respect to the "mean motion" of the system.
- TEF for conservative system (i.e., zero damping)

$$\delta_o = \frac{1}{M_T}\sum_{i=1}^{m} M_i\delta_i \quad \text{With center of speed as } \omega_o = \frac{1}{M_T}\sum_{i=1}^{m} M_i\omega_i$$

where $M_T = \sum_{i=1}^{m} M_i$. We then transform the variables to the COI variables as $\theta_i = \delta_i - \tilde{\delta}_o$, $\omega_i = \omega_i - \omega_o$.

It is easy to verify $\dot{\theta}_i = \dot{\delta}_i - \dot{\delta}_o = \omega_i - \tilde{\omega}_o \triangleq \omega_i$

TEF

- We consider the general case in which all M_i's are finite. We have two sets of differential equations:

$$M_i \frac{d\tilde{\omega}_i}{dt} = f_i^F(\theta) \quad 0 < t \le t_{cl} \qquad (Faulted)$$

$$\frac{d\theta_i}{dt} = \tilde{\omega}_i, \quad i = 1, 2, \ldots, m$$

And

$$M_i \frac{d\tilde{\omega}_i}{dt} = f_i(\theta) \quad t > t_{cl} \qquad (Post\ fault)$$

$$\frac{d\theta_i}{dt} = \tilde{\omega}_i, \quad i = 1, 2, \ldots, m$$

- Let the post fault system has a SEP at $\boldsymbol{\theta} = \boldsymbol{\theta}^s, \tilde{\boldsymbol{\omega}} = \mathbf{0}$
- This SEP is found by solving

$$\mathbf{f}_i(\boldsymbol{\theta}) = \mathbf{0}, \; i = 1, .., m$$

TEF, part 2

- Steps for computing the critical clearing time are:
  1. Construct a Lyapunov (energy) function for the post-fault system.
  2. Find the critical value of the Lyapunov function (critical energy) for a given fault
  3. Integrate the faulted equations until the energy is equal to the critical energy; this instant of time is called the critical clearing time
- Idea is once the fault is cleared the energy can only decrease, hence the critical clearing time is determined directly
- Methods differ as to how to implement steps 2 and 3.

Potential Energy Boundary Surface

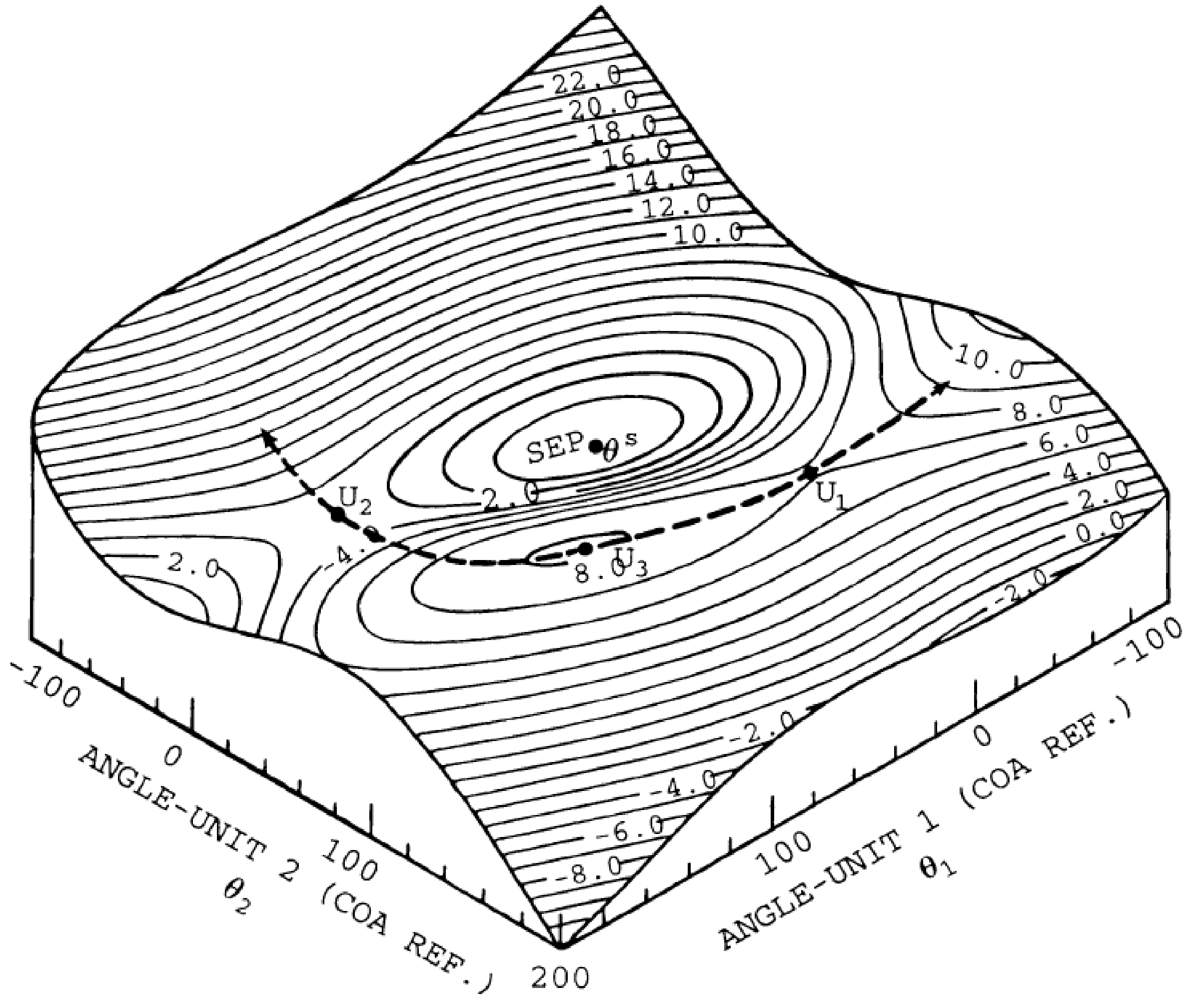


Figure 9.10: The potential energy boundary surface (reproduced from [97]

Figure from course textbook

TEF, part 3

- Integrating the equations between the post-fault SEP and the current state gives

$$V(\theta,\omega) = \frac{1}{2}\sum_{i=1}^{m} M_i\omega_i^2 - \sum_{i=1}^{m}\int_{\theta_i^s}^{\theta_i} f_i(\theta)d\theta_i$$

$$= \frac{1}{2}\sum_{i=1}^{m} M_i\omega_i^2 - \sum_{i=1}^{m} P_i(\theta_i - \theta_i^s) - \sum_{i=1}^{m-1}\sum_{j=i+1}^{m}[C_{ij}(\cos\theta_{ij} - \cos\theta_{ij}^s)]$$

$$-\int_{\theta_i^s+\theta_j^s}^{\theta_i+\theta_j} D_{ij}\cos\theta_{ij} d(\theta_i - \theta_j)]$$

$$= V_{KE}(\omega) + V_{PE}(\theta)$$

C_{ij} are the susceptance terms, D_{ij} the conductance terms; the conductance term is path dependent

TEF, part 4

- $V(\boldsymbol{\theta}, \tilde{\boldsymbol{\omega}})$ contains path dependent terms.
- Cannot claim that $V(\boldsymbol{\theta}, \tilde{\boldsymbol{\omega}})$ is p.d.
- If conductance terms are ignored, then it can be shown to be a Lyapunov function
- Methods to compute the UEPS are
  a) Potential Energy Boundary Surface (PEBS) method.
  b) Boundary Controlling Unstable (BCU) equilibrium point method.
  c) Other methods (Hybrid, Second-kick, etc.)

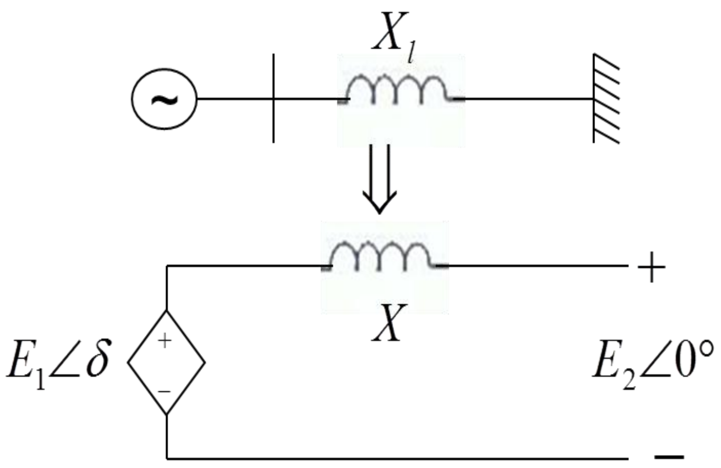
(a) and (b) are the most important ones.

Equal Area Criterion and TEF

- For an SMIB system with classical generators this reduces to the equal area criteria
  - TEF is for the post-fault system
  - Change notation from T_m to P_m

$$M\frac{d^2\delta}{dt^2} = P_m - P_e^{\max}\sin\delta \qquad (1)$$

$$P_e^{\max} = \frac{E_1 E_2}{X^I}\sin\delta \qquad (2)$$



$$X = X^F \quad (Faulted)$$

$$X = X^I \quad (Post-fault)$$

$$P_e = \frac{E_1 E_2}{X^F}\sin\delta \quad (Faulted)$$

$$P_e = \frac{E_1 E_2}{X^I}\sin\delta \quad (Post-fault)$$

TEF for SMIB System

$$M\frac{d^2\delta}{dt^2} = P_m - P_e^{\max}\sin\delta \qquad (1)$$

The right hand side of (1) can be written as $-\frac{\partial V_{PE}}{\partial \delta}$, where

$$V_{PE}(\delta) = -P_m\delta - P_e^{\max}\cos\delta \qquad (2)$$

Multiplying (1) by $\frac{d\delta}{dt}$, re-write

$$\frac{d}{dt}\left[\frac{M}{2}\left(\frac{d\delta}{dt}\right)^2 + V_{PE}(\delta)\right] = 0$$

since $\frac{d\delta}{dt} = \omega$

$$\frac{d}{dt}\left[\frac{1}{2}M\omega^2 + V_{PE}(\delta)\right] = 0$$ i.e

$$\frac{d}{dt}[V(\delta,\omega)] = 0$$ i.e

Hence, the energy function is

$$V(\delta,\omega) = \frac{1}{2}M\omega^2 + V_{PE}(\delta)$$

TEF for SMIB System (contd)

- The equilibrium point is given by

$$0 = P_m - P_e^{\max} \sin \delta \qquad (1)$$

$$\delta^s = \sin^{-1}\left(\frac{P_m}{P_e^{\max}}\right) \qquad (2)$$

- This is the stable e.p.
- Can be verified by linearizing.
- Eigenvalues on $j\omega$ axis. (Marginally Stable)
- With slight damping eigenvalues are in L.H.P.
- TEF is still constructed for undamped system.

TEF for SMIB System, (cont2)

- The energy function is

$$V(\delta,\omega) = V_{KE} + V_{PE}(\delta) = \tfrac{1}{2} M\omega^2 - P_m\delta - P_e^{\max}\cos\delta$$

- There are two UEP: $\delta^{u1} = \pi - \delta^s$ and $\delta^{u2} = -\pi - \delta^s$
- A change in coordinates sets $V_{PE}=0$ for $\delta=\delta^s$

$$V_{PE}(\delta,\delta^s) = -P_m(\delta-\delta^s) - P_e^{\max}(\cos\delta - \cos\delta^s)$$

- With this, the energy function is

$$V(\delta,\omega) = \tfrac{1}{2} M\omega^2 - P_m(\delta-\delta^s) - P_e^{\max}(\cos\delta - \cos\delta^s)$$

$$= V_{KE} + V_{PE}(\delta,\delta^s)$$

- The kinetic energy term is

$$V_{KE} = \tfrac{1}{2} M\omega^2$$

Equal-Area Criterion

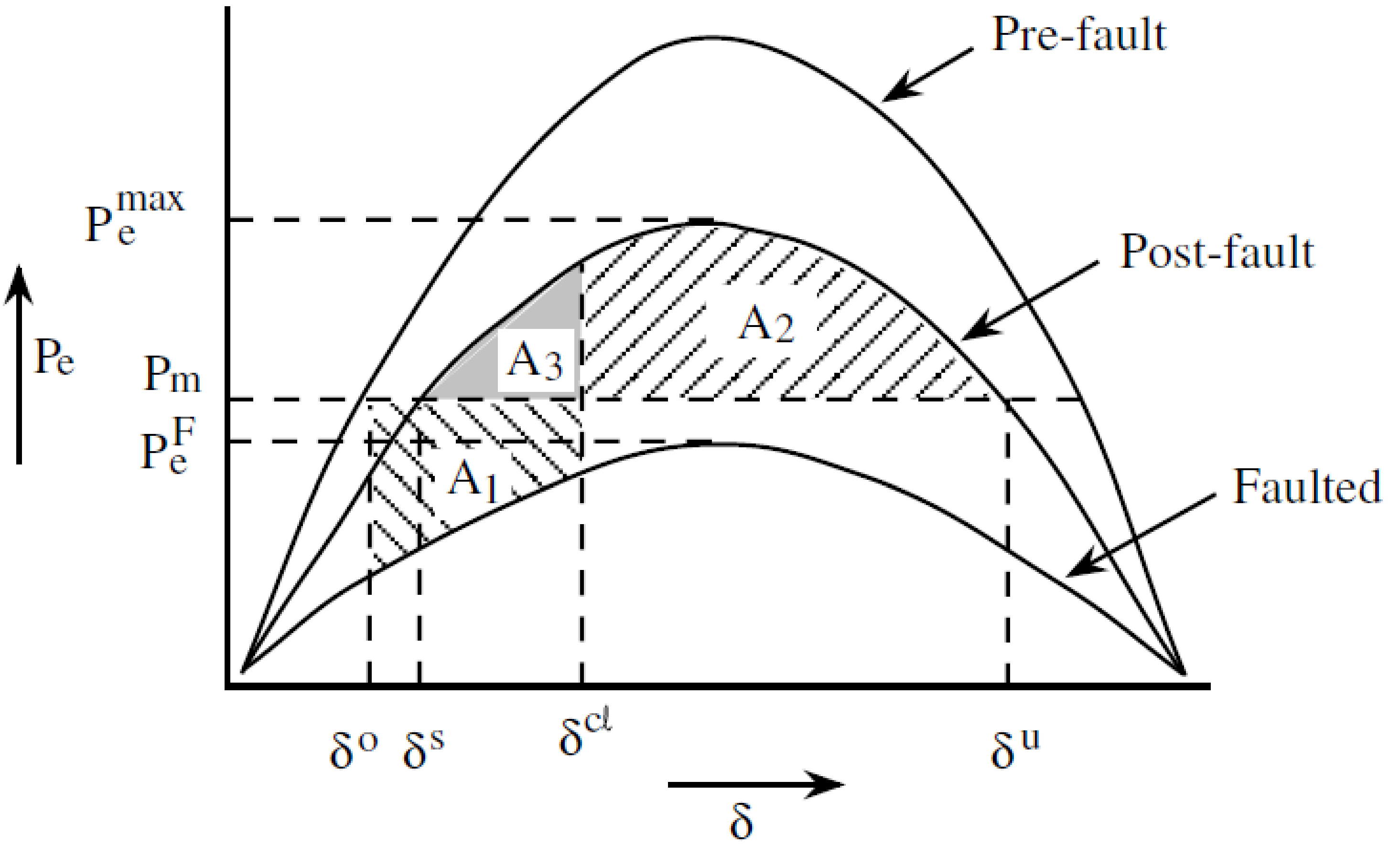


Figure 9.9: Equal-area criterion for the SMIB case

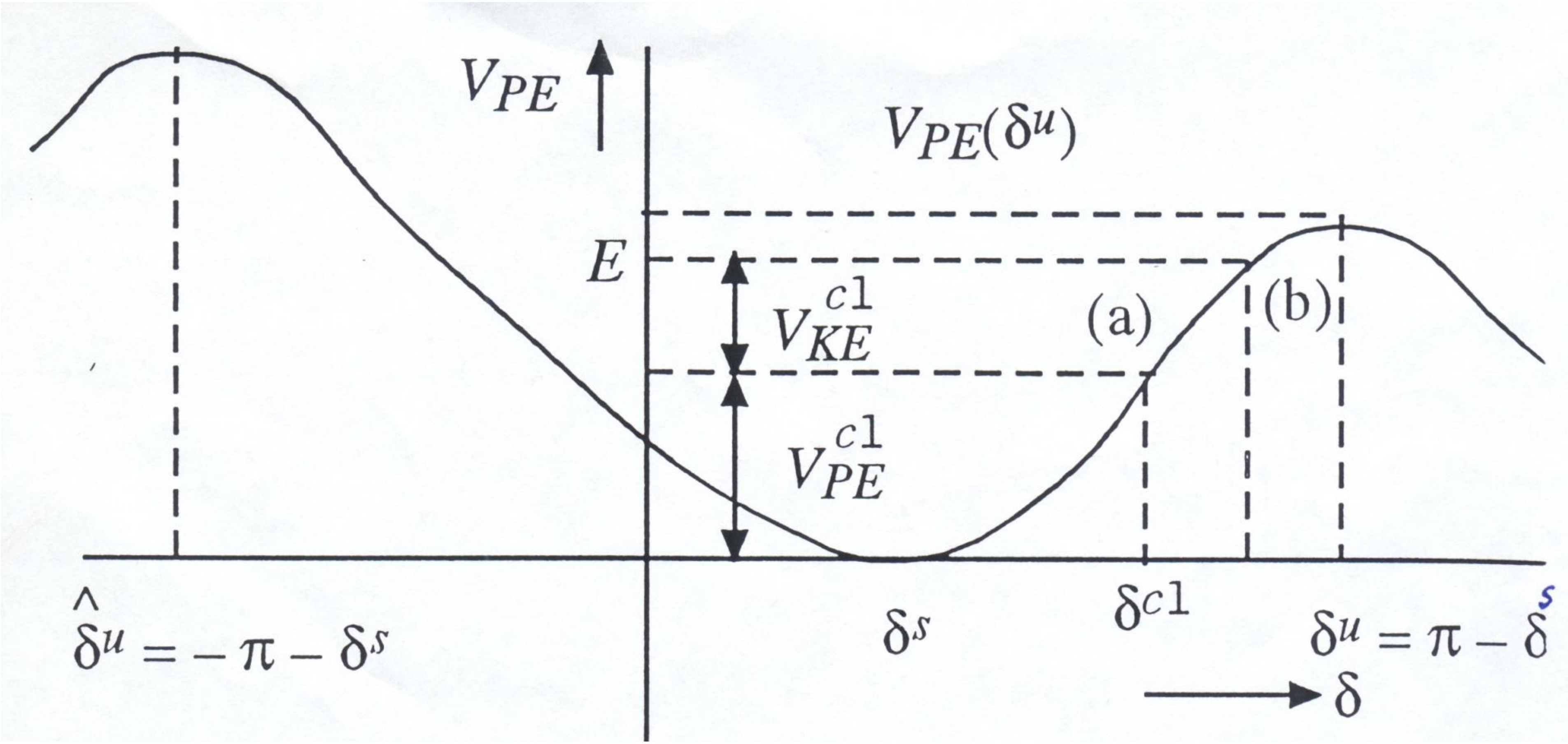
During the fault A_1 is the gain in the kinetic energy and A_3 the gain in potential energy

Figure from course textbook

Energy Function for SMIB System

- $V(\delta,\omega)$ is equal to a constant E, which is the sum of the kinetic and potential energies.
- It remains constant once the fault is cleared since the system is conservative (with no damping)
- $V(\delta,\omega)$ evaluated at $t=t_{cl}$ from the fault trajectory represents the total energy *E* present in the system at $t=t_{cl}$
- This energy must be absorbed by the system once the fault is cleared if the system is to be stable.
- The kinetic energy is always positive, and is the difference between *E* and $V_{PE}(\delta, \delta^s)$

Potential Energy Well for SMIB System



We need to compute the energy (E) and the boundary energy

Energy Functions for a Large System

- Need an energy function that at least approximates the actual system dynamics
  - This can be quite challenging!
- In general there are many UEPs; need to determine the UEPs for closely associated with the faulted system trajectory (known as the controlling UEP)
- Energy of the controlling UEP can then be used to determine the critical clearly time (i.e., when the fault-on energy is equal to that of the controlling UEP)
- For on-line transient stability, technique can be used for fast screening