

# ECEN 667

## Power System Stability

### Lecture 7: Synchronous Machine Modeling, Part 3

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# Announcements

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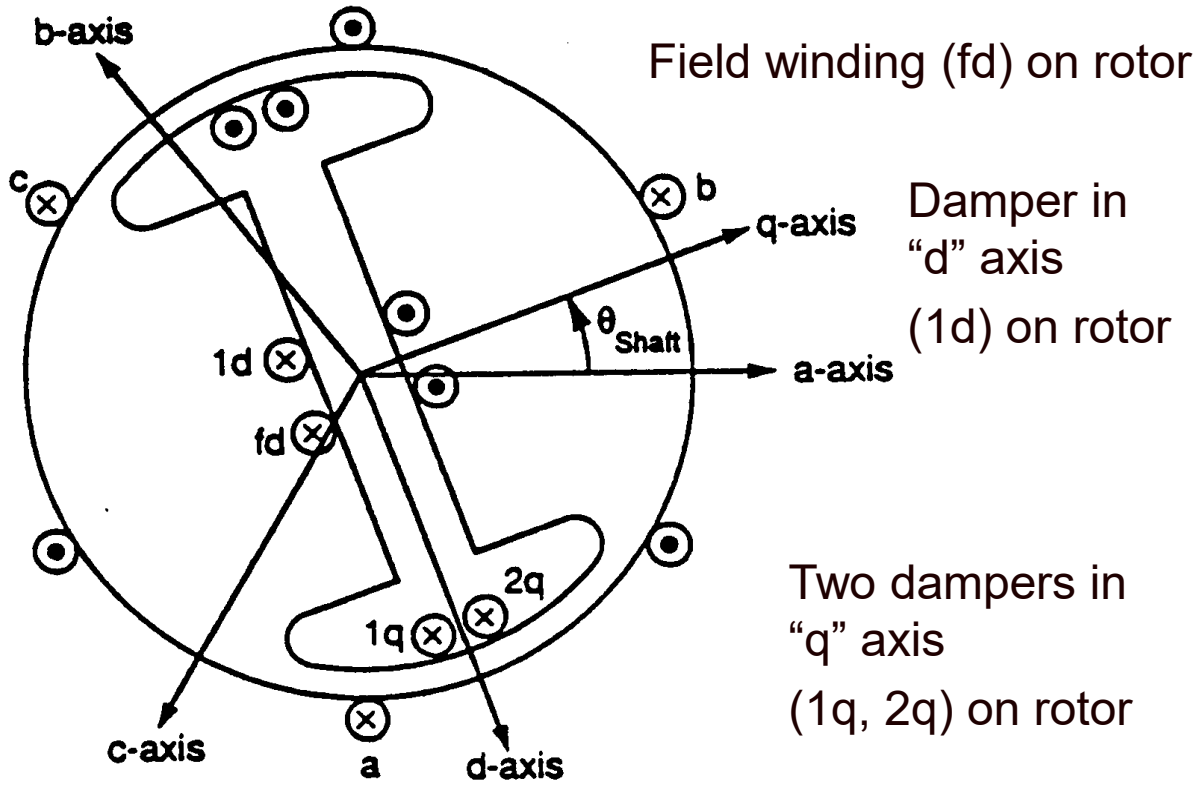


- Homework Assignment #2 is due Thursday, Sept. 25<sup>th</sup> at 8 AM. Email me your solution as a single PDF.
- Read book chapters 3, 4, and 5
- Review the slides and PowerWorld examples

# Recall, The Main Diagram



3 $\phi$  bal. windings (a,b,c) – stator



# Model Without Stator Transients, No Saturation



$$0 = V_d + I_d R_s - X_q'' I_q + (1 + \omega) \psi_q''$$

$$0 = V_q + I_q R_s + X_d'' I_d - (1 + \omega) \psi_d''$$

Network Interface

$$T_{do}' \frac{dE_q'}{dt} = -E_q' - (X_d - X_d') \left[ I_d - \frac{(X_d' - X_d'')}{(X_d' - X_{\ell s})^2} (\psi_{1d} + (X_d' - X_{\ell s}) I_d - E_q') \right] + E_{fd}$$

$$T_{do}'' \frac{d\psi_{1d}}{dt} = -\psi_{1d} + E_q' - (X_d' - X_{\ell s}) I_d$$

Rotor Windings

$$T_{qo}' \frac{dE_d'}{dt} = -E_d' + (X_q - X_q') \left[ I_q - \frac{X_q' - X_q''}{(X_q' - X_{\ell s})^2} (\psi_{2q} + (X_q' - X_{\ell s}) I_q + E_d') \right]$$

$$T_{qo}'' \frac{d\psi_{2q}}{dt} = -\psi_{2q} - E_d' - (X_q' - X_{\ell s}) I_q$$

$$\frac{d\delta}{dt} = \omega \cdot \omega_s$$

$$2H \frac{d\omega}{dt} = T_M + (\psi_q'' I_d - \psi_d'' I_q) - T_{FW}$$

Mechanical equations

$$\psi_d'' = \frac{X_d'' - X_{\ell s}}{X_d' - X_{\ell s}} E_q' + \frac{X_d' - X_d''}{X_d' - X_{\ell s}} \psi_{1d}$$

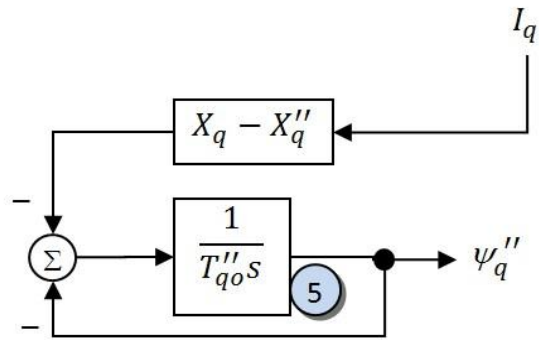
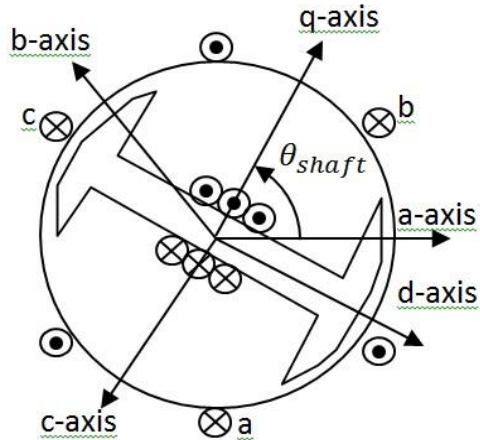
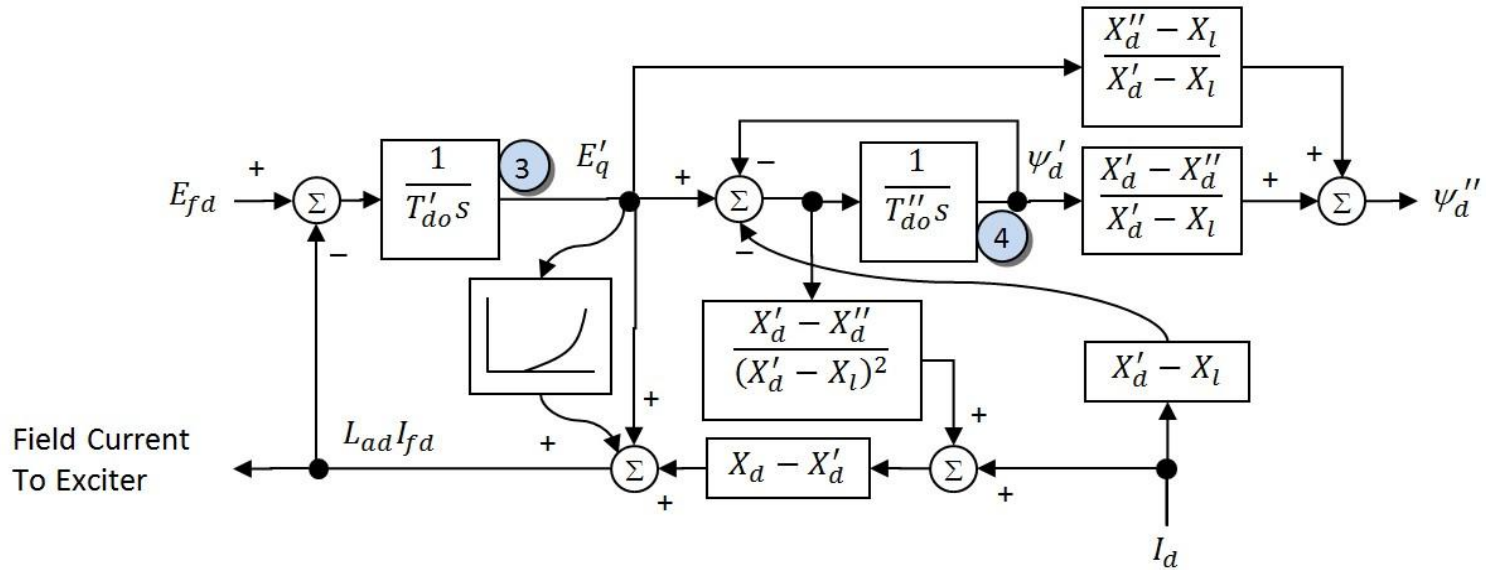
$$\psi_q'' = -\frac{X_q'' - X_{\ell s}}{X_q' - X_{\ell s}} E_d' + \frac{X_q' - X_q''}{X_q' - X_{\ell s}} \psi_{2q}$$

Flux definitions

# GENSAL Definition



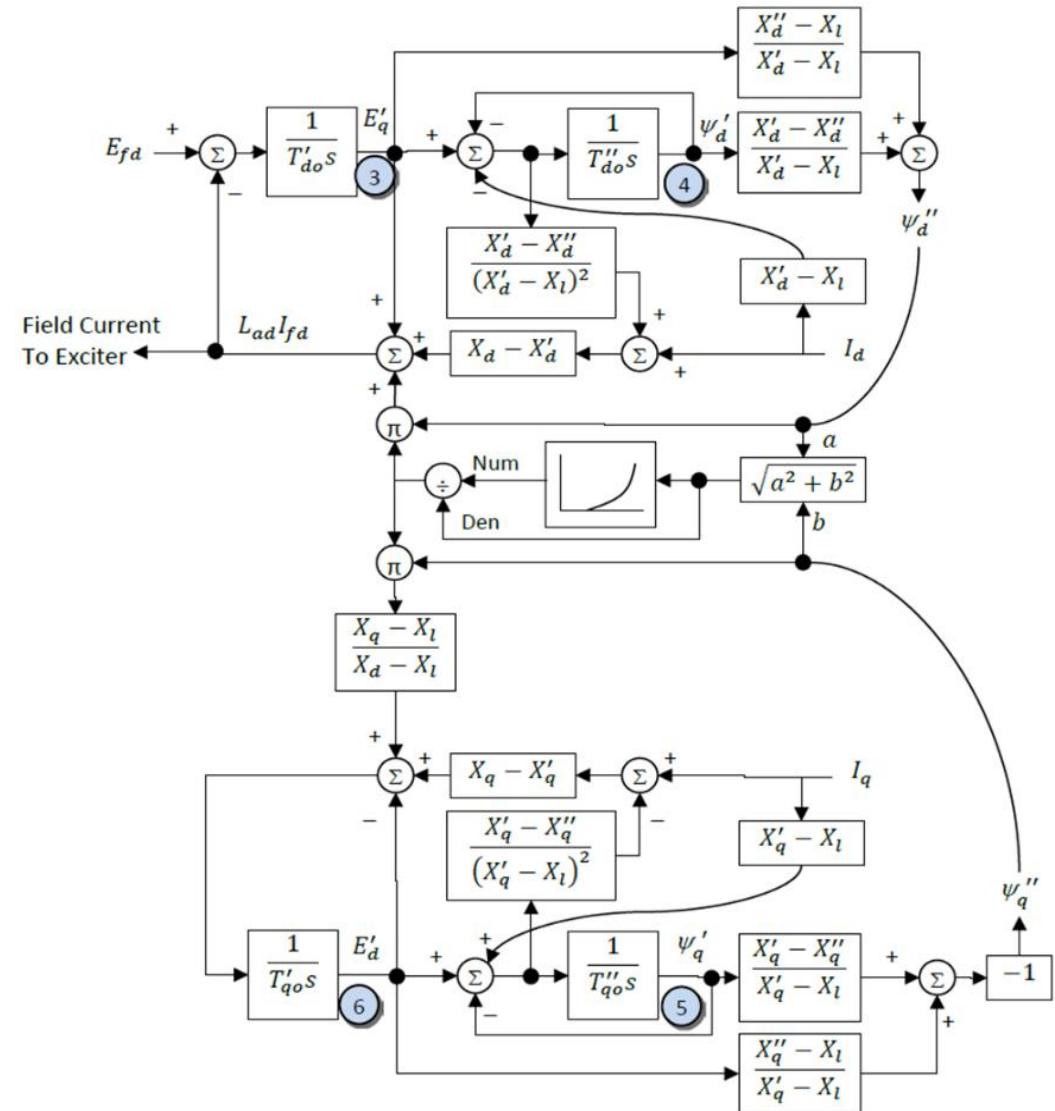
- Block diagram shows rotor equations
- Same machine interface equations and mechanical equations as the “model without stator transients” we developed above.
- For initialization, saturation only impacts calculation of  $E_{fd}$



# GENROU



- Widely used to model round rotor machines
- Same network interface and mechanical equations as GENSAL
- 6th order – includes both q-axis dampers
- Saturation is assumed to occur on both the d axis and the q axis
- This makes initialization more difficult



# GENROU Initialization



1. Recognize that the saturation is independent of  $\delta$

$$|\psi''| = |\bar{V} + (R_s + jX'')\bar{I}|$$

2. Finding  $\delta$  is the key. First get a guess of  $\delta$  using the unsaturated approach

$$E \angle \delta = \bar{V} + (R_s + jX_q)\bar{I}$$

3. Then solve 5 nonlinear equations for 5 unknowns.

- The 5 unknowns are  $\delta$ ,  $E'_q$ ,  $E'_d$ ,  $\psi'_q$  and  $\psi'_d$
- 3 of the equations come from differential equations (2 on q-axis and 1 on d-axis)
- The other 2 equations come from the definition of  $\psi''_d$  and  $\psi''_q$  as a function of the power flow voltage, current, and  $\delta$
- Use Newton's method

4. Finally, with  $\delta$  known, convert to  $V_d$ ,  $V_q$ ,  $I_d$ ,  $I_q$  and find the rest of the variables and  $E_{fd}$  and  $T_M$ .

# GENROU Initialization Example



- Use the B4 case as before (B4\_GENROU\_NoSat)
    - $H = 3.0, R_s = 0, X_d = 2.1, X_q = 2.0, X'_d = 0.3, X'_q = 0.5, T'_{do} = 7.0, T'_{qo} = 0.75, X''_d = X''_q = 0.28, X_l = 0.13, T''_{do} = 0.073, T''_{qo} = 0.07$
  - For comparison initially assume no saturation
    - Get our guess of  $\delta$  the same way we did before:  $\delta = 52.1^\circ$
    - This gives network current and voltage in the dq reference frame
 
$$V_d + jV_q = (V_s \angle \theta_s) e^{-j(\delta - \frac{\pi}{2})} = 1.09 \angle (11.59^\circ - 52.1^\circ + 90^\circ) = 0.7107 + j0.8326$$

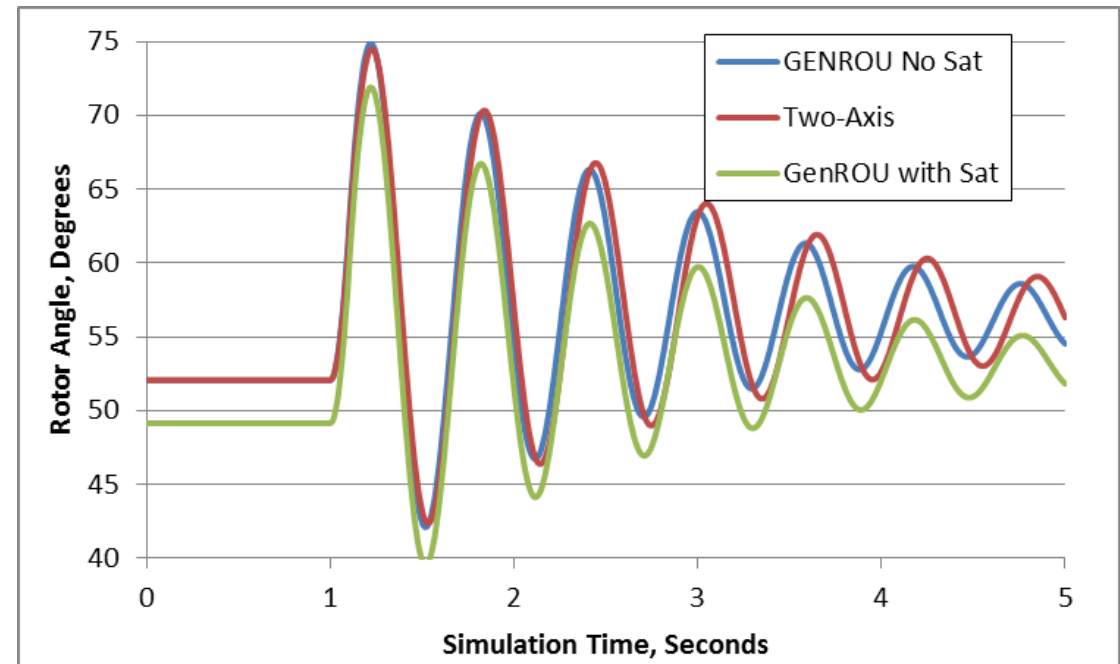
$$I_d + jI_q = (I_s \angle \phi_s) e^{-j(\delta - \frac{\pi}{2})} = 1.0526 \angle (-18.2^\circ - 52.1^\circ + 90^\circ) = 0.9909 + j0.3553$$
    - And from this we can get initial subtransient fluxes
 
$$0 = V_d + I_d R_s - X''_q I_q + (1 + \omega) \psi''_q \rightarrow \psi''_q = -0.611$$

$$0 = V_q + I_q R_s + X''_d I_d - (1 + \omega) \psi''_d \rightarrow \psi''_d = 1.110$$
    - Other values will be
 
$$E'_q = 1.1298, E'_d = 0.533, \psi'_q = 0.6645, \psi'_d = 0.9614, E_{fd} = 2.9133$$
- (See the block diagram)

# GENROU Initialization Example, with Saturation



- Assume prior example but with  $S(1.0) = 0.05$  and  $S(1.2) = 0.2$
- Initial values change to  $\delta = 49.2^\circ$ ,  $E'_q = 1.1591$ ,  $E'_d = 0.4646$ ,  $\psi'_q = 0.6146$ ,  $\psi'_d = 0.9940$ ,  $E_{fd} = 3.2186$
- Now run the same fault we've been running (B4\_GENROU\_Sat)

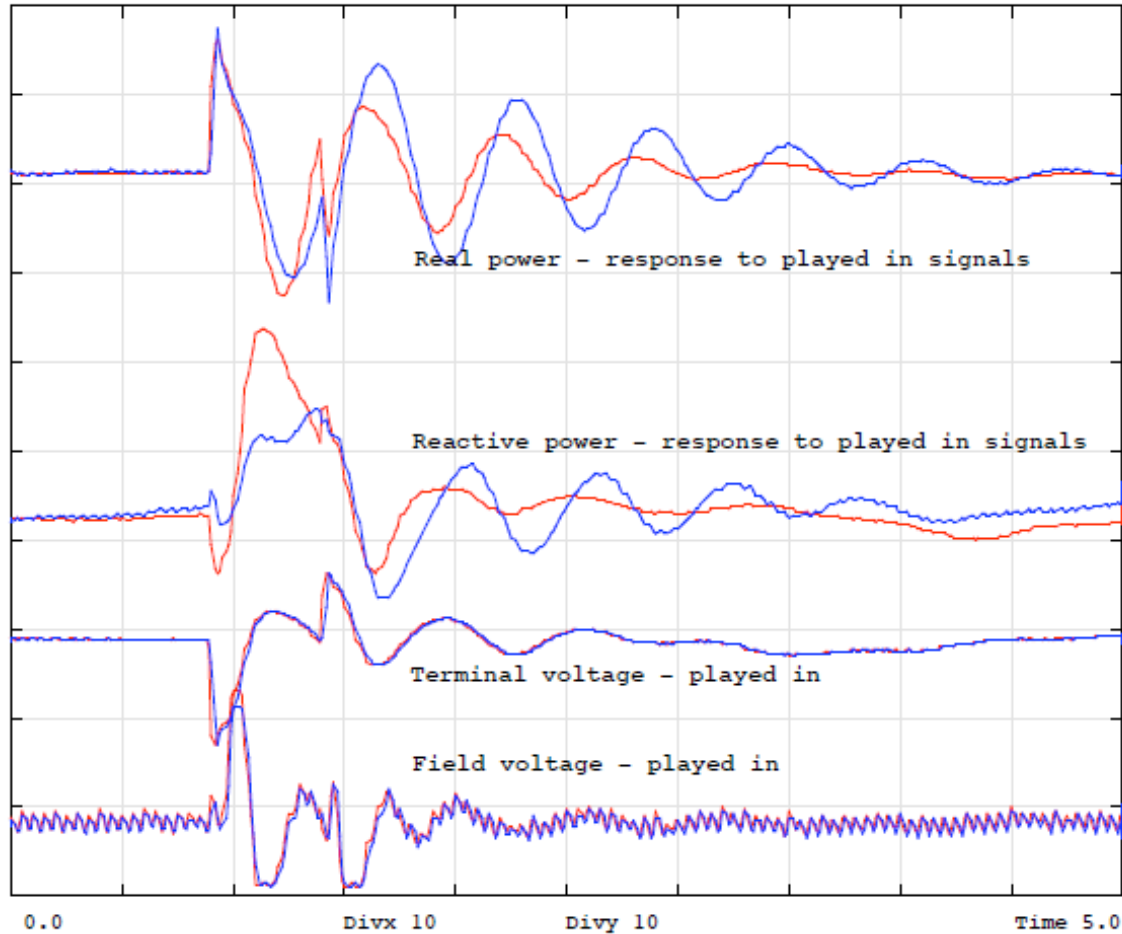


# GENTPF, GENTPJ, and GENQEC



- GENTPF and GENTPJ were introduced in 2009 to better match between simulated and actual system results for salient pole machines (replacing GENSAL)
  - Desire was to duplicate functionality from old BPA transient stability code
  - Allows for subtransient saliency, such that  $X_q'' \neq X_d''$
  - Can also be used for round rotor, hence can replace GENSAL and GENROU
- For more information
- <https://www.powerworld.com/knowledge-base/derivation-of-the-gentpjgentpf-model-from-the-genrougensal>
- Now these are being replaced by GENQEC, which is becoming the recommended model by WECC for new synchronous machines
- <https://www.wecc.org/sites/default/files/documents/meeting/2024/GENQEC%20Model%20Specification%20-%20R3.pdf>

# Motivation for the Change: GENSAL Results



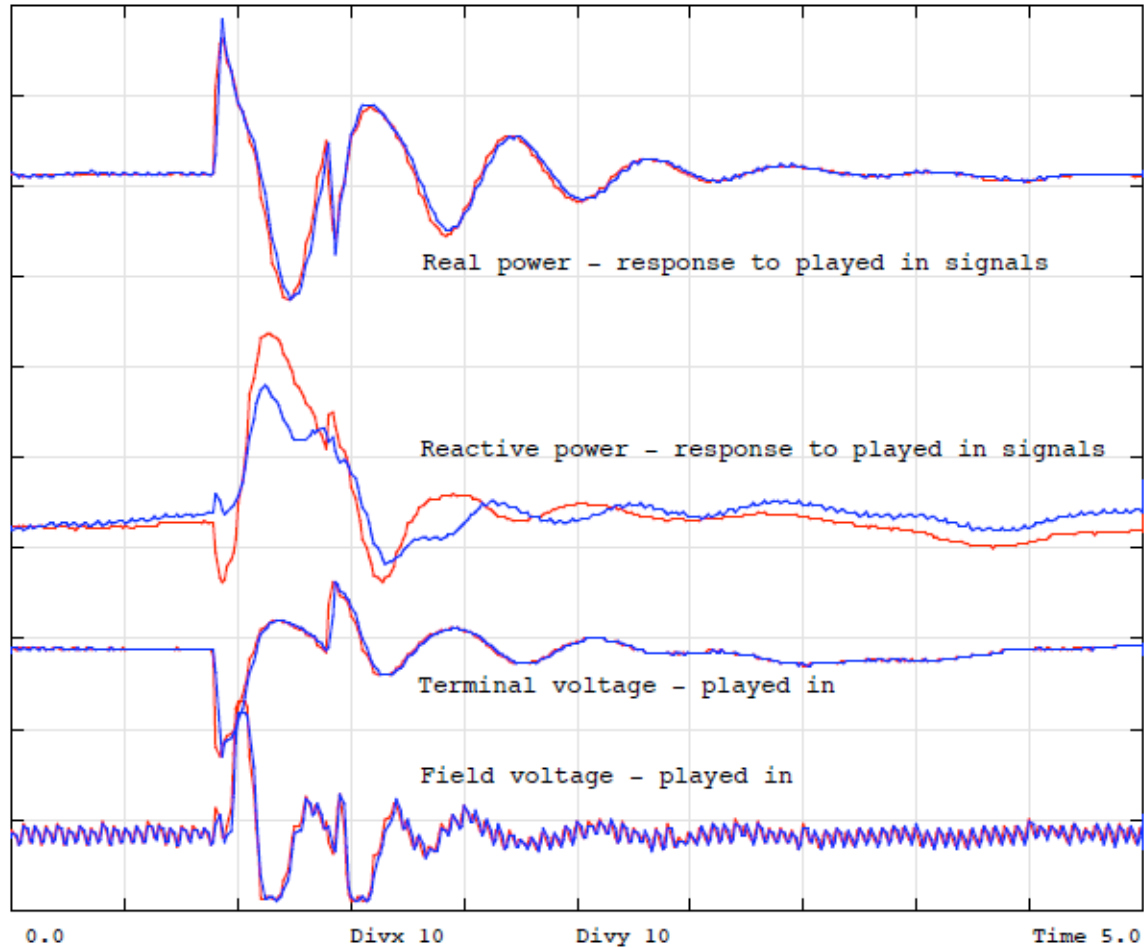
Chief Joseph disturbance  
playback

GENSAL BLUE = MODEL

RED = ACTUAL

(Chief Joseph is a 2620 MW  
hydro plant on the Columbia  
River in Washington)

# Motivation for the Change: GENTPJ Results



Chief Joseph  
disturbance  
playback  
GENTPJ  
BLUE = MODEL  
RED = ACTUAL

# GENQEC Definition



- Saturation terms now appear in lots of places around the rotor equations (notice the  $S_a$  terms)

$$S_a = \text{Sat}(\psi_{ag})$$

$$\psi_{ag} = \frac{1}{1 + \omega} \sqrt{V_{qag}^2 + V_{dag}^2}$$

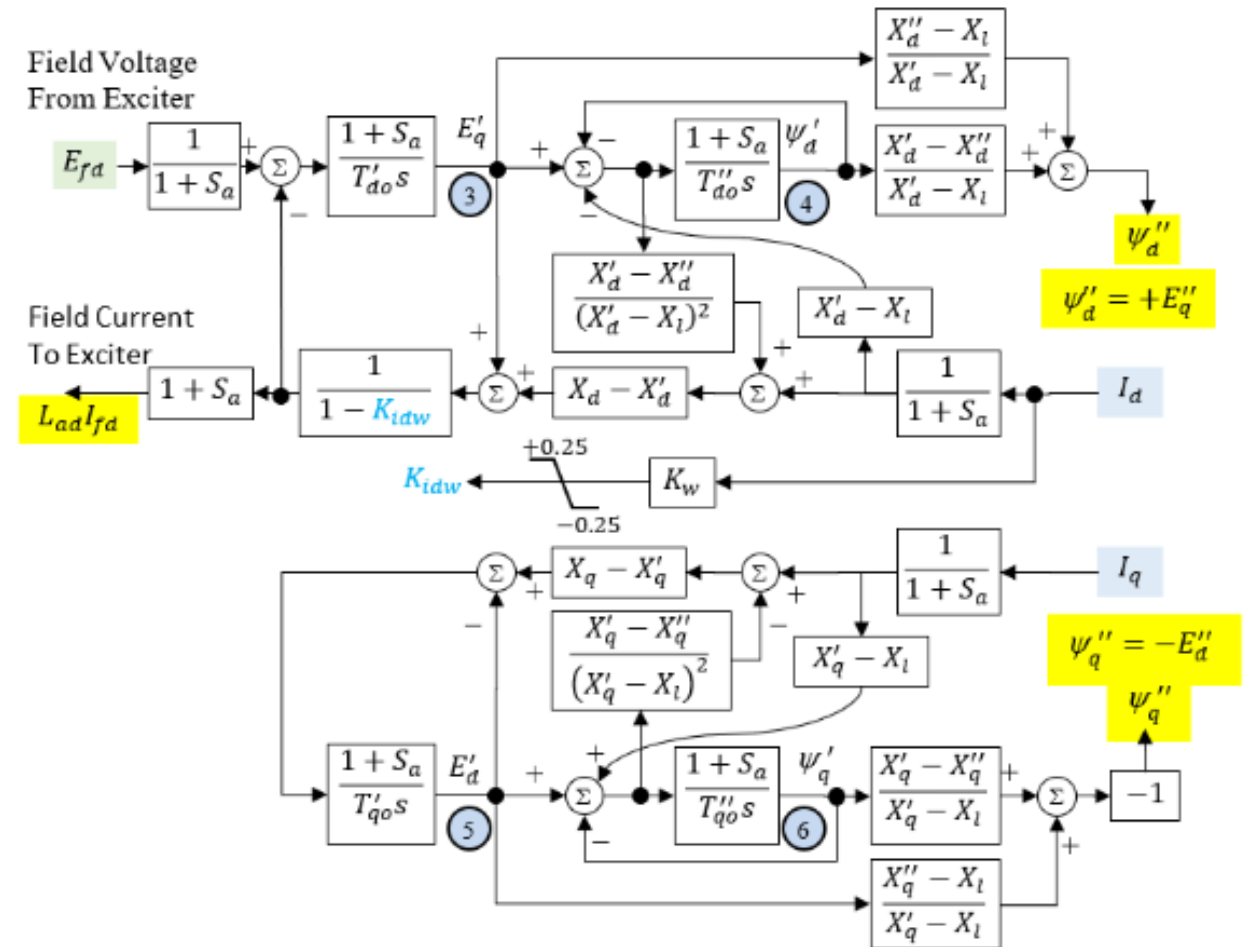
$$V_{qag} = V_q + I_q R_a + I_d X_\ell$$

$$V_{daq} = V_d + I_d R_a - I_q X_\ell$$

- We need to consider saturated transient reactances

$$X''_{dsat} = \frac{X'_d - X_\ell}{1 + S_a} + X_\ell$$

$$X''_{qsat} = \frac{X'_q - X_\ell}{1 + S_a} + X_\ell$$



# Subtransient Saliency



- Because  $X''_{qsat} \neq X''_{dsat}$ , there is no exact circuit model for the network interface. We need to use the following equations directly

$$\begin{aligned} V_d &= -(1 + \omega)\psi_q'' - R_a I_d + X''_{qsat} I_q \\ V_q &= (1 + \omega)\psi_d'' - X''_{dsat} I_d - R_a I_q \end{aligned}$$

- Combined with the network reference frame transformation based on delta
- For GENQEC, we use the following for electrical torque

$$\begin{aligned} T_{elec} &= \psi_d I_q - \psi_q I_d \\ \psi_d &= E_q'' - I_d X''_{dsat} \\ \psi_q &= -E_d'' - I_q X''_{qsat} \end{aligned}$$

# Summary: Synchronous Machine Models



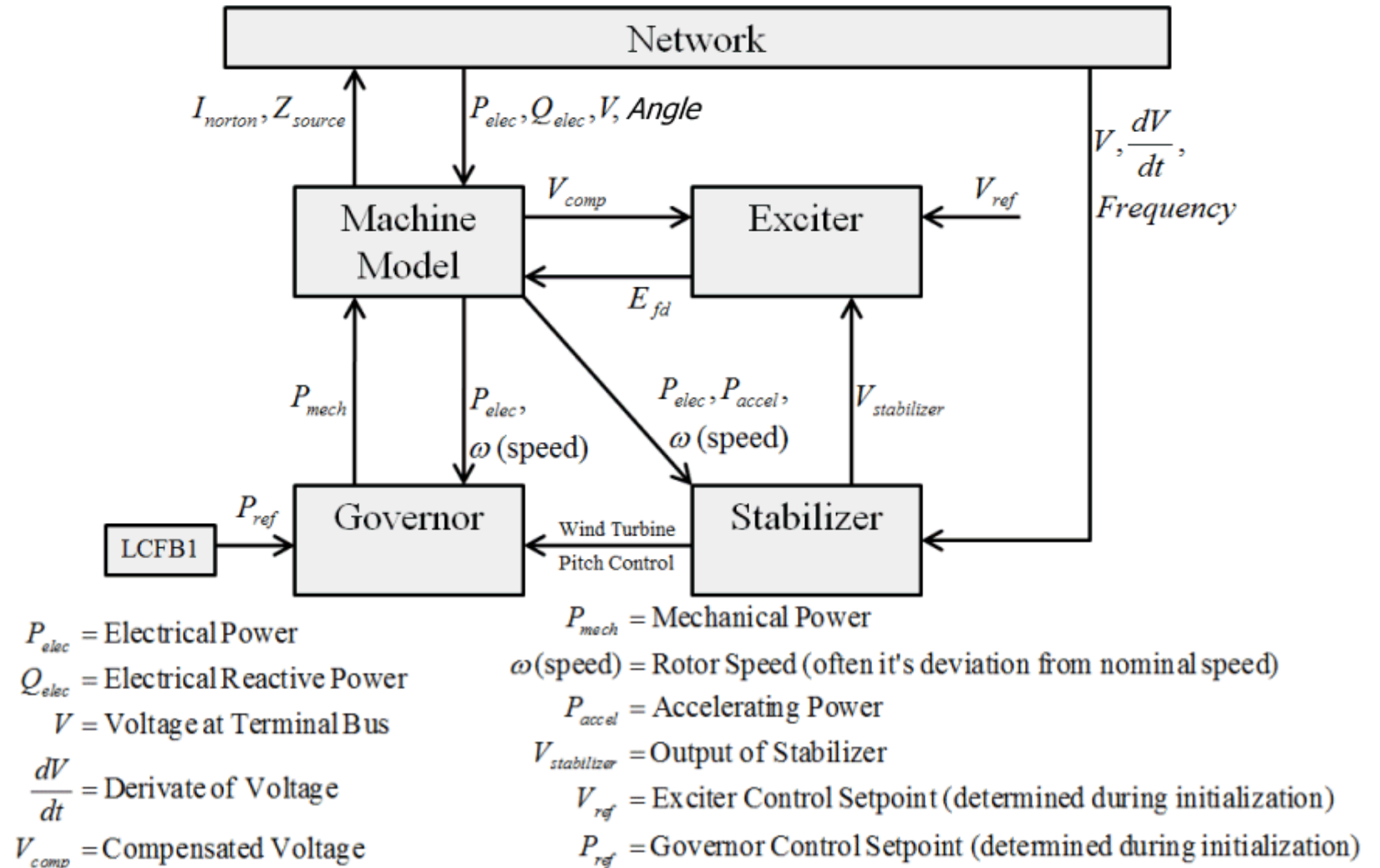
Model Name	Order	Saturation	Other Comments
Classical	2 <sup>nd</sup>	None	
Flux Decay	3 <sup>rd</sup>	None	
Two-Axis	4 <sup>th</sup>	None	
GENSAL	5 <sup>th</sup>	D-axis	
GENROU	6 <sup>th</sup>	D&Q	
GENTPF/J	6 <sup>th</sup>	D&Q	Allows subtransient saliency
GENQEC	6 <sup>th</sup>	D&Q	Allows subtransient saliency

- These models have the same parameter names, but they are not the same
  - Parameters are tuned to a specific model
  - It is not appropriate to just transfer parameters and call that a new model
  - When doing a new generator testing study, that is the time to update the model

# Synchronous Machine Controllers



- The machine model itself only models the physical stator and rotor
- Next we will be beginning discussion of various control systems associated with the synchronous machine



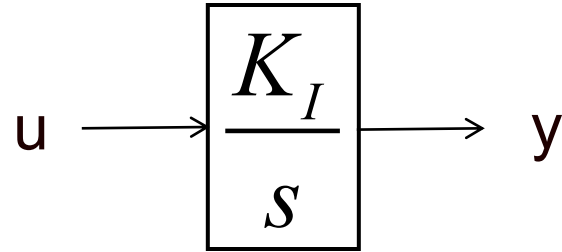
# Block Diagram Basics

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- The following slides will make use of block diagrams to explain some of the models used in power system dynamic analysis. The next few slides cover some of the block diagram basics.
- To simulate a model represented as a block diagram, the equations need to be represented as a set of first order differential equations
- Also the initial state variable and reference values need to be determined

# Integrator Block

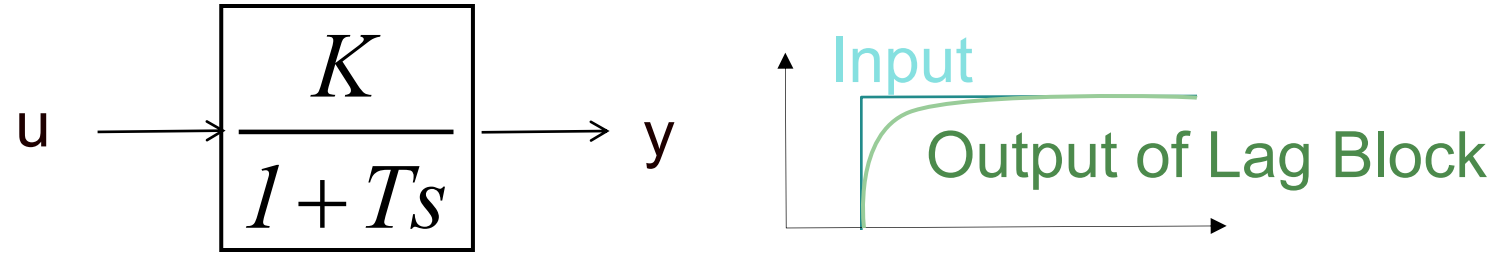


- Equation for an integrator with  $u$  as an input and  $y$  as an output is

$$\frac{dy}{dt} = K_I u$$

- In steady-state with an initial output of  $y_0$ , the initial state is  $y_0$  and the initial input is zero

# First Order Lag Block

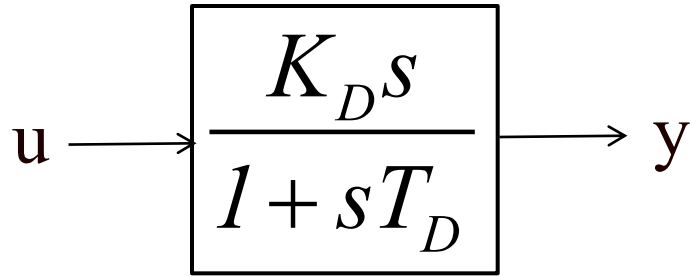


- Equation with  $u$  as an input and  $y$  as an output is

$$\frac{dy}{dt} = \frac{1}{T} (Ku - y)$$

- In steady-state with an initial output of  $y_0$ , the initial state is  $y_0$  and the initial input is  $y_0/K$
- Commonly used for measurement delay (e.g.,  $T_R$  block with IEEE T1)

# Derivative Block



- Block takes the derivative of the input, with scaling  $K_D$  and a first order lag with  $T_D$ 
  - Physically we can't take the derivative without some lag
  - An example is the feedback block in the IEEET1 model
- In steady-state the output of the block is zero
- State equations require a more general approach

# State Equations for More Complicated Functions

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- There is not a unique way of obtaining state equations for more complicated functions with a general form

$$\beta_0 u + \beta_1 \frac{du}{dt} + \cdots + \beta_m \frac{d^m u}{dt^m} =$$
$$\alpha_0 y + \alpha_1 \frac{dy}{dt} + \cdots + \alpha_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + d^n y / dt^n$$

- To be physically realizable we need  $n \geq m$

# General Block Diagram Approach



- One integration approach is illustrated in the below block diagram

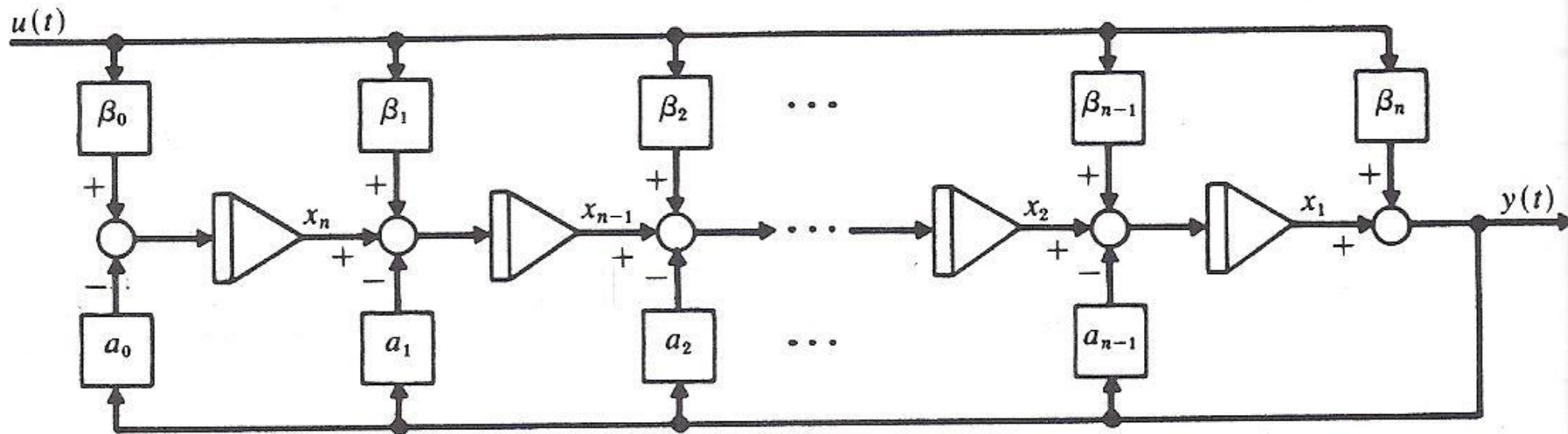


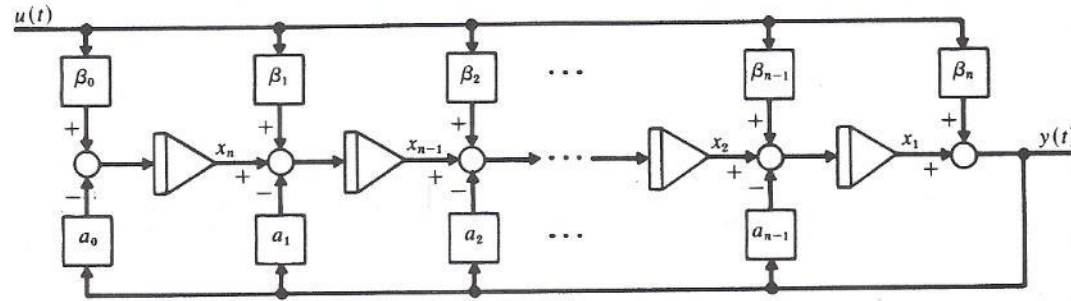
Image source: W.L. Brogan, *Modern Control Theory*, Prentice Hall, 1991, Figure 3.7

# Derivative Example



- Write in form

$$\frac{K_D / T_D s}{1/T_D + s}$$



- Hence  $\beta_0=0$ ,  $\beta_1=K_D/T_D$ ,  $\alpha_0=1/T_D$
- Define single state variable  $x$ , then

$$\frac{dx}{dt} = \beta_0 u - \alpha_0 y = -\frac{y}{T_D}$$

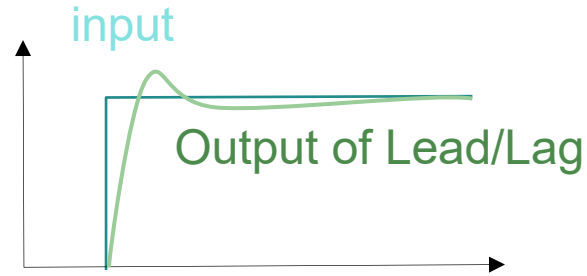
$$y = x + \beta_1 u = x + \frac{K_D}{T_D} u$$

Initial value of  $x$  is found by recognizing  $y$  is zero so  $x = -\beta_1 u$

# Lead-Lag Block



$$u \longrightarrow \boxed{\frac{1 + sT_A}{1 + sT_B}} \longrightarrow y$$



The steady-state requirement that  $u = y$  is readily apparent

- In exciters such as the EXDC1 the lead-lag block is used to model time constants inherent in the exciter; the values are often zero (or equivalently equal)
- In steady-state the input is equal to the output
- To get equations write in form with  $\beta_0 = 1/T_B$ ,  $\beta_1 = T_A/T_B$ ,  $\alpha_0 = 1/T_B$

$$\boxed{\frac{1 + sT_A}{1 + sT_B} = \frac{\frac{1}{T_B} + s\frac{T_A}{T_B}}{\frac{1}{T_B} + s}}$$

# Lead-Lag Block



- The equations are with

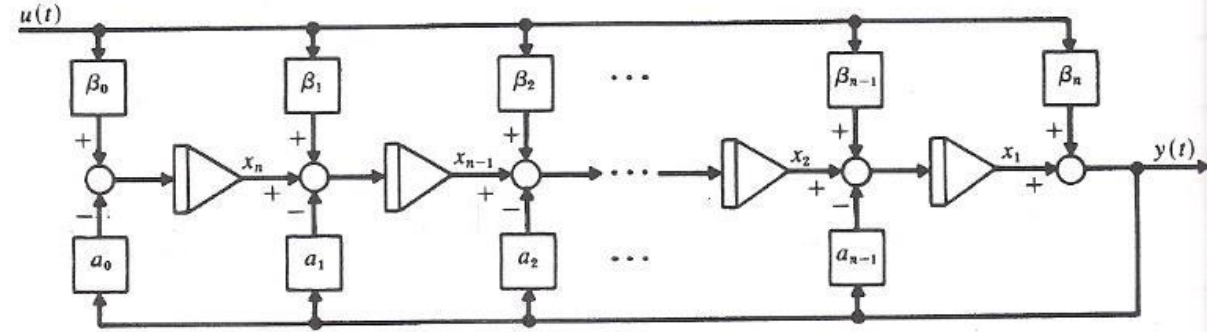
$$\beta_0 = 1/T_B, \quad \beta_1 = T_A/T_B,$$

$$\alpha_0 = 1/T_B$$

then

$$\frac{dx}{dt} = \beta_0 u - \alpha_0 y = \frac{1}{T_B} (u - y)$$

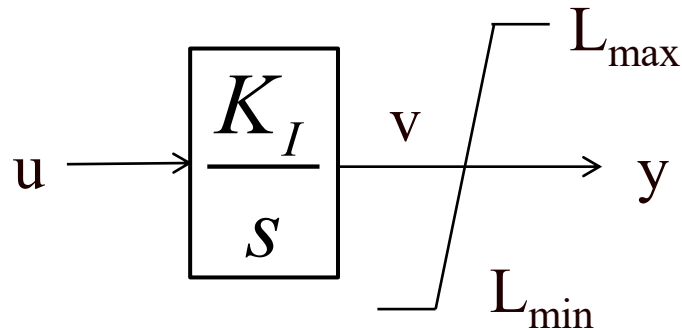
$$y = x + \beta_1 u = x + \frac{T_A}{T_B} u$$



# Limits: Windup versus Nonwindup



- When there is integration, how limits are enforced can have a major impact on simulation results
- Two major flavors: windup and non-windup
- Windup limit for an integrator block



The value of  $v$  is NOT limited, so its value can "windup" beyond the limits, delaying backing off of the limit

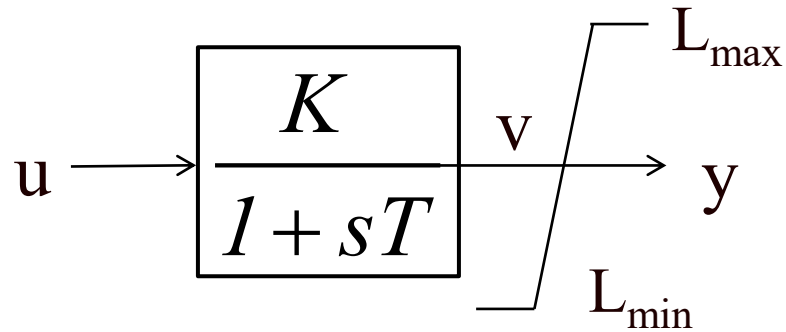
$$\frac{dv}{dt} = K_I u$$

If  $L_{\min} \leq v \leq L_{\max}$  then  $y = v$   
 else If  $v < L_{\min}$  then  $y = L_{\min}$ ,  
 else if  $v > L_{\max}$  then  $y = L_{\max}$

# Limits on First Order Lag



- Windup and non-windup limits are handled in a similar manner for a first order lag



$$\frac{dv}{dt} = \frac{1}{T}(Ku - v)$$

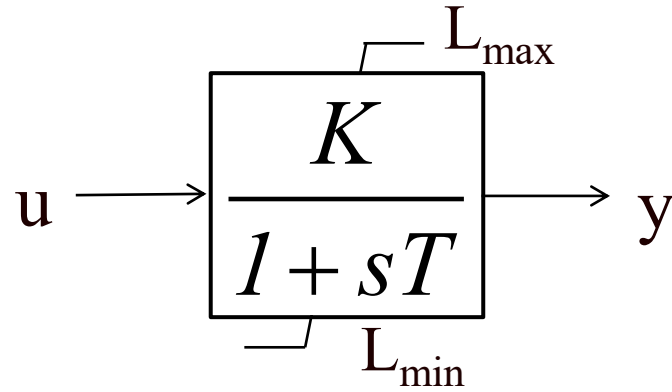
If  $L_{\min} \leq v \leq L_{\max}$  then  $y = v$   
 else If  $v < L_{\min}$  then  $y = L_{\min}$ ,  
 else if  $v > L_{\max}$  then  $y = L_{\max}$

Again the value of  $v$  is NOT limited, so its value can "windup" beyond the limits, delaying backing off of the limit

# Non-Windup Limit First Order Lag



- With a non-windup limit, the value of  $y$  is prevented from exceeding its limit



$$\frac{dy}{dt} = \frac{1}{T} (Ku - y)$$

(except as indicated below)

If  $L_{\min} \leq y \leq L_{\max}$  then normal  $\frac{dy}{dt} = \frac{1}{T} (Ku - y)$

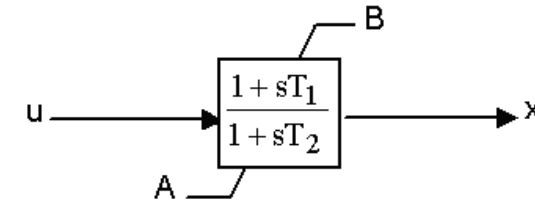
If  $y \geq L_{\max}$  then  $y=L_{\max}$  and if  $u > 0$  then  $\frac{dy}{dt} = 0$

If  $y \leq L_{\min}$  then  $y=L_{\min}$  and if  $u < 0$  then  $\frac{dy}{dt} = 0$

# Lead-Lag Non-Windup Limits



- There is not a unique way to implement non-windup limits for a lead-lag. This is the one from IEEE 421.5-1995 (Figure E.6)



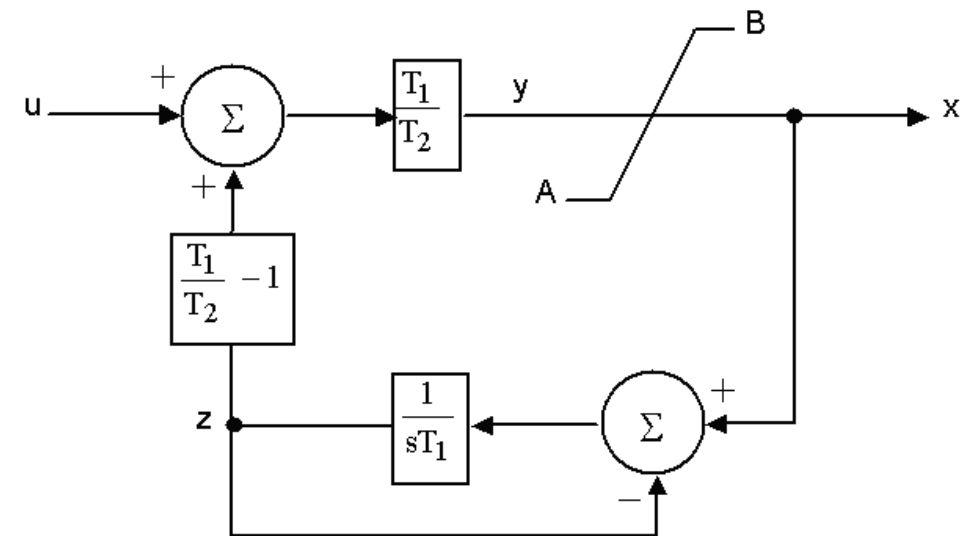
(a) Model

$$T_2 > T_1, T_1 > 0, T_2 > 0$$

$$\text{If } y > B, \text{ then } x = B$$

$$\text{If } y < A, \text{ then } x = A$$

$$\text{If } B \geq y \geq A, \text{ then } x = y$$

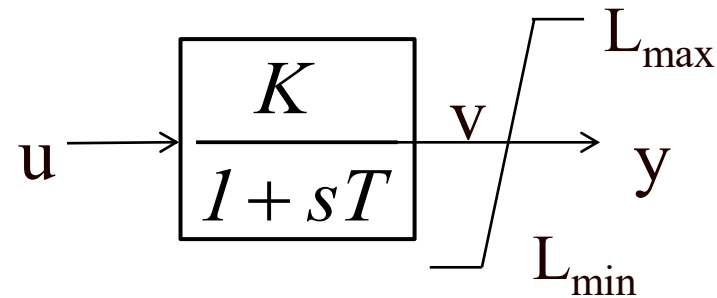


(b) Implementation

# Ignored States



- When integrating block diagrams often states are ignored, such as a measurement delay with  $T_R=0$
- In this case the differential equations just become algebraic constraints
- Example: For block at right, as  $T \rightarrow 0$ ,  $v=Ku$



- With lead-lag it is quite common for  $T_A=T_B$ , resulting in the block being ignored