ECEN 667
Power System Stability

Lecture 5: Synchronous Machine Modeling, Part 1

Prof. Adam Birchfield
Dept. of Electrical and Computer Engineering
Texas A&M University
abirchfield@tamu.edu

TEXAS A&M UNIVERSITY

Announcements

- Homework Assignment #1 is due Thursday, Sept. 11th at 8 AM. Email me your solution as a single PDF.
- Homework Assignment #2 is due Thursday, Sept. 25th at 8 AM.
- Read book chapters 3, 4, and 5
- Review the slides and PowerWorld examples

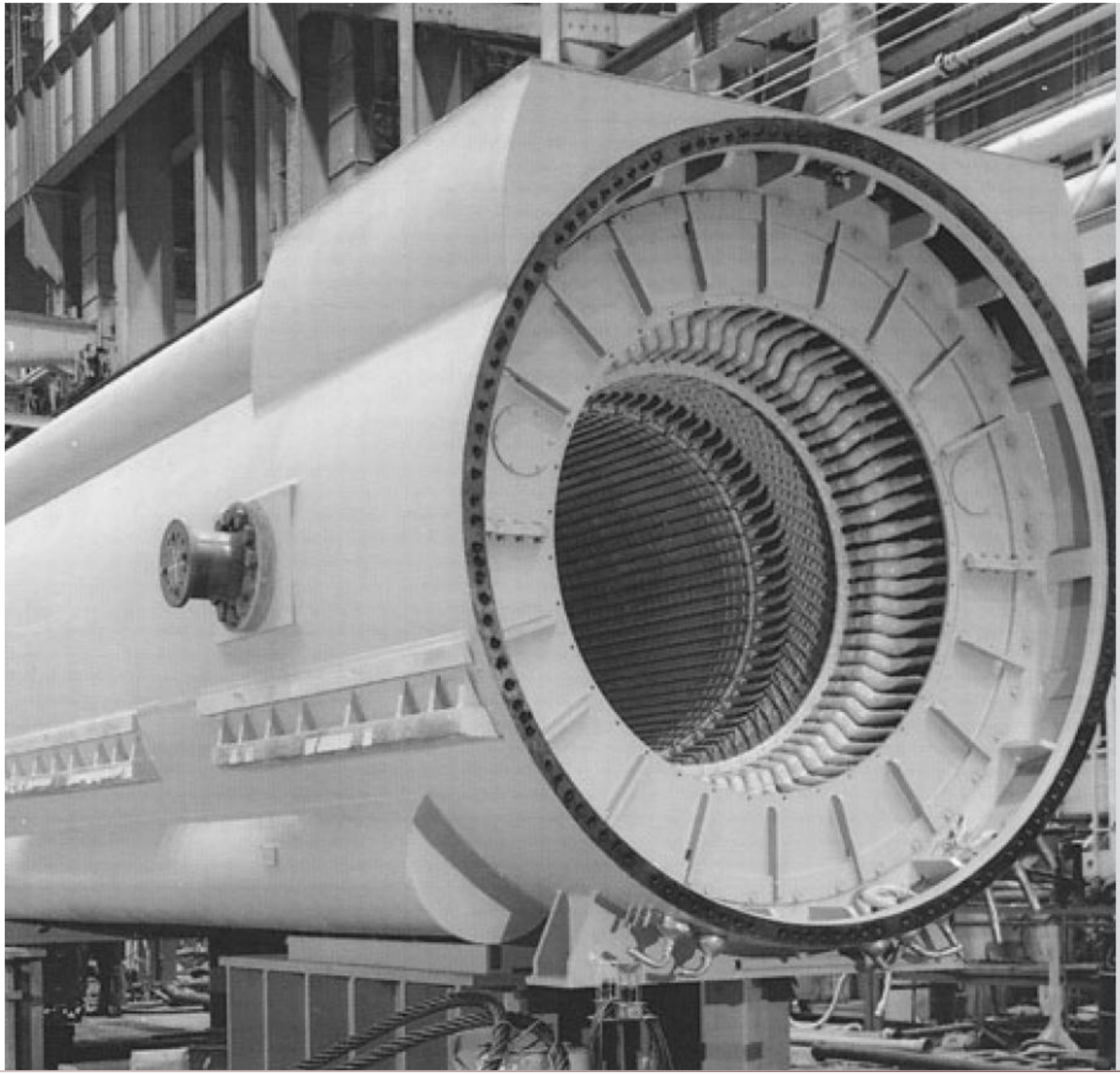
Synchronous Machines

- Steam, gas, and hydro turbines use synchronous machines to convert mechanical power to electric
- Hence, they have historically dominated power system dynamics
- The **classical model**, introduced before, is over-simplified and not used by industry very much
- We will now go through more detailed synchronous machine modeling for stability studies

Synchronous Machine Terms

- Two main components of a machine
  - **Stator** – the outer stationary part, containing armature windings arranged in 2 or more "poles". Machines with more poles spin slower.
  - **Rotor** – the inner, moving part, which contains a dc field winding powered by an exciter
- Two main types of rotors
  - **Round Rotor**
    - Air-gap is constant, used with higher speed machines
  - **Salient Rotor** (often called Salient Pole)
    - Air-gap varies circumferentially
    - Used with many pole, slower machines such as hydro
    - Narrowest part of gap in the d-axis and the widest along the q-axis

Synchronous Machine Stator

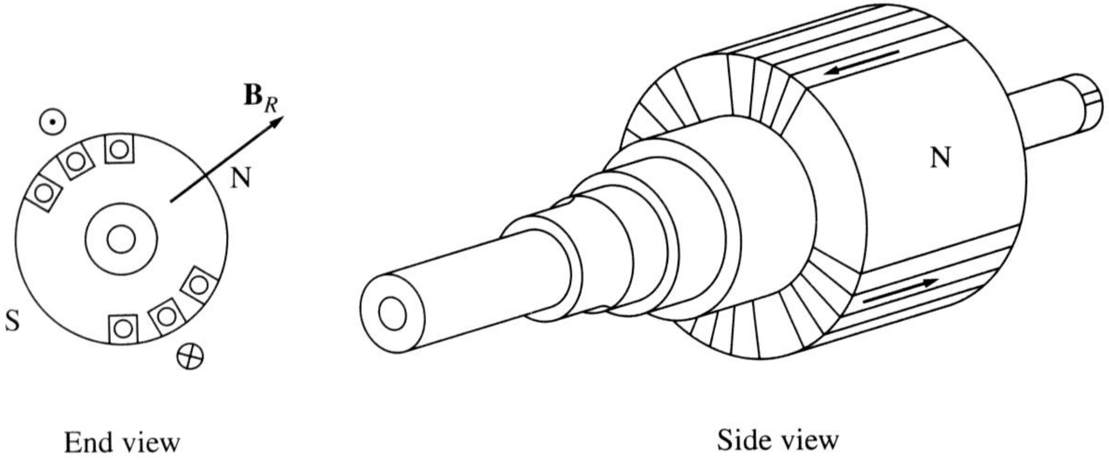


Generator stator showing completed windings for a 757-MVA, 3600-RPM, 60-Hz synchronous generator (Courtesy of General Electric.)

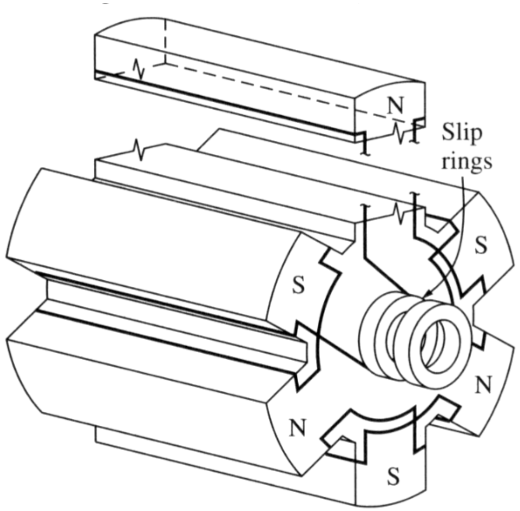
Image Source: Glover/Overbye/Sarma Book, Sixth Edition, Beginning of Chapter 8 Photo

Synchronous Machine Rotors

- Rotors are essentially electromagnets



Two pole (P) round rotor



Six pole salient rotor

Images Source: Dr. Gleb Tcheslavski, ee.lamar.edu/gleb/teaching.htm

Synchronous Machine Rotor

High pole salient rotor

Shaft

Part of exciter, which is used to control the field current

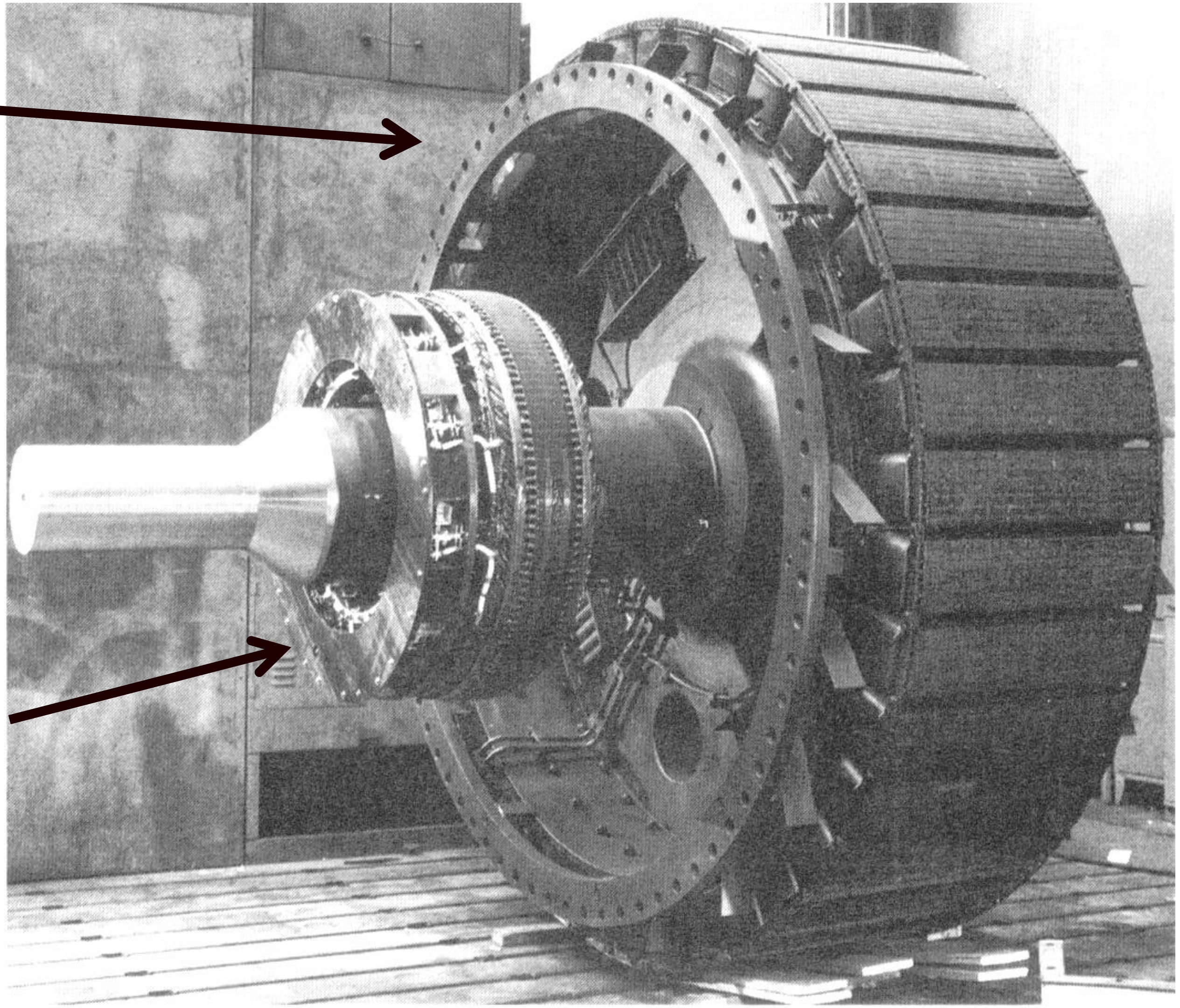
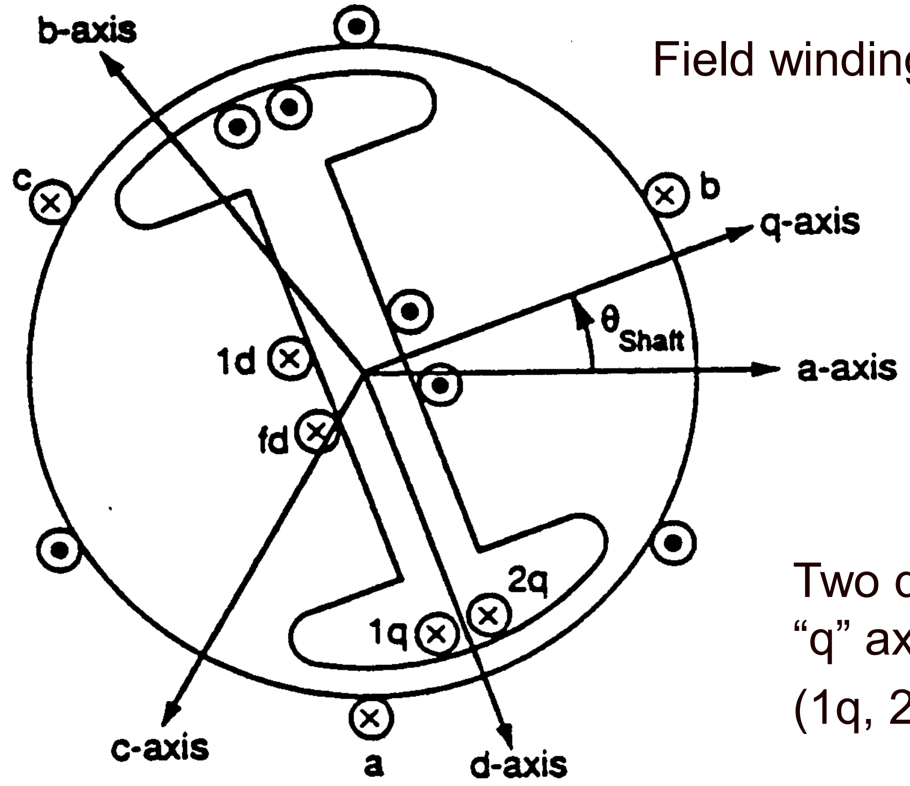


Image Source: Dr. Gleb Tcheslavski, ee.lamar.edu/gleb/teaching.htm

The Main Diagram

- Note that conventions vary in other textbooks – this can lead to confusion!
- On the stator (2 pole equivalent), we have three stationary phase windings (a, b, and c)
- On the rotor, we have two rotating "axes", direct (d) and quadrature (q), with one physical field winding (fd) and three damper windings that are possibly fictitious (1d, 1q, 2q)
- θ_{shaft} is the machine's angle

3ϕ bal. windings (a,b,c) – stator



Field winding (fd) on rotor

Damper in "d" axis (1d) on rotor

Two dampers in "q" axis (1q, 2q) on rotor

Park's Transformation

- The purpose of Park's transformation is to convert stator values (voltage, current, flux) into the rotating DQ reference frame of the rotor
- The matrix depends on θ_{shaft} and is specific to our diagram conventions

$$T(\theta_{shaft}) = \frac{2}{3}\begin{bmatrix} \sin\left(\frac{P}{2}\theta_{shaft}\right) & \sin\left(\frac{P}{2}\theta_{shaft} - \frac{2\pi}{3}\right) & \sin\left(\frac{P}{2}\theta_{shaft} + \frac{2\pi}{3}\right) \\ \cos\left(\frac{P}{2}\theta_{shaft}\right) & \cos\left(\frac{P}{2}\theta_{shaft} - \frac{2\pi}{3}\right) & \cos\left(\frac{P}{2}\theta_{shaft} + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

- Now, for example, we convert stator voltages on the a, b, and c windings to d, q, and 0 axes as follows

$$\begin{bmatrix} V_d \\ V_q \\ V_0 \end{bmatrix} = T(\theta_{shaft}) \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

- Similarly for current and flux linkage. The 0-axis represents the θ-independent component

Park’s Transformation Inverse

- The inverse is used to convert from DQ0 axis coordinates back to the stationary ABC reference frame

$$T^{-1} = \begin{bmatrix} \sin\left(\frac{P}{2}\theta_{shaft}\right) & \cos\left(\frac{P}{2}\theta_{shaft}\right) & 1 \\ \sin\left(\frac{P}{2}\theta_{shaft} - \frac{2\pi}{3}\right) & \cos\left(\frac{P}{2}\theta_{shaft} - \frac{2\pi}{3}\right) & 1 \\ \sin\left(\frac{P}{2}\theta_{shaft} + \frac{2\pi}{3}\right) & \cos\left(\frac{P}{2}\theta_{shaft} + \frac{2\pi}{3}\right) & 1 \end{bmatrix}$$

- For example, using current or flux linkage

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = T^{-1} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} \qquad \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix} = T^{-1} \begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_0 \end{bmatrix}$$

- A key idea in synchronous machine modeling is we do as much as possible in DQ0 coordinates

Winding Circuit Equations

- The voltage drop across each winding depends on both
  - The resistance in the copper conductors (Ohm's law) and
  - Induced electromotive force from the flux linkage (Faraday's law)
- So, we have the following equations for our 7 windings

$$v_a = i_a r_s + \frac{d\lambda_a}{dt} \qquad v_b = i_b r_s + \frac{d\lambda_b}{dt} \qquad v_c = i_c r_s + \frac{d\lambda_c}{dt}$$

$$v_{fd} = i_{fd} r_{rf} + \frac{d\lambda_{fd}}{dt} \qquad v_{1d} = i_{1d} r_s + \frac{d\lambda_{1d}}{dt}$$

$$v_{1q} = i_{1q} r_{1q} + \frac{d\lambda_{1q}}{dt} \qquad v_{2q} = i_{2q} r_s + \frac{d\lambda_{2q}}{dt}$$

- Generally, we assume that the damper windings are short-circuited, so that

$$v_{1d} = v_{1q} = v_{2q} = 0$$

Transforming the Stator Equations

- We have to be careful with the $\frac{d\lambda}{dt}$ terms because the transformation is a function of time

$$v_{abc} = r_s i_{abc} + \frac{d}{dt}\lambda_{abc}$$

$$T^{-1}v_{dq0} = r_s T^{-1} i_{dq0} + \frac{d}{dt}\left(T^{-1}\lambda_{dq0}\right)$$

$$v_{dq0} = r_s i_{dq0} + T\frac{d}{dt}\left(T^{-1}\lambda_{dq0}\right) = r_s i_{dq0} + T\left(T^{-1}\frac{d}{dt}\lambda_{dq0} + \left(\frac{d}{dt}T^{-1}\right)\lambda_{dq0}\right)$$

$$v_{dq0} = r_s i_{dq0} + \frac{d}{dt}\lambda_{dq0} + T\left(\frac{d}{dt}T^{-1}\right)\lambda_{dq0}$$

- You should verify the following for yourself

$$T\left(\frac{d}{dt}T^{-1}\right) = \begin{bmatrix} 0 & -\omega_{shaft} & 0 \\ \omega_{shaft} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

ω_{shaft} is the angular speed of the rotor, defined by

$$\frac{d\theta_{shaft}}{dt} = \frac{2}{P}\omega_{shaft}$$

- And we get the following transformed DQ0 stator equations

$$v_d = i_d r_s + \frac{d\lambda_d}{dt} - \omega_{shaft}\lambda_q \qquad v_q = i_q r_s + \frac{d\lambda_q}{dt} + \omega_{shaft}\lambda_d \qquad v_0 = i_0 r_s + \frac{d\lambda_0}{dt}$$

Power Analysis

- Total electric power P_t at the stator terminals is

$$P_t = v_a i_a + v_b i_b + v_c i_c = \left(T^{-1} v_{dq0}\right) \cdot \left(T^{-1} i_{dq0}\right) = \frac{3}{2} v_d i_d + \frac{3}{2} v_q i_q + 3 v_0 i_0$$

- Now plug in our circuit equations for the voltages from before

$$P_t = \underbrace{\frac{3}{2}\left(i_d \frac{d\lambda_d}{dt} + i_q \frac{d\lambda_q}{dt} + 2 i_0 \frac{d\lambda_0}{dt}\right)}_{\text{Power temporarily stored in the armature magnetic fields (averages out to zero)}} + \underbrace{\frac{3}{2}\left(\lambda_d i_q - \lambda_q i_d\right)\left(\frac{P}{2}\right)\omega_{shaft}}_{\text{Power transferred across the “air gap” between the rotor and the stator}} + \underbrace{\frac{3}{2}\left(i_d^2 + i_q^2 + 2 i_0^2\right) r_s}_{\text{Power lost in armature resistances}}$$

The Machine's Equation of Motion

- The middle term must correspond to the electrical torque (an alternative argument is given in our Sauer/Pai book, this one was from Kundur)

$$T_e = -\frac{3}{2}\left(\lambda_d i_q - \lambda_q i_d\right)\left(\frac{P}{2}\right)$$

- Newton's Second Law gives the machine's equations of motion

$$J\frac{2}{P}\frac{d\omega_{shaft}}{dt} = T_m - T_e - T_{fw}$$

- T_m is the mechanical torque
- T_e is the electrical torque
- T_{fw} is the torque lost due to friction and windage
- J is the mechanical moment of inertia of the rotor.
- This equation is also known as the swing equation.

Magnetic Coupling Between Windings

- Until this point, the winding equations have been mostly separate
- What couples them is the fact that currents cause flux linkages not just for their own windings, but for others as well
- A very nice thing about the DQ0 coordinates is that they are orthogonal, so currents only link along the same axis
- We will start with linear relationships, but saturation causes nonlinearities which we will consider shortly

$$\begin{bmatrix} \lambda_d \\ \lambda_{fd} \\ \lambda_{1d} \end{bmatrix} = \begin{bmatrix} L_{\ell s} + L_{md} & L_{sfd} & L_{s1d} \\ \frac{3}{2} L_{sfd} & L_{fdfd} & L_{fd1d} \\ \frac{3}{2} L_{s1d} & L_{fd1d} & L_{1d1d} \end{bmatrix} \begin{bmatrix} i_d \\ i_{fd} \\ i_{1d} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \lambda_q \\ \lambda_{1q} \\ \lambda_{2q} \end{bmatrix} = \begin{bmatrix} L_{\ell s} + L_{mq} & L_{s1q} & L_{s2q} \\ \frac{3}{2} L_{s1q} & L_{1q1q} & L_{1q2q} \\ \frac{3}{2} L_{s2q} & L_{1q2q} & L_{2q2q} \end{bmatrix} \begin{bmatrix} i_q \\ i_{1q} \\ i_{2q} \end{bmatrix} \quad \text{and} \quad \lambda_0 = L_{\ell s} i_0$$

- All the L's represent constant physical inductances among windings
- $L_{\ell s}$ is the “leakage inductance” of stator windings onto themselves

Summary So Far

- We have 7 winding circuit equations (transformed)

$$v_d = i_d r_s + \frac{d\lambda_d}{dt} - \omega_{shaft}\lambda_q \qquad v_q = i_q r_s + \frac{d\lambda_q}{dt} + \omega_{shaft}\lambda_d \qquad v_0 = i_0 r_s + \frac{d\lambda_0}{dt}$$

$$v_{fd} = i_{fd} r_{rf} + \frac{d\lambda_{fd}}{dt} \qquad v_{1d} = i_{1d} r_s + \frac{d\lambda_{1d}}{dt}$$

$$v_{1q} = i_{1q} r_{1q} + \frac{d\lambda_{1q}}{dt} \qquad v_{2q} = i_{2q} r_s + \frac{d\lambda_{2q}}{dt}$$

- Two mechanical equations

$$\frac{d\theta_{shaft}}{dt} = \frac{2}{P}\omega_{shaft} \qquad J\frac{2}{P}\frac{d\omega_{shaft}}{dt} = T_m - \left(-\frac{3}{2}\left(\lambda_d i_q - \lambda_q i_d\right)\left(\frac{P}{2}\right)\right) - T_{fw}$$

- And 7 magnetic coupling equations

$$\begin{bmatrix}\lambda_d\\ \lambda_{fd}\\ \lambda_{1d}\end{bmatrix} = \begin{bmatrix} L_{\ell s}+L_{md} & L_{sfd} & L_{s1d}\\ \frac{3}{2}L_{sfd} & L_{fdfd} & L_{fd1d}\\ \frac{3}{2}L_{s1d} & L_{fd1d} & L_{1d1d}\end{bmatrix}\begin{bmatrix} i_d\\ i_{fd}\\ i_{1d}\end{bmatrix} \quad \text{and} \quad \begin{bmatrix}\lambda_q\\ \lambda_{1q}\\ \lambda_{2q}\end{bmatrix} = \begin{bmatrix} L_{\ell s}+L_{mq} & L_{s1q} & L_{s2q}\\ \frac{3}{2}L_{s1q} & L_{1q1q} & L_{1q2q}\\ \frac{3}{2}L_{s2q} & L_{1q2q} & L_{2q2q}\end{bmatrix}\begin{bmatrix} i_q\\ i_{1q}\\ i_{2q}\end{bmatrix} \quad \text{and} \quad \lambda_0 = L_{\ell s} i_0$$

Next Steps

- The prior slide model is the full detail (without saturation) model based on analysis of the physics, in physical units, which is closer to modeling for time-domain (EMT) programs
- We are going to make some major modifications through mathematical manipulation, to produce a model more aligned with industry stability studies
  1. **We are going to normalize all variables with the use of the per-unit system**
  2. **We are going to make several equivalent substitutions. Instead of θ_{shaft} and ω_{shaft} we will have δ and ω; instead of λ_{fd}, λ_{1q}, v_{fd} we will have E'_q , E'_d, and E_{fd}; instead of resistances (except r_s) we will have time constants; instead of the physical reactances X we will have measured reactances X', X'', etc.**
  3. **We will eliminate damper winding voltages and assume some additional symmetry between the winding reactances.**

Per-Unit Bases for Synchronous Machines

- Fundamental "arbitrary" choices for per-unit are frequency base $\omega_B = \omega_s$ (rated synchronous electrical speed), S_B (three-phase rated MVA base of the machine), and V_B (rated RMS line-to-neutral stator voltage).
- From these we calculate additional bases. $I_B = -\frac{S_B}{3V_B}$ and $\Lambda_B = \frac{V_B}{\omega_B}$
- Note the negative sign on I_B, which converts the current orientation to "generator notation", that is, out of the machine.
- On the rotor, we have additional current bases scaled by inductances. So,

$$I_{BFD} = -\frac{L_{md}}{L_{sfd}} I_B \qquad I_{B1D} = -\frac{L_{md}}{L_{s1d}} I_B \qquad I_{B1Q} = -\frac{L_{mq}}{L_{s1q}} I_B \qquad I_{B2Q} = -\frac{L_{mq}}{L_{s2q}} I_B$$

- And there are corresponding voltage and flux bases, and impedance bases $V_{BX} = \frac{S_B}{I_{BX}}$, $\Lambda_{BX} = \frac{V_{BX}}{\omega_B}$, and $Z_{BX} = \frac{V_{BX}^2}{S_B}$, where X is fd, 1d, 1q, and 2q.
- Finally, there is a mechanical torque base of $T_B = \frac{S_B}{\omega_B \frac{2}{P}}$

Scaled Variables

- Upper-case for normalized variables of voltage, current, and resistance
$$V_d, V_q, V_0, V_{fd}, V_{1d}, V_{1q}, V_{2q}, I_d, I_q, I_0, I_{fd}, I_{1d}, I_{1q}, I_{2q}, R_s, R_{fd}, R_{1d}, R_{1q}, R_{2q}$$
- Normalized fluxes use psi, $\psi_d, \psi_q, \psi_0, \psi_{fd}, \psi_{1d}, \psi_{1q}, \psi_{2q}$
- And for inductances, we divide by the impedance base, multiplying by ω_s, resulting in reactance quantities (this does not require assuming frequency will be ω_s, just using it to define the base).
$$X_{\ell s}, X_{md}, X_{mq}, X_{fd}, X_{1d}, X_{1q}, X_{2q}$$
- The following scale factors are used in the linear impedance model.
$$c_d = \frac{L_{fd1d}L_{md}}{L_{sfd}L_{s1d}} \qquad c_q = \frac{L_{1q2q}L_{mq}}{L_{s1q}L_{s2q}}$$ (we commonly assume $c_d = c_q = 1$)
- For torques, we capitalize the subscripts when normalizing: T_M, T_E, T_{FW}
- For inertia constant J, we replace it with $H = \frac{1}{2}J\frac{\left(\omega_B\frac{2}{P}\right)^2}{S_B}$

New Relative Variables

- We define a relative, normalized machine angle

$$\delta = \frac{1}{\omega_s}\frac{P}{2}\theta_{shaft} - t$$

- And a relative, normalized machine speed

$$\omega = \frac{\omega_{shaft} - \omega_s}{\omega_s}$$

- We also replace three variables with “E” variables as follows

$$E_q' = \frac{X_{md}}{X_{fd}}\psi_{fd}$$

$$E_d' = -\frac{X_{mq}}{X_{q1}}\psi_{1q}$$

$$E_{fd} = \frac{X_{md}}{R_{fd}}V_{fd}$$

Using Test-Based Parameters

- Instead of physical-based parameters (R and X before) we convert to the following variables which are more easily determined from machine tests
- Time constants for the four rotor windings

$$T'_{do} = \frac{X_{fd}}{\omega_s R_{fd}} \qquad T''_{do} = \frac{1}{\omega_s R_{1d}}\left(X_{\ell 1d} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{\ell fd}}}\right)$$

$$T'_{qo} = \frac{X_{1q}}{\omega_s R_{1q}} \qquad T''_{qo} = \frac{1}{\omega_s R_{2q}}\left(X_{\ell 2q} + \frac{1}{\frac{1}{X_{mq}} + \frac{1}{X_{\ell 1q}}}\right)$$

- And 6 new reactance parameters

$$X_d = X_{md} + X_{\ell s} \qquad X'_d = X_d - \frac{X_{md}^2}{X_{fd}} \qquad X''_d = X_{\ell s} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{\ell fd}} + \frac{1}{X_{\ell 1d}}}$$

$$X_q = X_{mq} + X_{\ell s} \qquad X'_q = X_q - \frac{X_{mq}^2}{X_{1q}} \qquad X''_q = X_{\ell s} + \frac{1}{\frac{1}{X_{mq}} + \frac{1}{X_{\ell 1q}} + \frac{1}{X_{\ell 2q}}}$$

Full Per-Unit Model (after the Dust Clears)

$$\frac{1}{\omega_s}\frac{d\psi_d}{dt} = V_d + I_d R_s + (\omega + 1)\psi_q$$

$$\frac{1}{\omega_s}\frac{d\psi_q}{dt} = V_q + I_q R_s - (\omega + 1)\psi_d$$

$$\frac{1}{\omega_s}\frac{d\psi_0}{dt} = V_0 + I_0 R_s$$

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - (X_d - X'_d)\left[I_d - \frac{(X'_d - X''_d)}{(X'_d - X_{\ell s})^2}\left(\psi_{1d} + (X'_d - X_{\ell s})I_d - E'_q\right)\right] + E_{fd}$$

$$T''_{do}\frac{d\psi_{1d}}{dt} = -\psi_{1d} + E'_q - (X'_d - X_{\ell s})I_d$$

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + \left(X_q - X'_q\right)\left[I_q - \frac{X'_q - X''_q}{(X'_q - X_{\ell s})^2}\left(\psi_{2q} + \left(X'_q - X_{\ell s}\right)I_q + E'_d\right)\right]$$

$$T''_{qo}\frac{d\psi_{2q}}{dt} = -\psi_{2q} - E'_d - \left(X'_q - X_{\ell s}\right)I_q$$

$$\frac{d\delta}{dt} = \omega$$

$$2H\frac{d\omega}{dt} = T_M + \left(\psi_q I_d - \psi_d I_q\right) - T_{FW}$$

$$\psi_d = -X''_d I_d + \frac{X''_d - X_{\ell s}}{X'_d - X_{\ell s}}E'_q + \frac{X'_d - X''_d}{X'_d - X_{\ell s}}\psi_{1d}$$

$$\psi_q = -X''_q I_q - \frac{X''_q - X_{\ell s}}{X'_q - X_{\ell s}}E'_d + \frac{X'_q - X''_q}{X'_q - X_{\ell s}}\psi_{2q}$$

$$\psi_0 = -X_{\ell s} I_0$$

Full Per-Unit Model, Labeled

Stator Windings

$$\frac{1}{\omega_s}\frac{d\psi_d}{dt} = V_d + I_d R_s + (\omega+1)\psi_q$$

$$\frac{1}{\omega_s}\frac{d\psi_q}{dt} = V_q + I_q R_s - (\omega+1)\psi_d$$

$$\frac{1}{\omega_s}\frac{d\psi_0}{dt} = V_0 + I_0 R_s$$

Rotor Windings

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - (X_d - X'_d)\left[I_d - \frac{(X'_d - X''_d)}{(X'_d - X_{\ell s})^2}\left(\psi_{1d} + (X'_d - X_{\ell s})I_d - E'_q\right)\right] + E_{fd}$$

$$T''_{do}\frac{d\psi_{1d}}{dt} = -\psi_{1d} + E'_q - (X'_d - X_{\ell s})I_d$$

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + (X_q - X'_q)\left[I_q - \frac{X'_q - X''_q}{(X'_q - X_{\ell s})^2}\left(\psi_{2q} + (X'_q - X_{\ell s})I_q + E'_d\right)\right]$$

$$T''_{qo}\frac{d\psi_{2q}}{dt} = -\psi_{2q} - E'_d - (X'_q - X_{\ell s})I_q$$

Mechanical equations

$$\frac{d\delta}{dt} = \omega$$

$$2H\frac{d\omega}{dt} = T_M + (\psi_q I_d - \psi_d I_q) - T_{FW}$$

Stator flux definitions

$$\psi_d = -X''_d I_d + \frac{X''_d - X_{\ell s}}{X'_d - X_{\ell s}}E'_q + \frac{X'_d - X''_d}{X'_d - X_{\ell s}}\psi_{1d}$$

$$\psi_q = -X''_q I_q - \frac{X''_q - X_{\ell s}}{X'_q - X_{\ell s}}E'_d + \frac{X'_q - X''_q}{X'_q - X_{\ell s}}\psi_{2q}$$

$$\psi_0 = -X_{\ell s} I_0$$