ECEN 667
Power System Stability

Lecture 11: Transient Stability, Part 1

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Announcements

- Homework Assignment #3 is online – do not need to turn in but please complete before the first exam.
- Exam 1 in class Thursday, October 9^{th}
- Review the slides and PowerWorld examples

Recall, Stability

- Definition:

  Starting with a certain power system operating condition,
  and subjecting the system to some disturbance,
  will the system regain an equilibrium point
  with all important variables within an acceptable range?

- Notice that with this definition, instability can be affected in multiple ways
  - If you subject the system to a sufficiently large disturbance, you will eventually cause an unstable response
  - Alternatively, if you operate the system in sufficiently stressed conditions, even a very small disturbance could cause an unstable response
  - Design of various control systems are crucial to help increase stability
  - The definition of what variables are important and what is an acceptable range also matters

Recall, How Do We Analyze Stability?

- Generally, the first step is some type of modeling, which usually results in a set of DAEs that describes the system
  - We also need to know the operating point, the disturbance(s) of interests, and desired system performance metrics
- From here there are two main categories of stability analysis techniques
  - **Analytical Stability Methods.** These look at the equations themselves, break them down, and attempt to make mathematical claims about their stability properties. For a linear system (or linearized non-linear system), this could involve eigenvalue analysis. For nonlinear systems, energy functions is an example of an approach. Sometimes a frequency sweep with phase-margin and gain-margin is used. We will talk about several of these approaches later in the semester.
  - **Time-Domain Numerical Methods.** We use numerical integration methods (as last class) to simulate disturbance responses in time. We then need to analyze the numerical results to determine the system's stability properties. This is the focus of today's lecture.

Recall, Time-Domain Stability Analysis Steps

1. Assemble a system model as a DAE set
2. Use Power Flow and equilibrium analysis to find initial conditions
3. Perform time integration using a numerical method
4. When a disturbance event occur, there will be discrete changes to the equations. Often there will be several events, such as fault apply/clear/open.
5. Assess the time-domain results to answer questions about system stability. For example: Did the system return to an equilibrium? How quickly did it reach equilibrium? Is the equilibrium within acceptable range? Did anything exceed acceptable bounds during the transient period? What frequencies of oscillation are present? What is the damping of oscillations?

Transient Stability, History

- Dyrkacz, Young, Maginniss, “A Digital Transient Stability Program Including the Effects of Regular, Exciter and Governor Response,” Proc. AIEE, Part 3, February 1961, pp. 1245-1254.

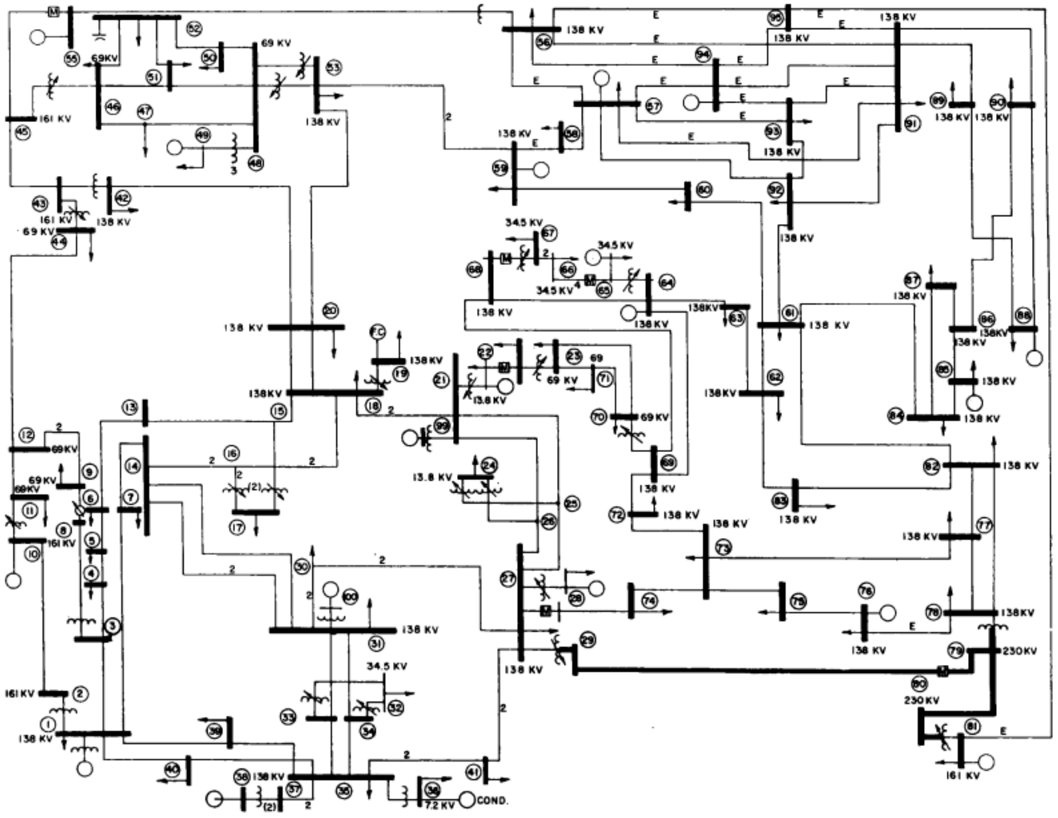
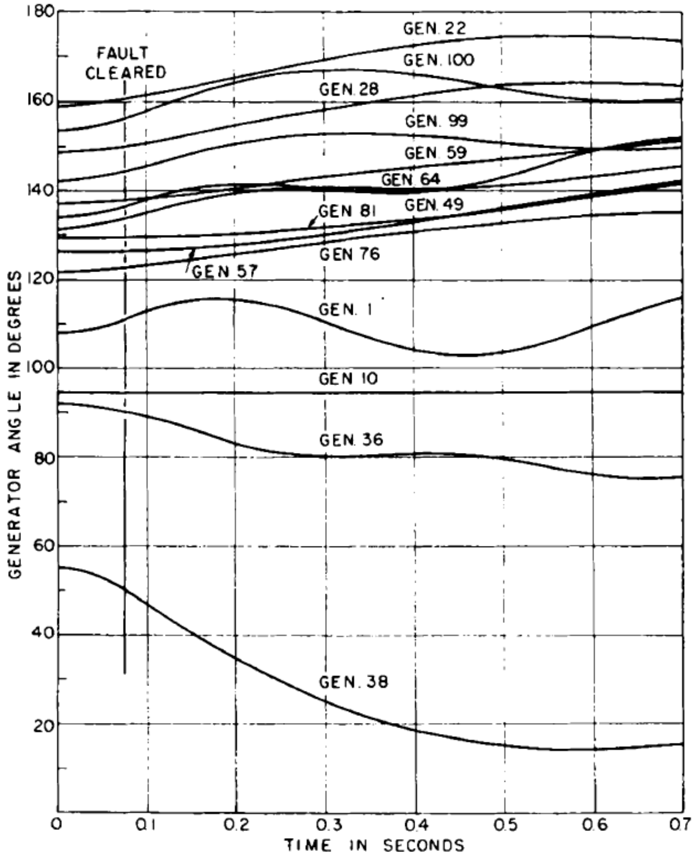


Fig. 4. One-line diagram of Union Electric Company system



Multi-Machine System Stability

- Multi-machine system stability simulation is structured as a set of differential-algebraic equations
$$\dot{x} = f(x, y)$$
$$0 = g(x, y)$$
- Differential equations f generally model the behavior of the machines (and some other devices)
- Algebraic equations g generally model the network constraints
- This differs from EMT-type studies, where line delays decouple the machines.

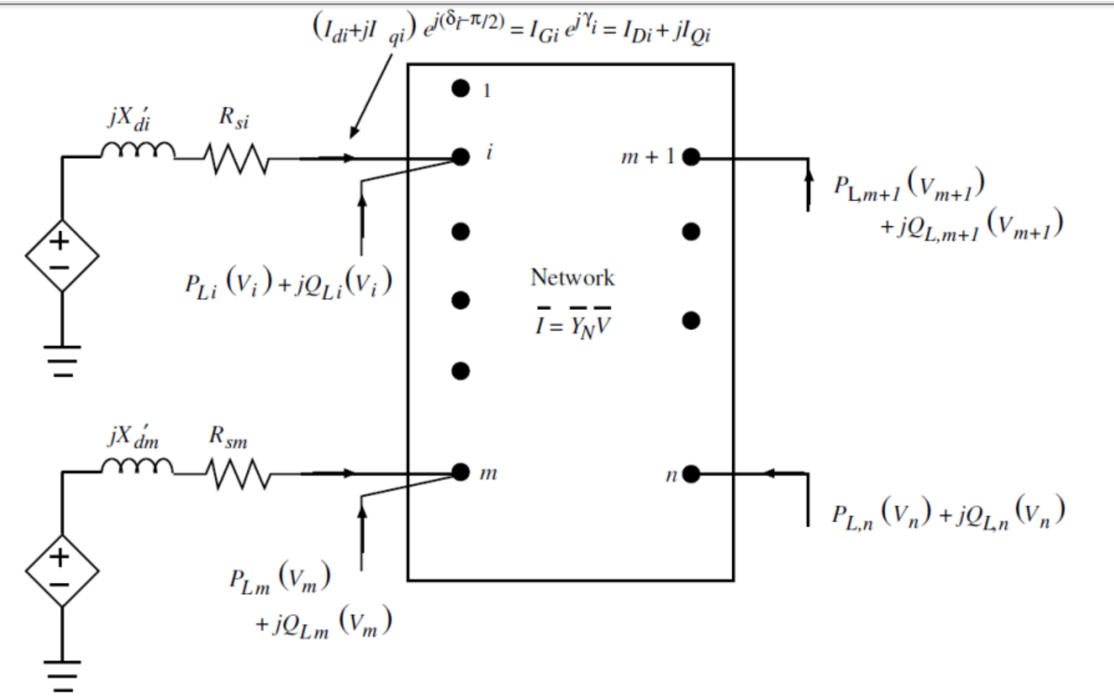


Figure 7.2: *Interconnection of synchronous machine dynamic circuit and the rest of the network*

Approach to Network/Algebraic Constraints

- We discussed before how the network constraints are modeled with

$$\bar{\mathbf{Y}}_{\mathbf{aug}}\bar{V} = \bar{I}_N$$

- Note that the $\bar{Y}_{aug}$ contains not only network admittances but also admittances associated with generators and loads (hence the “aug”)
- How is this different from power flow?
  - In power flow, we do not include generator and load admittances in Y
  - In power flow, the core equations are summing up powers (P and Q) not currents
  - But the power balance equations are derived from the current balance equations
  - Since transient stability models less commonly have power-oriented constraints, they tend to use current balance for the network constraints.

General DAE System Solutions

- To solve DAE problems of the form

$$\dot{x} = f(x, y)$$
$$0 = g(x, y)$$

There are a number of different approaches.

- We'll consider two broad categories of methods
  - **Simultaneous-Implicit Methods.** In these methods, an implicit numerical integration technique is used (such as trapezoidal) and applied to the DAE set, and then the f-g system is solved as one simultaneous implicit set at each time point.
  - **Partitioned-Explicit Methods.** In these methods, an explicit numerical integration technique is used (such as 2^{nd} order Runge-Kutta), and the algorithm alternates between implicitly solving for y using g and explicitly using f to find the derivatives needed for the explicit method. (This is the approach used by PowerWorld and many commercial tools today.)

Trapezoidal Simultaneous Implicit Approach

1. Apply the trapezoidal method to get the following equations for $t + \Delta t$

$$x(t+\Delta t) - x(t) = \frac{\Delta t}{2}\Big(f\big(x(t), y(t)\big) + f\big(x(t+\Delta t), y(t+\Delta t)\big)\Big)$$

$$0 = g(x(t+\Delta t), y(t+\Delta t))$$

2. Use Newton's method at each time step to solve this nonlinear algebraic system.

Newton update step: $\begin{bmatrix} x^{i+1}(t+\Delta t) \\ y^{i+1}(t+\Delta t) \end{bmatrix} = \begin{bmatrix} x^{i}(t+\Delta t) \\ y^{i}(t+\Delta t) \end{bmatrix} - J^{-1} r$

Where the residual, updated each iteration, is

$$r = \begin{bmatrix} -x^i(t+\Delta t) + x(t) + \frac{\Delta t}{2}\Big(f\big(x(t), y(t)\big) + f\left(x^i(t+\Delta t), y^i(t+\Delta t)\right)\Big) \\ g\big(x^i(t+\Delta t), y^i(t+\Delta t)\big) \end{bmatrix}$$

And the Jacobian J, updated each iteration, is the matrix of partial derivatives of r.

3. At each time step, you need to iterate until r is below some tolerance ϵ.

Return to Two-Machine Example

- The smallest multi-machine system is the two-bus, two-classical-machine system with the following equations we have derived before

Note: $g + jb = \frac{1}{R+jX}$

$$\dot{\delta}_1 = \omega_1 \omega_s$$

$$\dot{\omega}_1 = \frac{1}{2H_1}\left(\frac{P_{m1}}{\omega_1+1} - \frac{E_{p1}}{X'_{d1}}(V_{1r}\sin\delta_1 - V_{1i}\cos\delta_1)\right)$$

$$\dot{\delta}_2 = \omega_2 \omega_s$$

$$\dot{\omega}_2 = \frac{1}{2H_2}\left(\frac{P_{m2}}{\omega_2+1} - \frac{E_{p2}}{X'_{d2}}(V_{2r}\sin\delta_2 - V_{2i}\cos\delta_2)\right)$$

Differential equations f

$$0 = gV_{1r} + \left(\frac{1}{X'_{d1}} - b\right)V_{1i} - gV_{2r} + bV_{2i} + E_{p1}\sin\delta_1 \frac{1}{X'_{d1}}$$

$$0 = gV_{1i} + \left(b - \frac{1}{X'_{d1}}\right)V_{1r} - gV_{2i} - bV_{2r} - E_{p1}\cos\delta_1 \frac{1}{X'_{d1}}$$

$$0 = -gV_{1r} + bV_{1i} + gV_{2r} + \left(\frac{1}{X'_{d2}} - b\right)V_{2i} + E_{p2}\sin\delta_2 \frac{1}{X'_{d2}}$$

$$0 = -gV_{1i} - bV_{1r} + gV_{2i} + \left(b - \frac{1}{X'_{d2}}\right)V_{2r} - E_{p2}\cos\delta_2 \frac{1}{X'_{d2}}$$

Algebraic equations g

Differential variables x; Algebraic variables y

$$\begin{bmatrix} \delta_1 \\ \omega_1 \\ \delta_2 \\ \omega_2 \\ V_{1r} \\ V_{1i} \\ V_{2r} \\ V_{2i} \end{bmatrix}$$

The following is an equivalent form of the algebraic equations, similar to the Yaug approach:

$$\begin{bmatrix} (g+jb) - \frac{1}{jX'_{d1}} & -(g+jb) \\ -(g+jb) & (g+jb) - \frac{1}{jX'_{d2}} \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \end{bmatrix} = -\begin{bmatrix} \frac{E_{p1}\angle\delta_1}{jX'_{d1}} \\ \frac{E_{p2}\angle\delta_2}{jX'_{d2}} \end{bmatrix}$$

Simultaneous Implicit Approach to Two-Machine Example, Residuals and Jacobian

- There will be 8 residual equations and they will follow the pattern

$$r = \begin{bmatrix} -x^i(t+\Delta t) + x(t) + \frac{\Delta t}{2}\Big(f\big(x(t), y(t)\big) + f\left(x^i(t+\Delta t), y^i(t+\Delta t)\right)\Big) \\ g\big(x^i(t+\Delta t), y^i(t+\Delta t)\big) \end{bmatrix}$$

- For example, the first residual equation will be

$$r_1 = -\delta_1^i(t+\Delta t) + \delta_1(t) + \frac{\Delta t}{2}\big(\omega_1(t)\omega_s + \omega_1^i(t+\Delta t)\omega_s\big)$$

- And the Jacobian is the matrix of partial derivatives, with the first row all zeros except the first two entries.

$$\frac{\partial r_1}{\partial \delta_1^i(t+\Delta t)} = -1; \quad \frac{\partial r_1}{\partial \omega_1^i(t+\Delta t)} = \frac{\Delta t}{2}\omega_s$$

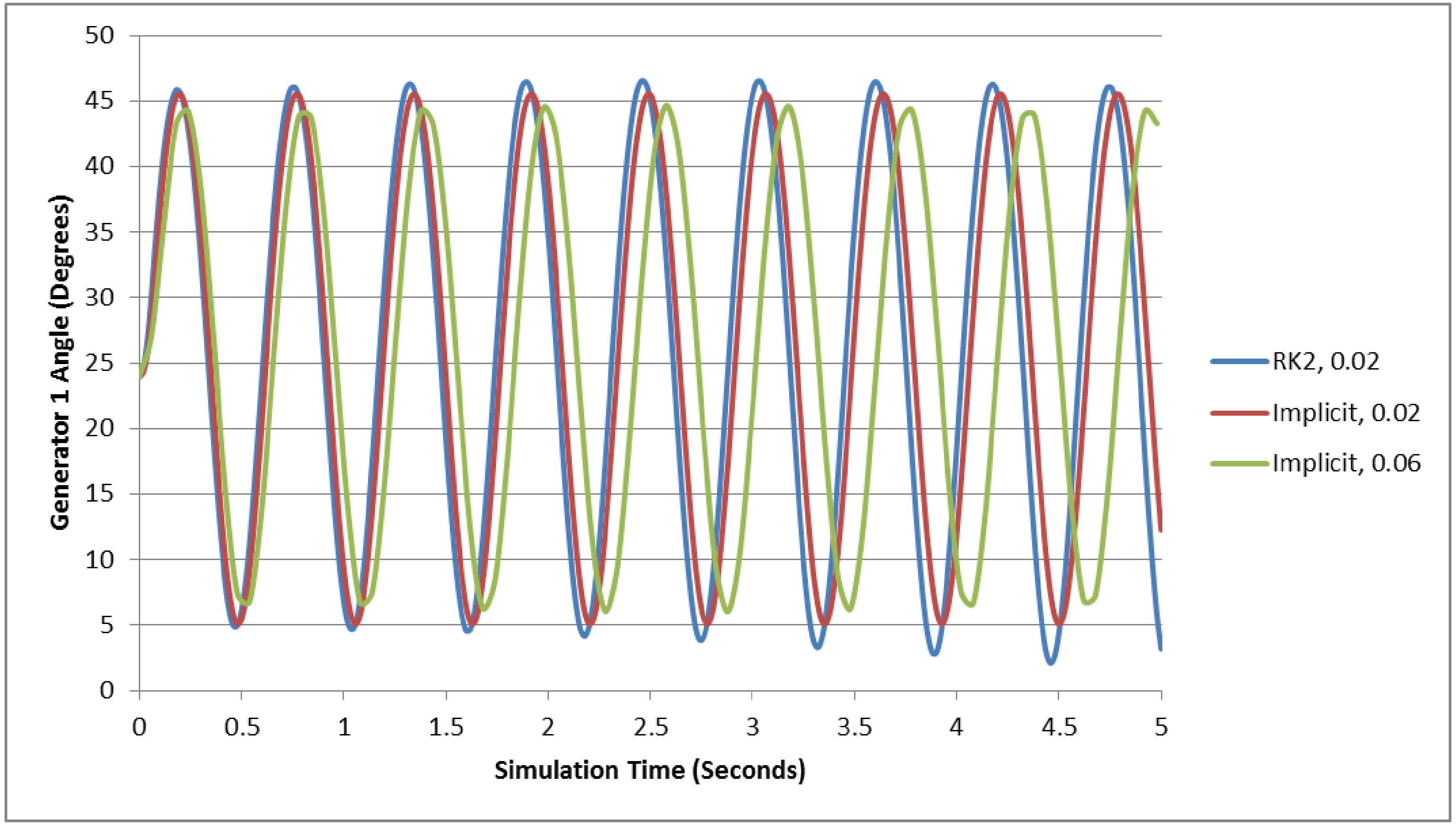
Two-Bus, Two-Machine Initial Jacobian

- For conceptual purposes – there are a few different assumptions in this matrix

$$\begin{bmatrix} & \delta_1 & \Delta\omega_1 & \delta_2 & \Delta\omega_2 & V_{D1} & V_{Q1} & V_{D2} & V_{Q2} \\ \dot{\delta}_1 & -1 & 3.77 & 0 & 0 & 0 & 0 & 0 & 0 \\ \Delta\dot{\omega}_1 & -0.0076 & -1 & 0 & 0 & -0.0029 & 0.0065 & 0 & 0 \\ \dot{\delta}_2 & 0 & 0 & -1 & 3.77 & 0 & 0 & 0 & 0 \\ \Delta\dot{\omega}_2 & 0 & 0 & -0.0039 & -1 & 0 & 0 & 0.0008 & 0.0039 \\ I_{D1} & -3.90 & 0 & 0 & 0 & 0 & 7.879 & 0 & -4.545 \\ I_{Q1} & -1.73 & 0 & 0 & 0 & -7.879 & 0 & 4.545 & 0 \\ I_{D2} & 0 & 0 & -4.67 & 0 & 0 & -4.545 & 0 & 9.545 \\ I_{Q2} & 0 & 0 & 1.00 & 0 & 4.545 & 0 & -9.545 & 0 \end{bmatrix}$$

Result Comparison

- The below graph compares the angle for the generator at bus 1 using Δt=0.02 between RK2 and the Implicit Trapezoidal; also Implicit with Δt=0.06



Partitioned-Explicit Approach with RK2

1. Explicitly use f to find the derivatives for $k_{1f} = \Delta t\, f\big(x(t), y(t)\big)$
2. Need to solve g implicitly to find the term $y(t) + k_{1g}$

   in $0 = g\big(x(t) + k_{1f}, y(t) + k_{1g}\big)$
3. Now use f again to find the derivatives for $k_{2f} = \Delta t\, f\big(x(t) + k_{1f}, y(t) + k_{1g}\big)$
4. New value of $x(t + \Delta t) = x(t) + \frac{1}{2}\big(k_{1f} + k_{2f}\big)$
5. Now implicitly solve g to find $y(t + \Delta t)$

   $0 = g\big(x(t + \Delta t), y(t + \Delta t)\big)$

More steps, but especially if g is linear you do not need a Jacobian or iterative approach.

Partitioned-Explicit Approach with Two-Machine Example

- Use the 4 differential and 4 algebraic equations above
- Parameters: $H_1 = 3, X_d' = 0.1, H_2 = 5, X_d' = 0.1, X = 0.22, R = 0.07$
  - Note that $g + jb = 1.3133 - j4.1276$
- Initial conditions: $V_{r1} = 1 \quad V_{i1} = 0 \quad V_{r2} = 0.9929 \quad V_{i2} = 0.1189$

$$\omega_1 = \omega_2 = 0 \quad \delta_1 = -0.04725 \quad \delta_2 = 0.169821$$

$$E_{p1} = 1.0197 \quad E_{p2} = 0.989 \quad P_{m1} = -0.48 \quad P_{m2} = 0.50$$

- Let's choose $\Delta t = 0.02$ seconds
- For the fault, a "solid" fault can be represented with a very large admittance such that the voltage at the bus becomes essentially zero. In this case, let's add $Y_{fault} = -j1000$ to the first entry of the Y-bus

$$\mathbf{Y_{aug}} = \begin{bmatrix} 1.3133 - j1014.1276 & -1.3133 + j4.1276 \\ -1.3133 + j4.1276 & 1.3133 - j14.1276 \end{bmatrix}$$

Partitioned-Explicit Approach with Two-Machine Example, Solution at t=0.02

Usually a time step begins by solving the differential equations. However, in the case of an event, such as the solid fault at the terminal of bus 1, the network equations need to be first solved $\mathbf{Y_{aug}}\bar{V} = \begin{bmatrix} \frac{E_{p1}\angle\delta_1}{jX'_{d1}} \\ \frac{E_{p2}\angle\delta_2}{jX'_{d2}} \end{bmatrix} \rightarrow \bar{V} = \begin{bmatrix} 0.0128 + 0.0006j \\ 0.6988 + 0.0547 \end{bmatrix}$

1. Applying f we get $k_{1f} = 0.02 \begin{bmatrix} \omega_1\omega_s \\ \frac{1}{2H_1}\left(\frac{P_{m1}}{\omega_1+1} - \frac{E_{p1}}{X'_{d1}}(V_{1r}\sin\delta_1 - V_{1i}\cos\delta_1)\right) \\ \omega_2\omega_s \\ \frac{1}{2H_2}\left(\frac{P_{m2}}{\omega_2+1} - \frac{E_{p2}}{X'_{d2}}(V_{2r}\sin\delta_2 - V_{2i}\cos\delta_2)\right) \end{bmatrix} = 0.02\begin{bmatrix} 0 \\ -0.078 \\ 0 \\ -0.0135 \end{bmatrix}$
2. Now solve g implicitly (Yaug equations) using $x + k_{1f}$. Since δ didn't change in step 1, the voltages (y) won't change here

Partitioned-Explicit Approach with Two-Machine Example, Solution at t=0.02, cont.

3. Now we use f again for $k_{2f} =$

$$0.02\begin{bmatrix} \omega_1\omega_s \\ \frac{1}{2H_1}\left(\frac{P_{m1}}{\omega_1+1} - \frac{E_{p1}}{X'_{d1}}(V_{1r}\sin\delta_1 - V_{1i}\cos\delta_1)\right) \\ \omega_2\omega_s \\ \frac{1}{2H_2}\left(\frac{P_{m2}}{\omega_2+1} - \frac{E_{p2}}{X'_{d2}}(V_{2r}\sin\delta_2 - V_{2i}\cos\delta_2)\right) \end{bmatrix} = 0.02\begin{bmatrix} -5.8811 \\ -0.0792 \\ -1.0179 \\ -0.0133 \end{bmatrix}$$

4. $x(t+\Delta t) = \begin{bmatrix} -0.04725 \\ 0 \\ 0.169821 \\ 0 \end{bmatrix} + \frac{0.02}{2}\left(\begin{bmatrix} 0 \\ -0.078 \\ 0 \\ -0.0135 \end{bmatrix} + \begin{bmatrix} -5.8811 \\ -0.0792 \\ -1.0179 \\ -0.0133 \end{bmatrix}\right) = \begin{bmatrix} -0.1061 \\ -0.0015 \\ 0.1596 \\ -0.000268 \end{bmatrix}$

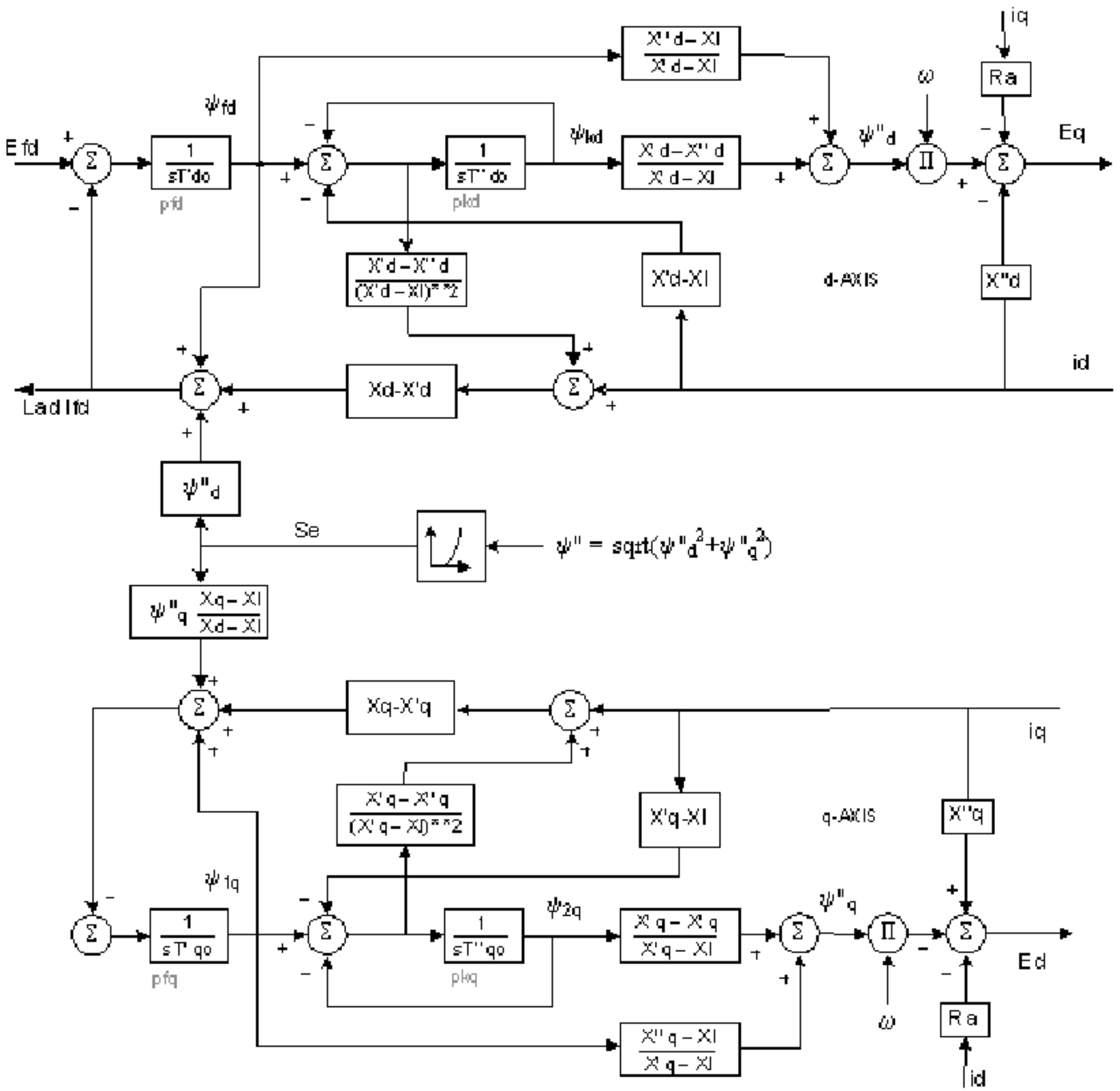
5. Solve voltages once again and get $\bar{V}_1 = 0.0128 + j0$; $\bar{V}_2 = 0.6993 + j0.0474$

Note on Angle Reference

- Initial angles are given from power flow, which is based on the slack bus's angle
- Notice both our angles are changing fairly rapidly, but that is because this is measured with respect to a synchronous reference frame
- Alternatively, other reference frames can be used:
  - Choose a reference generator, or
  - Present angles relative to the system average
- The solution is not affected by this choice

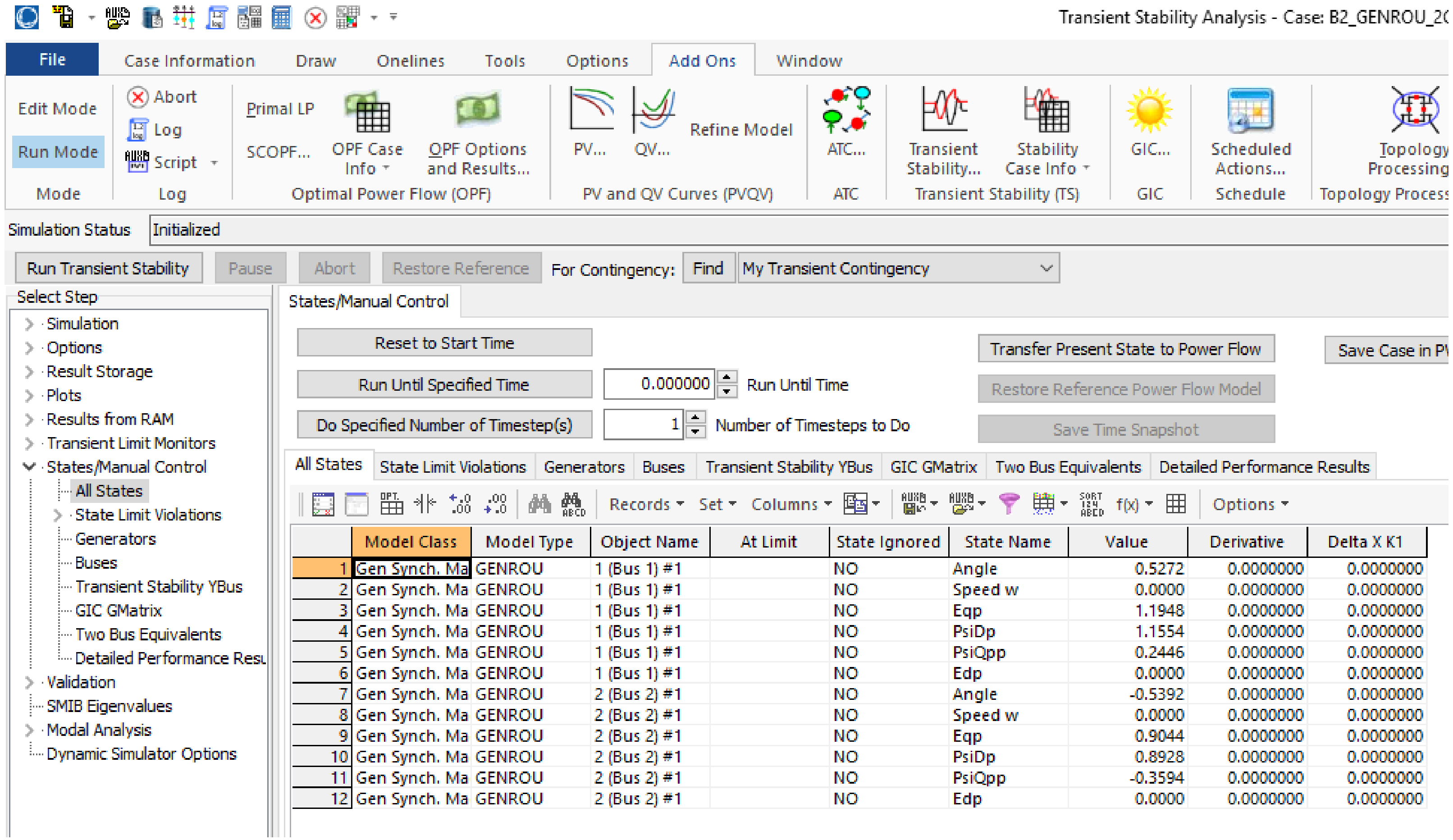
Two-Machine, GENROU Example

- Let’s use the same case as before, except with both generators modeled using GENROUs
  - For simplicity we'll make both generators identical except set H1=3, H2=6; other values are Xd=2.1, Xq=0.5, X'd=0.2, X'q=0.5, X"q=X"d=0.18, Xl=0.15, T'do = 7.0, T'qo=0.75, T"do=0.035, T"qo=0.05; no saturation
  - We have covered initialization before.



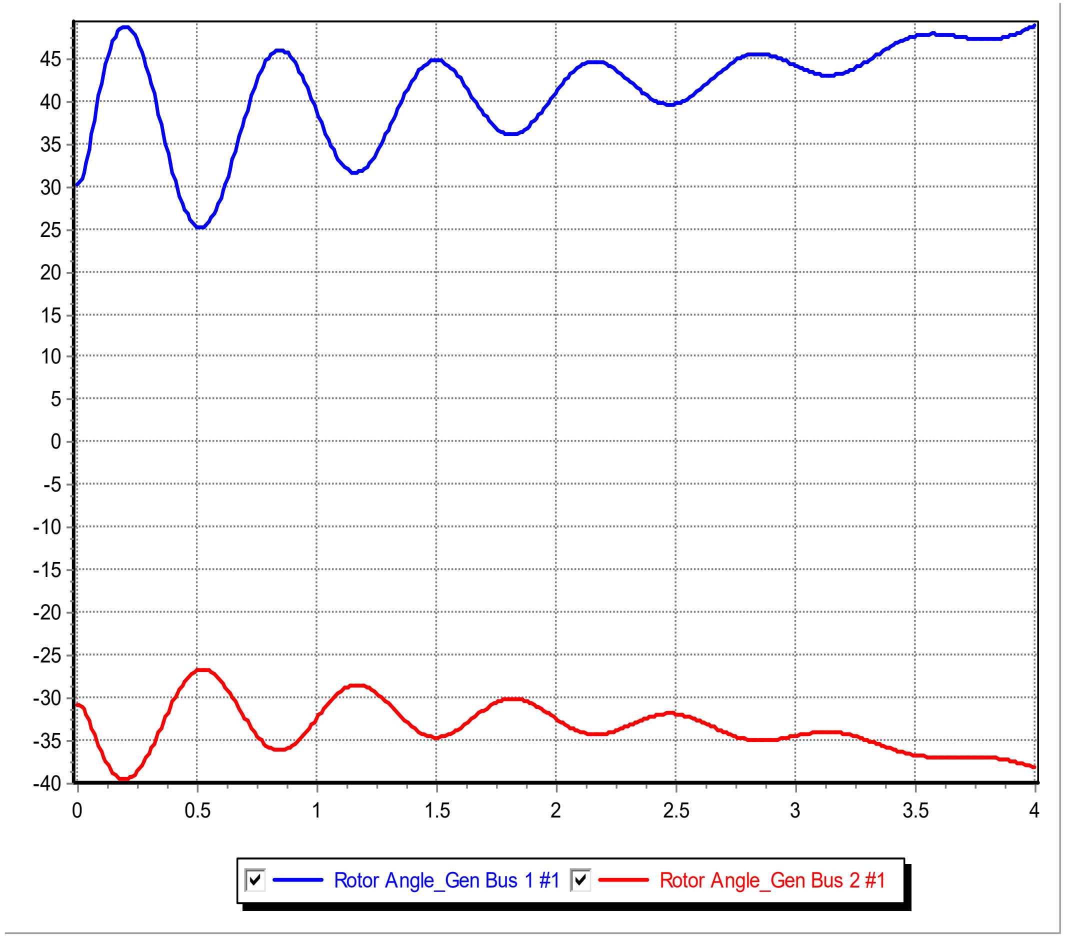
State Vector X for the 2-Machine GENROU Case

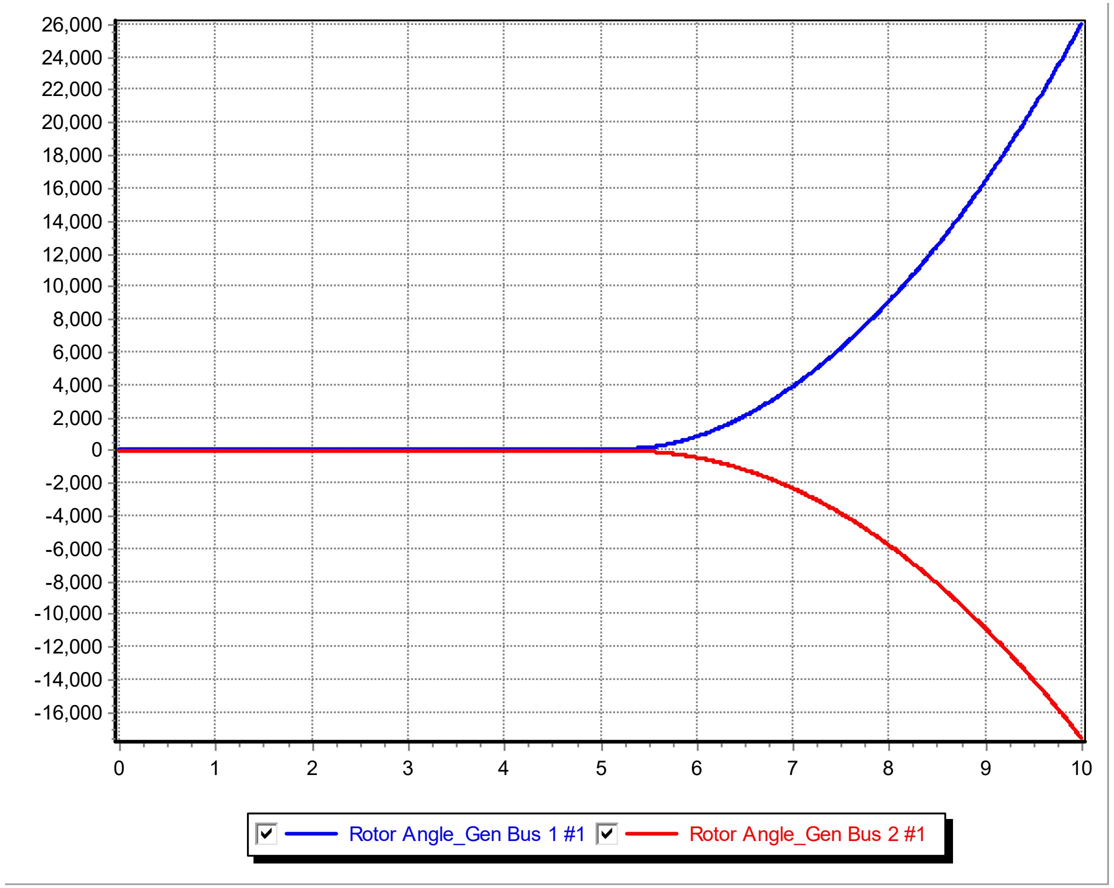
$$
\mathbf{x}(0) = \begin{bmatrix} \delta_1 \\ \Delta\omega_1 \\ E'_{q1} \\ \psi_{1d1} \\ \psi_{2q1} \\ E'_{d1} \\ \delta_2 \\ \Delta\omega_2 \\ E'_{q2} \\ \psi_{1d2} \\ \psi_{2q2} \\ E'_{d2} \end{bmatrix} = \begin{bmatrix} 0.5273 \\ 0.0 \\ 1.1948 \\ 1.1554 \\ 0.2446 \\ 0 \\ -0.5392 \\ 0 \\ 0.9044 \\ 0.8928 \\ -0.3594 \\ 0 \end{bmatrix}
$$



	Model Class	Model Type	Object Name	At Limit	State Ignored	State Name	Value	Derivative	Delta X K1
1	Gen Synch. Ma	GENROU	1 (Bus 1) #1		NO	Angle	0.5272	0.0000000	0.0000000
2	Gen Synch. Ma	GENROU	1 (Bus 1) #1		NO	Speed w	0.0000	0.0000000	0.0000000
3	Gen Synch. Ma	GENROU	1 (Bus 1) #1		NO	Eqp	1.1948	0.0000000	0.0000000
4	Gen Synch. Ma	GENROU	1 (Bus 1) #1		NO	PsiDp	1.1554	0.0000000	0.0000000
5	Gen Synch. Ma	GENROU	1 (Bus 1) #1		NO	PsiQpp	0.2446	0.0000000	0.0000000
6	Gen Synch. Ma	GENROU	1 (Bus 1) #1		NO	Edp	0.0000	0.0000000	0.0000000
7	Gen Synch. Ma	GENROU	2 (Bus 2) #1		NO	Angle	-0.5392	0.0000000	0.0000000
8	Gen Synch. Ma	GENROU	2 (Bus 2) #1		NO	Speed w	0.0000	0.0000000	0.0000000
9	Gen Synch. Ma	GENROU	2 (Bus 2) #1		NO	Eqp	0.9044	0.0000000	0.0000000
10	Gen Synch. Ma	GENROU	2 (Bus 2) #1		NO	PsiDp	0.8928	0.0000000	0.0000000
11	Gen Synch. Ma	GENROU	2 (Bus 2) #1		NO	PsiQpp	-0.3594	0.0000000	0.0000000
12	Gen Synch. Ma	GENROU	2 (Bus 2) #1		NO	Edp	0.0000	0.0000000	0.0000000

Results – Unstable

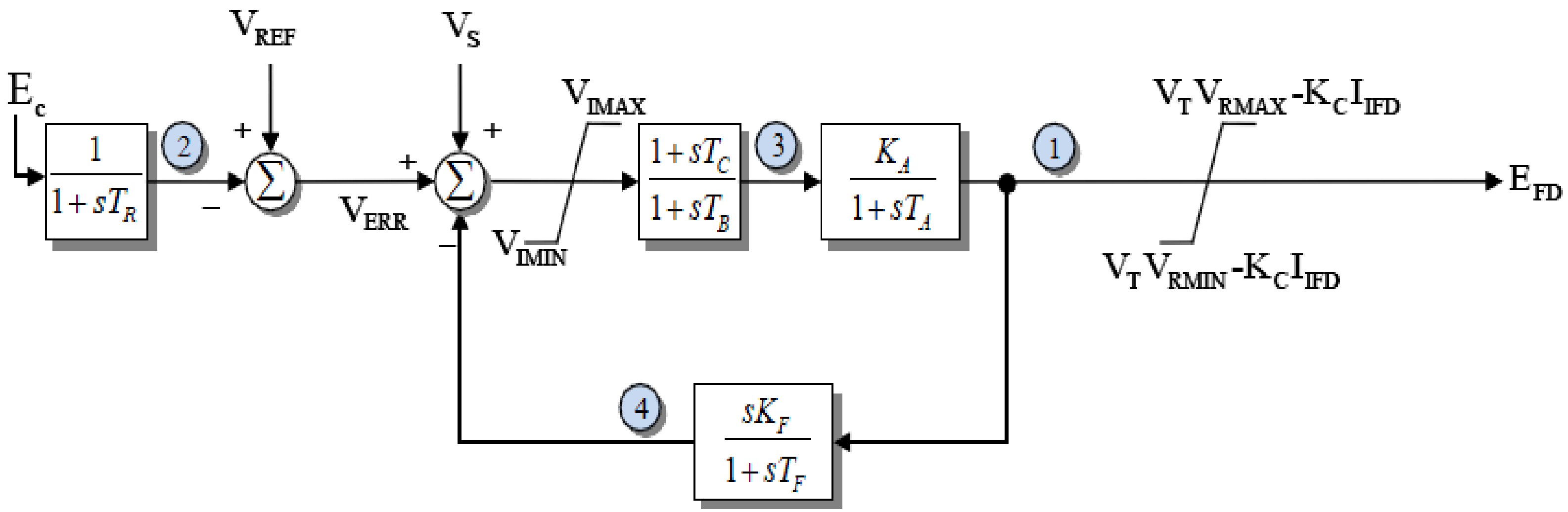




Adding EXST1 Exciter

- Let's add an EXST1 exciter to this case, with the following block diagram.
- Use the following parameters that simplify some of the blocks T_R=0, T_C=T_B=0, K_f=0, K_A=100, T_A=0.1
- Now there is only one differential equation per generator

$$\frac{dE_{fd}}{dt} = \frac{1}{T_A}\left(K_A(V_{REF} - |V_t|) - E_{fd}\right)$$



State Vector with Exciters

- 14 functional differential states; others are ignored or do not impact

All States | State Limit Violations | Generators | Buses | Transient Stability YBus | GIC GMatrix | Two Bus Equivalents

Records ▾ Set ▾ Columns ▾ f(x) ▾

	Model Class	Model Type	Object Name	At Limit	State Ignored	State Name	Value
1	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	Angle	0.5273
2	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	Speed w	0.0000
3	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	Eqp	1.1948
4	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	PsiDp	1.1554
5	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	PsiQpp	0.2446
6	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	Edp	0.0000
7	Gen Exciter	EXST1	1 (Bus 1) #1		NO	EField before lim	2.6904
8	Gen Exciter	EXST1	1 (Bus 1) #1		YES	Sensed Vt	1.0946
9	Gen Exciter	EXST1	1 (Bus 1) #1		YES	VLL	0.0269
10	Gen Exciter	EXST1	1 (Bus 1) #1		NO	VF	0.0000
11	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	Angle	-0.5392
12	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	Speed w	0.0000
13	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	Eqp	0.9044
14	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	PsiDp	0.8928
15	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	PsiQpp	-0.3594
16	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	Edp	0.0000
17	Gen Exciter	EXST1	2 (Bus 2) #1		NO	EField before lim	1.3441
18	Gen Exciter	EXST1	2 (Bus 2) #1		YES	Sensed Vt	1.0000
19	Gen Exciter	EXST1	2 (Bus 2) #1		YES	VLL	0.0134
20	Gen Exciter	EXST1	2 (Bus 2) #1		NO	VF	0.0000

Results with Exciters

- Now the case is stable