

Homework #1 – ECEN 667 Fall 2025

Due September 11, 2025 at 8 AM

Problem 1. Consider the following set of two differential equations

$$\begin{aligned}\dot{x}_1 &= \frac{3}{5}x_1 - \frac{8}{5}x_1x_2 \\ \dot{x}_2 &= x_1x_2 - 4x_2\end{aligned}$$

- Find all equilibrium points for the system.
- With a step size of $\Delta t = 0.1$ seconds, use Euler's method to determine the values of x_1 and x_2 at $t = 0.4$ seconds, with initial conditions $x_1(0) = x_2(0) = 1$.
- Repeat 1.b with the Second Order Runge-Kutta Method.
- Repeat 1.c with the Second Order Adams-Bashforth method. Use your solution from 1.c for the value at $t = 0.1$, then start your integration at $t = 0.2$.

Answer

- (0, 0) and (4, 0.375)
- $x_1(0.4) = 0.844, x_2(0.4) = 0.226$
- $x_1(0.4) = 0.8822, x_2(0.4) = 0.2968$
- $x_1(0.4) = 0.8903, x_2(0.4) = 0.3025$

Problem 2. Consider the following set of three differential equations

$$\begin{aligned}\dot{x}_1 &= 8(x_2 - x_1) \\ \dot{x}_2 &= x_1(28 - x_3) - x_2 \\ \dot{x}_3 &= x_1x_2 - \frac{4}{3}x_3\end{aligned}$$

- Find all equilibrium points for this system.
- Write a computer program using either Python or Matlab to solve the above set of three differential equations using the Second-Order Runge-Kutta method. Use $\Delta t = 0.01$ seconds. Initial conditions are $x_1 = x_2 = x_3 = 5$. Turn in your source code listing. You must code the method yourself. Do not use any packages such as built-in or third party differential equation solvers, symbolic algebra libraries, etc. You may use a package to plot the results.
- Using your program, integrate the equations long enough that you can describe the system behavior, including whether it converges to the equilibrium point.

Answer

- (0, 0, 0) and (27, 6, 6) and (27, -6, -6)
- Answers vary
- System behavior is chaotic. It does not converge to an equilibrium point, nor does it diverge into infinite values. It remains oscillating within a bounded region, but not in a periodic way.

Problem 3. Consider a two-bus, two-generator power system. Each of the two generators is modeled with a classical generator as discussed in class. For the first generator, $H = 3$ and $X'_d = 0.1$. For the second generator, $H = 5$ and $X'_d = 0.1$. There is a transmission line connecting the two lines which has $X = 0.22, R = 0.07, B = 0$. Assume that in the initial steady-state conditions the generator at bus 2 is producing 50 MW and the voltage magnitude at both buses is 1.0 per-unit. Then assume a 3 cycle solid three-phase fault at the first bus occurs at $t = 1$.

- Build this case in PowerWorld and solve the power flow to find the initial voltage magnitudes and angles at each bus.

- Write the set of differential and algebraic equations that represent this system before the fault occurs.
- Solve for the initial steady-state values of each variable in your equations.
- Add the generator stability models to PowerWorld and add the fault event in the Transient Stability dialog. Run the simulation for 30 seconds and create a plot of the two machines' speeds during the simulation. Turn in a plot of the speeds along with a brief description, including whether the response is stable or unstable.
- Why is the solution to this system different from Example 1 as given in class?

Answer

a. $\bar{V}_1 = 1\angle 0^\circ$, $\bar{V}_2 = 1\angle 6.83^\circ$

b. System equations are as follows

$$\begin{aligned}\dot{\delta}_1 &= \omega_1 \omega_s & \dot{\omega}_1 &= \frac{1}{2H_1} \left(\frac{P_{m1}}{\omega_1 + 1} - \frac{E_{p1}}{X'_{d1}} (V_{1r} \sin \delta_1 - V_{1i} \cos \delta_1) \right) & E_{p1} \angle \delta_1 &= \bar{V}_1 - jX'_{d1} \bar{I}_1 \\ \dot{\delta}_2 &= \omega_2 \omega_s & \dot{\omega}_2 &= \frac{1}{2H_2} \left(\frac{P_{m2}}{\omega_2 + 1} - \frac{E_{p2}}{X'_{d2}} (V_{2r} \sin \delta_2 - V_{2i} \cos \delta_2) \right) & E_{p2} \angle \delta_2 &= \bar{V}_2 - jX'_{d2} \bar{I}_2 \\ & & \bar{I}_1 &= -\bar{I}_2 = \frac{\bar{V}_1 - \bar{V}_2}{R + jX}\end{aligned}$$

c. Variable initial values are as follows

$$\begin{aligned}\omega_1 &= \omega_2 = 0 & \delta_1 &= -2.707^\circ & \delta_2 &= 9.73^\circ & E_{p1} &= 1.0197 & E_{p2} &= 0.989 \\ P_{m1} &= -0.48 & P_{m2} &= 0.50 & \bar{I}_1 &= -0.482 - j0.186 & \bar{I}_2 &= 0.482 + j0.186\end{aligned}$$

- Both machine speeds diverge from synchronous during the fault and begin to oscillate following the clearing of the fault. The oscillations are undamped and begin to grow, leading to an unstable response.
- In Example 1 in class, there is an infinite bus which provides variable power supply and provides improved stability characteristic.