

Name: _____ UIN: _____ Section: _____ Score: _____

A 60 Hz generator is supplying 150 MW and 0 Mvar to an infinite bus (with 1.0 per-unit) through two parallel transmission lines. Each transmission line has a per-unit impedance (with 100 MVA base) of $j0.05$. The per-unit transient reactance for the machine is $j0.02$, and its inertia constant is 8 seconds. At time = 0, one of the lines opens and is removed from the system (no fault, just a line opening). There is no damping.

$$\begin{aligned}\dot{\delta} &= \omega \\ \frac{2H}{\omega_s} \dot{\omega} &= T_M - \frac{E'V_s}{X'_d + X_{ep}} \sin(\delta - \theta_{vs}) - D \cdot \omega\end{aligned}$$

Using the classical generator model with equations given above, determine the pre-event internal voltage magnitude and angle of the generator $E' \angle \delta$, and find the pre-event initial values of ω and the mechanical torque T_M . Then write the dynamics of the system after the event as two differential equations with two variables δ and ω , and use Euler's method to solve for these variables for two time steps, with $\Delta t = 0.01$.

$$\begin{aligned}\text{Euler's method} \\ x(t + \Delta t) &= x(t) + \Delta t \cdot f(x) \\ \text{Where } f(x) &= \dot{x}\end{aligned}$$

Before the event

The generator connects to a Thev. equiv. with $V_s \angle \theta_{vs} = 1.0 \angle 0$ and $X_{ep} = j0.05 || j0.05 = j0.025$. Solve the circuit for the generator internal voltage, which will give you E' and initial value of δ :

$$I = \left(\frac{S}{V}\right)^* = \left(\frac{1.5}{1.0}\right)^* = 1.5$$

$$E' \angle \delta = 1.0 + (1.5)(j0.02 + j0.025) = 1.00228 \angle 0.0674 \text{ (rad)}$$

Now consider steady-state, to get $\omega = 0$

$$\text{And } T_M = \frac{E'V_s}{X'_d + X_{ep}} \sin(\delta - \theta_{vs}) = \frac{1.00228 \cdot 1}{0.02 + 0.025} \sin(0.0674) = 1.5$$

After the event

The Thevenin equivalent changes. Since there is only one line, $X_{ep} = 0.05$, $V_s = 1.0$

$$\text{Now rewrite the swing equation: } \frac{2 \cdot 8}{2\pi 60} \dot{\omega} = 1.5 - \frac{1.00228}{0.07} \sin \delta$$

So, using Euler's method starting with $\delta(0) = 0.0674$ and $\omega(0) = 0$

$$\delta(0.01) = \delta(0) + \Delta t \left(\dot{\delta}(0)\right) = 0.0674 + 0.01 \cdot (0) = 0.0674$$

$$\omega(0.01) = \omega(0) + \Delta t \left(\dot{\omega}(0)\right) = 0 + 0.01 \cdot \left(\frac{2\pi 60}{16} \left(1.5 - \frac{1.00228}{0.07} \sin 0.0674\right)\right) = 0.1262$$

$$\delta(0.02) = \delta(0.01) + \Delta t \left(\dot{\delta}(0.01)\right) = 0.0674 + 0.01 \cdot 0.1262 = 0.0687$$

$$\omega(0.02) = \omega(0.01) + \Delta t \left(\dot{\omega}(0.01)\right) = 0.1262 + 0.01 \cdot \left(\dot{\omega}(0.01)\right) = 0.2524$$