The general power balance equations:

$$
\begin{aligned}
& \mathrm{P}_{\mathrm{i}}=\sum_{k=1}^{n}\left|V_{i}\right|\left|V_{k}\right|\left(g_{i k} \cos \theta_{i k}+b_{i k} \sin \theta_{i k}\right)=P_{G i}-P_{D i} \\
& \mathrm{Q}_{\mathrm{i}}=\sum_{k=1}^{n}\left|V_{i}\right|\left|V_{k}\right|\left(g_{i k} \sin \theta_{i k}-b_{i k} \cos \theta_{i k}\right)=Q_{G i}-Q_{D i}
\end{aligned}
$$

## Example 1

For the two bus power system shown below, use the Newton-Raphson power flow to determine the voltage magnitude and angle at bus two. Assume that $\mathrm{S}_{\text {Base }}=100 \mathrm{MVA}$.


Steps:

1. Make the system admittance (Y-bus) matrix.
2. For each bus, list the bus type and which variables $(P, Q, V, \theta)$ are known and unknown.
3. Write the power balance equations for this system.
4. Make the power flow Jacobian matrix.
5. Start with an initial flat start guess (voltages are 1 pu , angles are 0 )
6. Use the Newton-Raphson equation $x^{(v+1)}=x^{(v)}-\left[J\left(x^{(v)}\right)\right]^{-1} f\left(x^{(v)}\right)$ to get successive values of $x$.
7. Continue until the mismatch is less than 0.01 .
8. Calculate the generator real and reactive power output given the bus voltages.

## Power Flow Examples

## Example 2

Following the same steps as Example 1, solve for all bus voltages and line flows.


## Example 3

For the following 5-bus system, assume bus 1 is the slack bus. Generator 1 is regulating the voltage to 1.0 per-unit, and generator 3 is regulating the voltage to 1.05 per-unit. Line and transformer data are given below.


1. Make the system admittance y-bus matrix.
2. Make a table of system buses, explaining what the bus type of each will be, which variables are known (and the values of the known variables) and which variables are unknown for each bus.
