HW 5 Solutions

6.9 $x_2 - 3x_1 + 1.9 = 0$

$x_2 + x_1^2 - 3.0 = 0$

Rearrange to solve for x_1 and x_2:

$$x_1 = \frac{1}{3}x_2 + 0.6333$$

$$x_2 = 3.0 - x_1^2$$

Starting with an initial guess of $x_1(0) = 1$ and $x_2(0) = 1$

$$x_1(1) = \frac{1}{3}x_2(0) + 0.6333 = 0.9667$$
$$x_2(1) = 3 - [x_1(0)]^2 = 2$$

We repeat this procedure using the general equations

$$x_1(n+1) = \frac{1}{3}x_2(n) + 0.6333$$
$$x_2(n+1) = 3 - [x_1(n)]^2$$

n	0	1	2	3	. . .	47	48	49	50
x_1	1	0.9667	1.3	1.3219	. . .	1.1746	1.1735	1.1734	1.1743
x_2	1	2.0	2.0656	1.31	. . .	1.6205	1.6202	1.6229	1.6231

After 50 iterations, x_1 and x_2 have a precision of 3 significant digits.

$$x_1 = 1.17$$
$$x_2 = 1.62$$

6.12 Rewriting the given equations,

$$x_1 = \frac{x_2}{3} + 0.633; \quad x_2 = 1.8 - x_1^2$$

With an initial guess of $x_1(0) = 1$ and $x_2(0) = 1$,

update x_1 with the first equation above, and x_2 with the second equation.

Thus $$x_1 = \frac{x_2(0)}{3} + 0.633 = \frac{1}{3} + 0.633 = 0.9663$$

and $$x_2 = 1.8 - x_1(0)^2 = 1.8 - 1 = 0.8$$

In succeeding iterations, compute more generally as

$$x_1(n+1) = \frac{x_2(n)}{3} + 0.633$$

and $$x_2(n+1) = 1.8 - x_1^2(n)$$

After several iterations, $x_1 = 0.938$ and $x_2 = 0.917$.

After a few more iterations, $x_1 = 0.93926$ and $x_2 = 0.9178$.

However, note that an "uneducated guess" of initial values, such as $x_1(0) = x_2(0) = 100$, would have caused the solution to diverge.

6.18 $x^3 + 8x^2 + 2x - 40 = 0$

$x(0) = 1$

$$J(i) = \left.\frac{df}{dx}\right|_{x=x(i)} = [3x^2 + 16x + 2]_{x=x(i)}$$

In the general form:

$$x(i+1) = x(i) + \frac{1}{3x(i)^2 + 16x(i) + 2} \times (-x(i)^3 - 8x(i)^2 - 2x(i) + 40)$$

Using $x(0) = 1$, we arrive at

i	0	1	2	3	4
x	1	2.381	1.9675	1.9116	1.9106
ε	1.381	0.1737	0.0284	5.19E-4	

After 4 iterations, $\varepsilon < 0.001$.

$\therefore x = 1.9106$

6.25 $10x_1 \sin x_2 + 2 = 0$

$10x_1^2 - 10x_1 \cos x_2 + 1 = 0$

$$J(i) = \left.\frac{df}{dx}\right|_{x=x(i)} = \begin{bmatrix} 10\sin x_2 & 10x_1 \sin x_2 \\ 20x_1 - 10\cos x_2 & 10x_1 \sin x_2 \end{bmatrix}_{x=x(i)}$$

In the general form

$$x(i+1) = x(i) - J^{-1}(i) f(x)$$

Solving using MATLAB with $x_1(0) = 1,\ x_2(0) = 0,\ \varepsilon = 1\text{E}-4$:

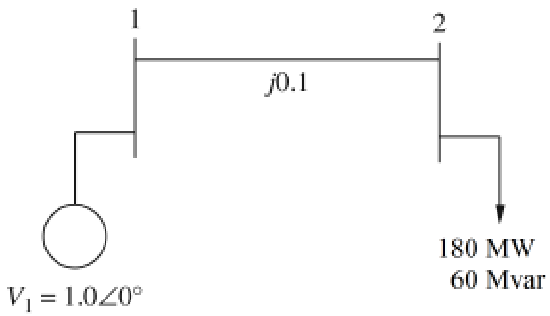
i	0	1	2	3	4
x_1	1	0.90	0.8587	0.8554	0.8554
x_2	0	− 0.20	− 0.2333	− 0.2360	− 0.2360
ε	∞	0.1667	0.01131	7.0E-5	

After 4 iterations, Newton-Raphson converges to

$x_1 = 0.8554$ rad

$x_2 = -0.2360$ rad

6.38



First, convert all values to per-unit.

$$P_{\text{pu}} = \frac{P}{S_{\text{base}}} = \frac{-180\text{ MW}}{100\text{ MVA}} = -1.8\text{ p.u.}$$

$$Q_{\text{pu}} = \frac{Q}{S_{\text{base}}} = \frac{-60\text{ MW}}{100\text{ MVA}} = -0.6\text{ p.u.}$$

$$\Delta y_{\text{max,pu}} = \frac{\Delta y_{\text{max}}}{S_{\text{base}}} = \frac{0.1\text{ MVA}}{100\text{ MVA}} = 1\text{E-4 p.u.}$$

Since there are 2 buses, we need to solve $2(n-1) = 2$ equations.

Therefore, J has dimension 2×2.

Using Table 6.5

$$J1_{22} = -V_2 V_{21} V_1 \sin(\delta_2 - \delta_1 - \theta_{21})$$

$$J2_{22} = V_2 Y_{22} \cos(\theta_{22}) + Y_{21} V_1 \cos(\delta_2 - \delta_1 - \theta_{21}) + Y_{22} V_2 \cos(-\theta_{22})$$

$$J3_{22} = V_2 Y_{21} V_1 \cos(\delta_2 - \delta_1 - \theta_{21})$$

$$J4_{22} = -V_2 Y_{22} \sin\theta_{22} + Y_{21} V_1 \sin(\delta_2 - \delta_1 - \theta_{21}) + Y_{22} + V_2 \sin(-\theta_{22})$$

Also,

$$Y_{\text{bus}} = \begin{bmatrix} -j10 & j10 \\ j10 & -j10 \end{bmatrix} = \begin{bmatrix} 10\angle -90 & 10\angle 90 \\ 10\angle 90 & 10\angle -90 \end{bmatrix}$$

Finally, using Eqs. (6.6.2) and (6.6.3),

$$P_2(\delta_2, V_2) = V_2\left[Y_{21} V_1 \cos(\delta_2 - \delta_1 - \theta_{21}) + Y_{22} V_2 \cos(-\theta_{22})\right]$$

$$Q_2(\delta_2, V_2) = V_2\left[Y_{21} V_1 \sin(\delta_2 - \delta_1 - \theta_{21}) + Y_{22} V_2 \sin(-\theta_{22})\right]$$

Solving using MATLAB with $V_2(0) = 1.0\angle 0$,

i	0	1	2	3
V_2	$1.0\angle 0$	$0.94\angle -10.313°$	$0.9158\angle -11.308°$	$0.9149\angle -11.347°$

After 3 iterations

$V_2 = 0.9149\angle -11.347°$