Class 1: Three-Phase AC Power Calculations – Selected Solutions

Homework #1 due Tuesday, Jan 23rd

For more help, read Chapters 1 and 2 in the textbook, view the videos and slides on the website, and take advantage of office hours of TAs and the instructor.

1. Solutions not given.

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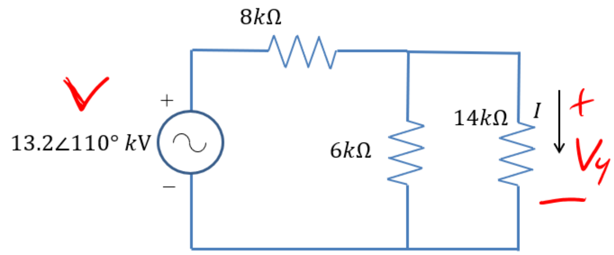
2. Solve for I as a phasor using any method. Assuming the frequency is 60 Hz, write the time signals for V and I.

- Solve for I as a phasor using any method
- Assuming the frequency is 1 kHz, write the time signals for V and I



$I_4 = \frac{V_s}{12.2k\Omega}$

$V_4 = 4k \cdot I_4 = V_s \frac{4}{12.2}$

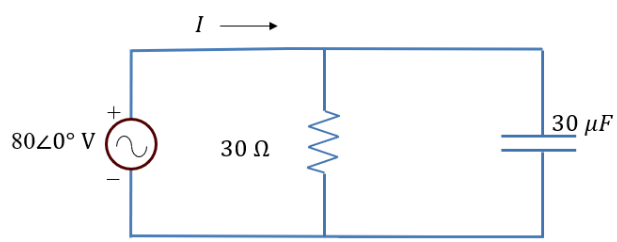
$I = V/R = V_s \frac{4}{12.2 \cdot 14k}$

$I = 0.324\angle 110°$ A

$$v(t) = 13.2\sqrt{2}\cos(2\pi 60t + 110°)\ \text{kV}$$
$$i(t) = 0.324\sqrt{2}\cos(2\pi 60t + 110°)\ \text{kV}$$

3.

- Find the current phasor I for this 1 kHz circuit
- Impedance of the resistor?
  $Z = 30\ \Omega$
- Impedance of the capacitor?
  $$Z = \frac{1}{j\omega C} = \frac{1}{j2\pi 1k \cdot 30\mu} = -j5.3\Omega$$
- You can treat impedances just like complex resistances! The capacitor and resistor are in parallel
  $$Z = \frac{1}{\frac{1}{30} + \frac{1}{-j5.3}} = (0.91 - j5.14)\ \Omega$$
  $$I = \frac{V}{Z} = \frac{80}{0.91 - j5.14} = 15.32\angle 80°\text{A}$$



What would this be as a time function?
What would it be in a lower frequency?

4.

Solve for the phasor V_c for this circuit
if it is operated at 400 Hz

$Z_R = 50\,\Omega$

$Z_L = j12.566\,\Omega$

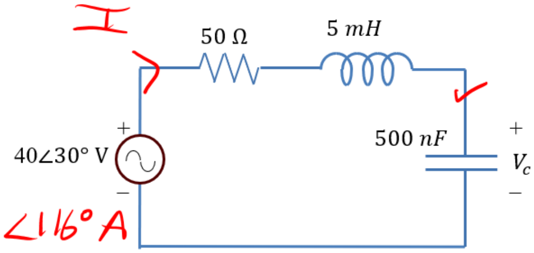
$Z_C = -j795.77\,\Omega$

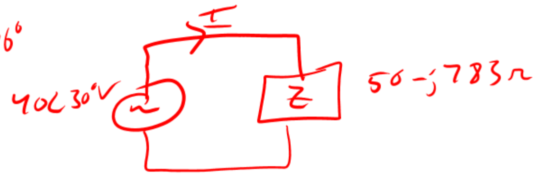
$Z = 50 - j783.2\,\Omega$

$I = \frac{V}{Z} = \frac{40\angle 30°\,V}{50 - j783\,\Omega} = .051\angle 116°\,A$

$784\angle -86°$

$V_C = I \cdot Z_C = (.051\angle 116°)(-j795) = 40.5\angle 26°\ V$





5. Solution not given.

6.

Calculate for the voltage source

1. Current I
2. Complex power S
3. Active power P
4. Reactive power Q
5. Apparent power $|S|$
6. Power factor angle θ_s
7. Power factor

$Z_{Total} = 20 + j6.4 - j1.4 = 20 + 5j$

$I = \frac{V}{Z} = \frac{120\angle 30}{20.6\angle 14} = 5.82\angle 16°\,A$

$S = V I^* = 698\angle 14°\ VA = 677 + j169$

$(120\angle 30)(5.82\angle 16)^* = S$

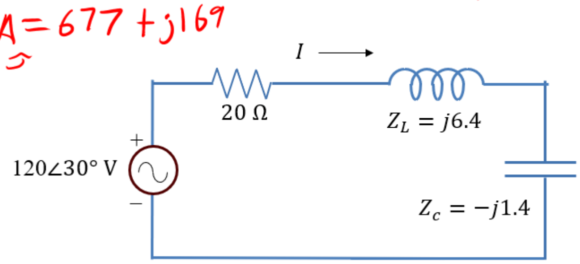
677 W

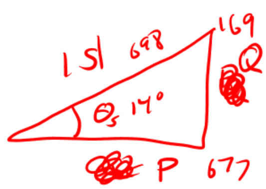
168.9 var

698 VA

14°

0.97 lagging





Bonus: Can you verify conservation of both active and reactive power?

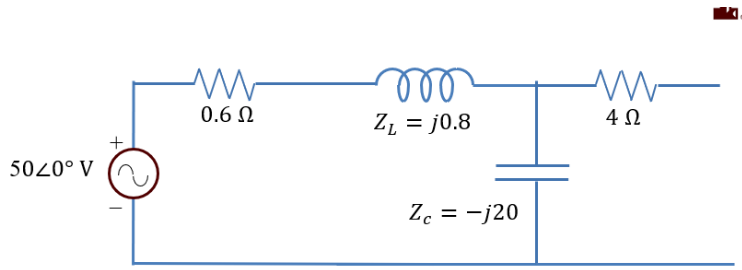
7. Solutions not given – quite similar to online video.

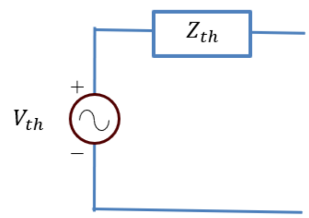
8.

We want to find

(1) The Thevenin equivalent for this circuit, as shown.

(2) The maximum power that could be delivered to a load with impedance Z_{load}





Maximum power transfer occurs when $Z_{load} = Z_{th}^*$

$$V_{th} = 52.06\angle -1.79° \text{ V}$$
$$Z_{th} = 4.65 + j0.81\ \Omega$$
$$S_{\text{Max}} = 145.69 - j25.47 \text{ W}$$

Homework #2 due Tuesday, Jan 30th

Textbook problems:

2.9

$$KVL: \quad 120\angle 0° = (60\angle 0°)(0.1 + j0.5) + \overline{V}_{LOAD}$$
$$\therefore \overline{V}_{LOAD} = 120\angle 0° - (60\angle 0°)(0.1 + j0.5)$$
$$= 114.1 - j30.0 = 117.9\angle -14.7° \text{ V}$$

2.22

(a) $\overline{Y}_1 = \dfrac{1}{\overline{Z}_1} = \dfrac{1}{(4+j5)} = \dfrac{1}{6.4\angle 51.34^\circ} = 0.16\angle -51.34^\circ$

$= (0.1 - j0.12)\text{S}$

$\overline{Y}_2 = \dfrac{1}{\overline{Z}_2} = \dfrac{1}{10} = 0.1\text{S}$

$P = V^2(G_1 + G_2) \Rightarrow V = \sqrt{\dfrac{P}{G_1+G_2}} = \sqrt{\dfrac{1000}{(0.1+0.1)}} = 70.71\text{ V}$

$P_1 = V^2 G_1 = (70.71)^2\, 0.1 = 500\text{ W}$

$P_2 = V^2 G_2 = (70.71)^2\, 0.1 = 500\text{ W}$

(b) $\overline{Y}_{eq} = \overline{Y}_1 + \overline{Y}_2 = (0.1 - j0.12) + 0.1 = 0.2 - j0.12$

$= 0.233\angle -30.96^\circ\text{ S}$

$I_S = V\, Y_{eq} = 70.71(0.233) = 16.48\text{ A}$

2.28

$\overline{S}_1 = 15 + j6.667$

$\overline{S}_2 = 3(0.96) - j3\left[\sin\left(\cos^{-1} 0.96\right)\right] = 2.88 - j0.84$

$\overline{S}_3 = 15 + j0$

$\overline{S}_{TOTAL} = \overline{S}_1 + \overline{S}_2 + \overline{S}_3 = (32.88 + j5.827)\text{kVA}$

(i) Let $\overline{Z}$ be the impedance of a series combination of R and X

Since $\overline{S} = \overline{V}\,\overline{I}^* = \overline{V}\left(\dfrac{\overline{V}}{\overline{Z}}\right)^* = \dfrac{V^2}{\overline{Z}^*}$, it follows that

$$\overline{Z}^* = \frac{V^2}{\overline{S}} = \frac{(240)^2}{(32.88 + j5.827)10^3} = (1.698 - j0.301)\,\Omega$$

$$\therefore \overline{Z} = (1.698 + j0.301)\Omega \quad \leftarrow$$

(ii) Let $\overline{Z}$ be the impedance of a parallel combination of R and X

Then $R = \dfrac{(240)^2}{(32.88)10^3} = 1.7518\,\Omega$

$X = \dfrac{(240)^2}{(5.827)10^3} = 9.885\,\Omega$

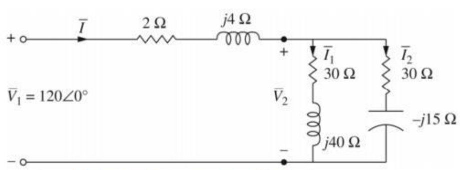
$$\therefore \overline{Z} = (1.7518 \| j9.885)\,\Omega \quad \leftarrow$$

2.43

(a) Transforming the Δ-connected load into an equivalent Y, the impedance per phase of the equivalent Y is

$$\bar{Z}_2 = \frac{60 - j45}{3} = (20 - j15)\,\Omega$$

With the phase voltage $V_1 = \frac{120\sqrt{3}}{\sqrt{3}} = 120\,\text{V}$ taken as a reference, the per-phase equivalent circuit is shown below:



Total impedance viewed from the input terminals is

$$\bar{Z} = 2 + j4 + \frac{(30 + j40)(20 - j15)}{(30 + j40) + (20 - j15)} = 2 + j4 + 22 - j4 = 24\,\Omega$$

$$\bar{I} = \frac{\bar{V}_1}{\bar{Z}} = \frac{120\angle 0°}{24} = 5\angle 0°\,\text{A}$$

The three-phase complex power supplied = $\bar{S} = 3\bar{V}_1\,\bar{I}^* = 1800\,\text{W}$

$P = 1800\,\text{W}$ and $Q = 0\,\text{VAR}$ delivered by the sending-end source

(d) The three-phase complex power absorbed by each load is

$$\bar{S}_1 = 3\bar{V}_2\bar{I}_1^* = 430\,\text{W} + j600\,\text{VAR}$$
$$\bar{S}_2 = 3\bar{V}_2\bar{I}_2^* = 1200\,\text{W} - j900\,\text{VAR}$$

The three-phase complex power absorbed by the line is

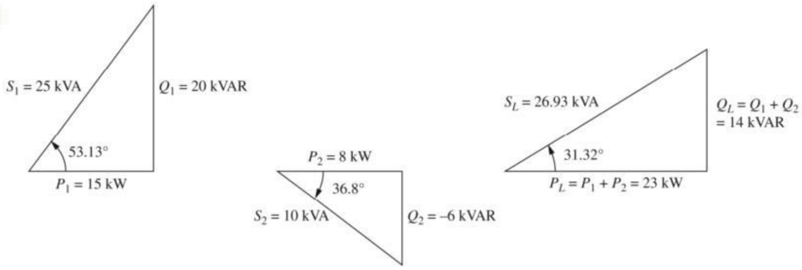
$$\bar{S}_L = 3(R_L + jX_L)I^2 = 3(2 + j4)(5)^2 = 150\,\text{W} + j300\,\text{VAR}$$

The sum of load powers and line losses is equal to the power delivered from the supply:

$$\bar{S}_1 + \bar{S}_2 + \bar{S}_L = (450 + j600) + (1200 - j900) + (150 + j300)$$
$$= 1800\,\text{W} + j0\,\text{VAR}$$

2.48

(a)



(b) $pf = \cos 31.32° = 0.854$ Lagging

(c) $I_L = \frac{S_L}{\sqrt{3} V_{LL}} = \frac{26.93 \times 10^3}{\sqrt{3}(480)} = 32.39\,\text{A}$

(d) $Q_C = Q_L = 14 \times 10^3\ \text{VAR} = 3(V_{LL})^2 / X_\Delta$

$$X_\Delta = \frac{3(480)^2}{14 \times 10^3} = 49.37\,\Omega$$

(e) $I_C = V_{LL} / X_\Delta = 480 / 49.37 = 9.72\,\text{A}$

$$I_{LINE} = \frac{P_L}{\sqrt{3}\,V_{LL}} = \frac{23 \times 10^3}{\sqrt{3}\,480} = 27.66\,\text{A}$$

6. Solutions not given.