Class 1: Three-Phase AC Power Calculations

- A **phasor** is a complex number that represents a cosine-valued AC function
- The Root Mean Square (RMS) for cosine is found by dividing the maximum value by $\sqrt{2}$
- In polar form, $R\angle\theta$, a phasor represents the RMS voltage or current and phase angle
  - $R\angle\theta \rightarrow \sqrt{2}\, R\cos(2\pi f t + \theta)$
- Conversions to rectangular form: a+jb, and back can be done with these identities:
  - $R = \sqrt{a^2 + b^2}$ $\quad\theta = \tan^{-1}\left(\frac{b}{a}\right)$ $\quad a = R\cos\theta$ $\quad b = R\sin\theta$
- Complex number addition can be done in rectangular form, and complex number multiplication can be done in polar form.
- Phasor diagrams have the real part on the x axis and imaginary part on the y axis.
- The angular frequency is $\omega = 2\pi f$.
- KVL, KCL, and Ohm's law all apply with AC phasor analysis exactly as with DC.
- The effect of resistors, inductors, and capacitors upon phasors is handled with **impedance (Z),** which acts like complex resistance. $V = I \cdot Z$
- The impedance of inductors and capacitors depends on frequency
- Instantaneous power from the time signal $p(t) = v(t)i(t)$

Element	Time Domain	Phasor Domain	Z (impedance)
Resistor	$v(t) = Ri(t)$	$\boldsymbol{V} = \boldsymbol{I}R$	R
Inductor	$v(t) = L\frac{di(t)}{dt}$	$\boldsymbol{V} = j\omega L\boldsymbol{I}$	$j\omega L$
Capacitor	$i(t) = C\frac{dv(t)}{dt}$	$\boldsymbol{V} = \frac{1}{j\omega C}\boldsymbol{I}$	$\frac{1}{j\omega C}$

- **Complex power**: $S = VI^* = \lvert S\rvert\angle\theta_s = P + jQ$ Don't forget the conjugate!! (*)
- Average power or active power or real power: $\mathrm{Re}[S] = P = \lvert S\rvert\cos\theta_s$
  - This is what's normally thought of as "power". Units are W.
  - It's also what you get if you take the average value of the instantaneous power
- Reactive power: $\mathrm{Im}[S] = Q = \lvert S\rvert\sin\theta_s$ Units are "var".
- Apparent power: $\lvert S\rvert = \lvert V\rvert \cdot \lvert I\rvert$ Units are "VA".
- Power factor angle: θ_s is the angle of S or $\theta_v - \theta_i$
- **Power factor**: $\cos(\theta_s) = P/\lvert S\rvert$. It must be indicated as "leading" (negative θ_s) or "lagging" (positive θ_s). A unity (1) power factor indicates zero reactive power and is neither leading nor lagging.
- **At every node in the system, both active (real) and reactive power are conserved**
- Inductors only absorb reactive power, capacitors only produce reactive power
- Capacitor banks are used for power factor correction by supplying reactive power locally
- A three-phase system is **balanced** if (1) all voltages are equal in magnitude and shifted in phase by 120°, (2) loads are equal on each phase, (3) impedances are equal on each phase.
- Line-to-line voltages are related to line-to-neutral (phase) voltages as: $V_{phase} = \frac{V_{line}}{\sqrt{3}\angle 30°}$
- Delta-connected loads can be replaced with wye-connected loads: $Z_Y = \frac{1}{3}Z_\Delta$
- Delta-connected sources can be replaced by wye-connected sources $V_{phase} = \frac{V_{line}}{\sqrt{3}\angle 30°}$
- Per-phase analysis works for balanced systems if there is no mutual inductance between phases. Steps: (1) convert delta sources and loads to equivalent wye (2) solve phase "a" circuit independent of other phases (3) total system power is $3V_aI_a^*$ (4) if needed, go back to original circuit to find "b" or "c" values and internal Δ values.

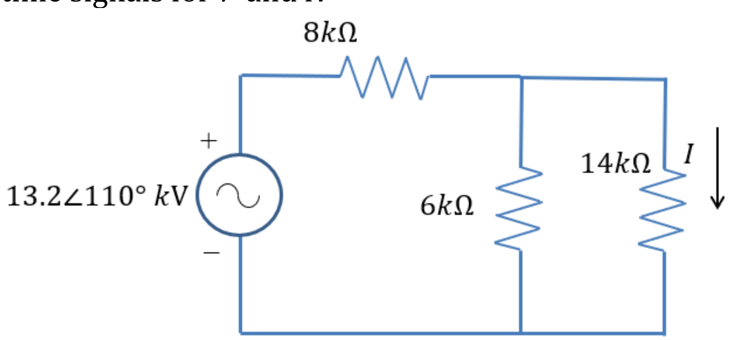
Homework #1 due Tuesday, Jan 23rd

For more help, read Chapters 1 and 2 in the textbook, view the videos and slides on the website, and take advantage of office hours of TAs and the instructor.

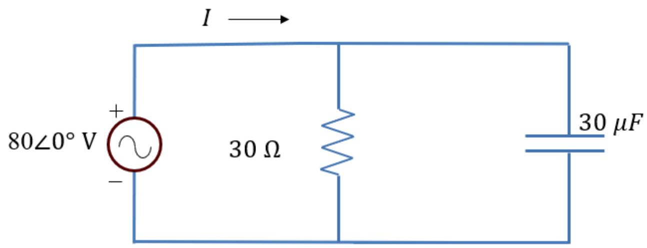
1. Practice problems for phasor conversion
   a. Convert $5\angle 12°$ A to rectangular form.
   b. Convert $14\angle 20°$ V to the cosine time function, assuming a frequency of 14 kHz.
   c. Find the polar form phasor for $20\cos(377t - 40°)$ kV.
   d. Convert the phasor 12-j3 A to polar form.
   e. Sketch a time plot of the phasor $18\angle 12°$ mA, assuming a frequency of 100 MHz.
   f. Draw a phasor diagram for the phasor $35\angle -110°$ V.
   g. Convert $24\angle -60°$ A to rectangular form.
   h. Convert $30\angle 0°$ V to the cosine time function, assuming a frequency of 50 Hz.
   i. Find the rectangular form phasor for $20\cos((6.28 \times 10^6)t + 18°)$ kV.
   j. Convert the phasor 30+j30 kA to polar form.
   k. Sketch a time plot of the phasor $1.32\angle 10°$ MV, assuming a frequency of 60 Hz.
   l. Draw a phasor diagram for the two phasors $3.5\angle 10°\ A$ and $2.7\angle 40°\ A$.
   m. Convert $16\angle -90°$ A to rectangular form.
   n. Convert $100.5\angle 0°$ V to the cosine time function, assuming a frequency of 400 Hz.
   o. Find the polar form phasor for $200\sin(377t)$ kV.
   p. Convert the phasor j5 V to polar form.
   q. Sketch a time plot of the phasor $300\angle -90°$ V, assuming a frequency of 10 Hz.
   r. Draw a phasor diagram for the phasor $3.25\angle 0°$ V.
   s. Convert $2.0\angle 90°$ MV to rectangular form.
   t. Convert $74.5\angle 14°$ V to the cosine time function, assuming a frequency of 2500 Hz.
   u. Find the rectangular form phasor for $55\cos(10^9 t - 108°)$ V. What is the frequency?
   v. Convert the phasor 10-j30000 kA to polar form.
   w. Sketch a time plot of the phasor $100\angle 90°$ A, assuming a frequency of 6000 Hz.
   x. Draw a phasor diagram for the two phasors $90\angle 90°\ A$ and $90\angle -90°\ A$.

---

2. Solve for I as a phasor using any method. Assuming the frequency is 60 Hz, write the time signals for V and I.

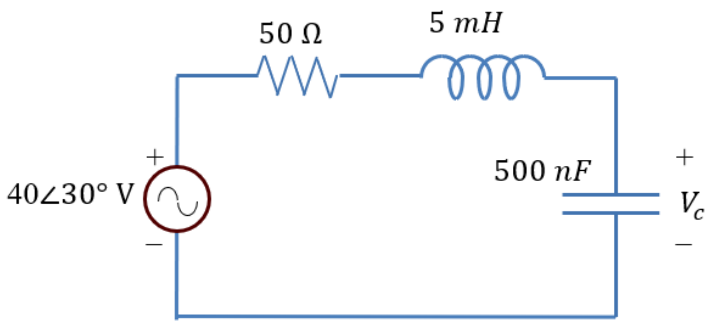


3. Find the current phasor I for this 1 kHz circuit



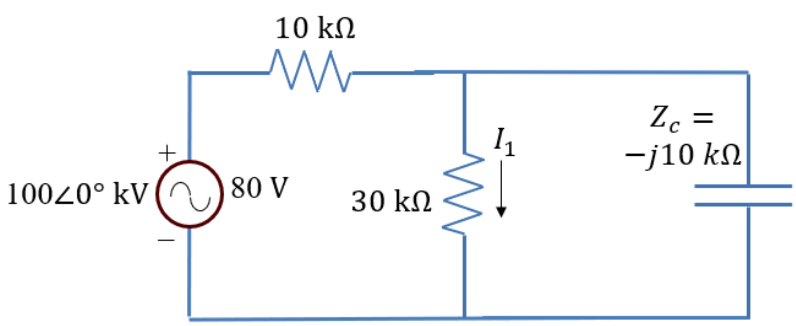
---

4. Solve for the phasor V_c for this circuit if it is operated at 400 Hz



---

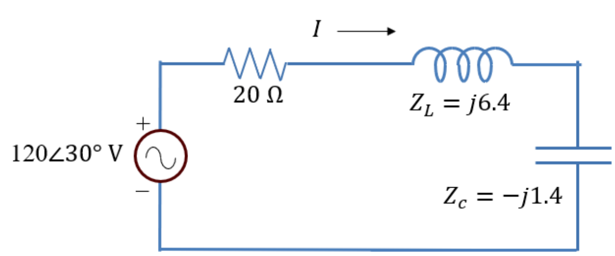
5. Solve for the phasor I_1. The impedance for Z_c is given so you don't need the frequency.



6.

Calculate for the voltage source

1. Current I
2. Complex power S
3. Active power P
4. Reactive power Q
5. Apparent power $|S|$
6. Power factor angle θ_s
7. Power factor



7. A factory is acting like a 250 Ω resistor in parallel with a 500 mH inductor. A single-phase power distribution line supplying the factory from the substation can be modeled as a 65 mH inductor. At the substation, the voltage is 12∠6.5° kV.

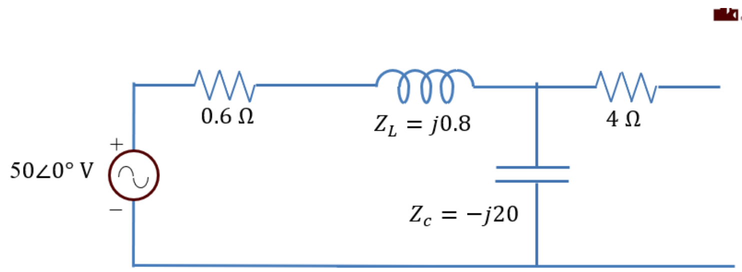
a. Draw this circuit and find the impedance of the circuit elements. The substation can be modeled as a voltage source. The frequency is 60 Hz.
b. What is the voltage at the factory?
c. How much active and reactive power is the factory absorbing?
d. What is the factory's power factor?
e. If a 50 μF capacitor is installed in parallel with the factory, how do the voltage and power factor change?
f. What should the value of the capacitor be so that the power factor is 0.95 lagging?

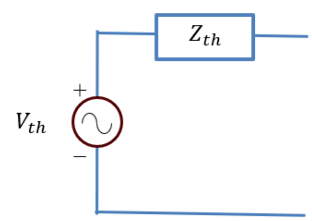
8.

We want to find

(1) The Thevenin equivalent for this circuit, as shown.

(2) The maximum power that could be delivered to a load with impedance Z_{load}





Maximum power transfer occurs when $Z_{load} = Z_{th}^*$

Homework #2 due Tuesday, Jan 30th

Textbook problems: 2.9, 2.22, 2.28, 2.43, 2.48

6. For a balanced 3-phase system, there is a load with the phase-A to neutral voltage $V_a = 12\angle 0°\ kV$. The load is a wye-connected impedance consuming a total of 2 + j1.5 MVA

a. Draw a diagram of this load and label the voltages and currents.
b. What are the phase B and C phase voltages to neutral?
c. What are the line-to-line voltages V_{ab}, V_{bc}, and V_{ca}?
d. How much complex power is each phase of the load consuming?
e. What is the power factor of the load? Apparent power?
f. What is the current in each of the load phases: I_{an}, I_{bn}, I_{cn}?
g. What is the total neutral current of this wye-connected load (hint: balanced) $I_a + I_b + I_c$
h. What is the wye-connected impedance of each phase, Z_Y?
i. What would be the equivalent delta-connected impedance of each phase, Z_Δ?
j. If this load were converted to delta, what would the delta currents be I_{ab}, I_{bc}, I_{ca}?
k. What would be the complex power consumed by each leg of the delta and the total?