Techniques for Creating Synthetic Combined Electric and Natural Gas Transmission Grids

Yousef A. Abu-Khalifa, *Student Member, IEEE,* and Adam B. Birchfield, *Member, IEEE*

***Abstract*—This paper presents a methodology for the creation of a synthetic combined electric and natural gas transmission network, along with representative benchmark results. The systems do not contain actual, confidential network data, but are synthetic, meaning they are built to capture the behavior of a combined network that is geographically constrained. First, natural gas loads are placed in a selected area. Work already done in building synthetic electric grids aids in this process, where the natural gas-powered generators are modeled as loads in the natural gas system. Publicly available data is then used to place the remaining gas loads and the gathering plant. Next, a method is introduced to construct a pipeline network connecting the loads and processing plant, which acts as the source. The combined electric-gas system is then solved for the nodal pressures, pipeline flow rates, and electric state variables. A 51-node gas test case with 49 pipes, 23 loads, 2 compressor stations and a loop is constructed and solved in combination with a 173-bus electric system, designed to aid with developing and validating analysis techniques for combined electric-gas systems.**

***Index Terms*—Natural gas pipeline system, synthetic network, power flow, Weymouth equation, compressor stations**

I. INTRODUCTION

NATURAL gas makes up 40% of the United States' electricity generation (roughly 1,000 billion kilowatt hours generated per year) and is projected to increase to almost 2,000 billion kilowatt hours by the year 2050 [2]. Furthermore, natural gas-fired generators account for about 40% of approximately 1,000 gigawatts of projected capacity additions through 2050, with other renewable sources accounting for the remaining 60% [2]. The electric grid is becoming more dependent on the availability of natural gas and the ability of generators to produce electricity based off of gas production and distribution. No event in recent times is more evident of this than the February 2021 cold weather outages in Texas and the South Central region of the United States. Due to the extremely cold weather caused by Winter Storm Uri, the Electric Reliability Council of Texas (ERCOT) averaged 34,000 MW of generation that was unavailable [3]. With this shortage of generating capacity, 23,000 MW of load had to be manually shed. Taking into account the number of unplanned outages, more than 4.5 million Texas residents lost power during the event and 210 people died, with most of the deaths attributed to power outages [3].

One of the challenges in the research area of combined electric-gas energy systems is that actual data and models are not publicly available, but are considered Critical Energy Infrastructure Information (CEII). The projected increase in the use of natural gas for electricity generation, in addition to the events of the 2021 winter storm, has driven interest in enhancing combined electric-gas simulation capabilities and studying the system inter-dependencies. Despite the increasing importance of combined electric-natural gas studies, the lack of publicly available test cases poses a challenge in providing a realistic and comprehensive combined simulation for research in this area. Having a combined simulation of real electric-natural gas systems is critical in understanding the behavior of these complex energy systems and provides a close representation of the actual network. The objective here is to create a synthetic natural gas system which is representative of a real system and can be used for research in combined electric-gas studies.

The research on applications of combined electric-gas networks has notably increased over the past couple of years. Methods have already been presented in previous works, such as [4] and [5], which study the inter-dependencies between electric and natural-gas systems. The authors use pre-existing, established methods that simulate the natural-gas [6] and electric [7] transmission systems individually. The authors present a single integrated formulation that solves the non-linear equations associated between the combined systems; where the links between the systems are at the compressor stations, gas-powered generators and power-to-gas units. The studies proved to be successful as the solutions were able to converge to a stable point while obeying the fundamental equations and without violating any of the defined constraints. The studies, however, were conducted on test cases that are not representative of any real networks. Work has been done in previous literature, such as [8], [9] and [10], in using publicly available data to solve real natural gas pipeline systems for simulation purposes with electric systems. References [8] and [9] use similar methods as those seen in [4] and [5] to model the dependence between electric and natural gas systems in a unified formulation. The studies are carried out on existing, simplified version of the Belgian natural gas pipeline system. Reference [10] takes this a step further by using data made public via the Homeland Infrastructure Foundation-Level Data (HIFLD) [11] to build a geographically accurate pipeline network to implement an integrated system formulation of both systems.

The main motivation for this work is that detailed simulation models of actual electric and gas systems, specifically for the state of Texas, are not publicly available, which limits their ability to be used for research in this area. A full-scale, combined electric-gas model which contains all the

The authors are with the Department of Computer and Electrical Engineering, Texas A&M University.

data necessary for steady-state and dynamic simulations is not readily available for public research. There are research gaps in the references mentioned in the previous paragraph. The networks used in [4] and [5] are benchmark test cases that are not geographically constrained. Additionally, no system conditions or parameters for the natural gas systems are given ; which makes it difficult or near impossible to re-create any results for future studies. As stated earlier, the systems used in [8], [9] are on simplified models of an existing Belgian natural gas transmission systems. A methodology to create a synthetic natural gas transmission system from scratch is not presented. Some public geographical data that can help with this is available through the EIA [12], the Texas Railroad Commission [13] and the HIFLD [11], but the data is not complete or entirely accurate. Reference [10] does build upon this by using publicly available locational pipeline data to build their system, but it is relatively simple compared to the pipeline network of Texas. The methods presented cannot confidently be implement for the complex, intricate Texas natural gas pipeline transmission network.

This work fills these research gaps by presenting a methodology to systematically create a synthetic natural gas pipeline network of a section of the Texas natural gas pipeline system, specifically in Travis County, only using public data and by providing a readily available test case which can be used for future combined electric-gas studies. This paper builds on prior efforts towards building and validating synthetic electric grids which aid in power systems research and development, such as in [14]. The methodology builds a test case that is consistent with industry standard principles for steady-state and dynamic electric [7] and natural gas [6] modeling. The final results should be consistent with prior works, such as [4] and [5], which outline different approaches to describe various aspects of the relationship between the electric and natural gas systems working in unison. The contributions of this paper can be outlined as follows:

- Present a systematic methodology to create geographically constrained synthetic grid of the natural-gas pipeline system of Travis county, Texas using only publicly available data and publish a complete natural-gas data set.
- Demonstrate the engineering application of a combined electric-natural gas system simulation using the constructed synthetic grid.
- Validate the numerical solution of the constructed system by implementing the solution algorithms seen in [6] and [17].
- Validate the topological and operational characteristics of the network by comparing it to available data from previous literature.

This paper serves as an extension to [1] towards creating and testing a combined electric and natural gas transmission system in four ways. First, compressors stations are added to the system, which introduces a pressure dependence between nodes. Second, loops are added to the system, which introduces non-linearity to the network. With loops and compressor stations playing a major role in everyday natural gas pipeline operations [15], it is crucial to include them in the simulations to present a more realistic model. Reference [1] does not include these pipeline elements in the system nor does it present a methodology to solve a pipeline system taking into account loops and compressor stations. Since these elements were not accounted for and a strict tree-branch structure with no compressor stations was used, a simpler methodology was implemented to solve for the pipeline flows and nodal pressures. The methodology implements a direct, analytical approach. With the inclusion of loops and compressor stations, the pipeline system is no longer linear and a numerical solution algorithm is needed.

The third addition is that this paper provides a case study along with the results obtained. Reference [1] simply provided the network properties of the joint electric-gas system and some of the results obtained from the linear solution. This case study builds upon this and provides an example of how the system can be used in research in the operation and planning of an integrated electric and natural gas network with concrete results. The final addition is that this paper compares the accuracy of the implemented solution algorithm and the constructed system with systems seen in previous literature.

The paper continues in Section II which outlines the mathematical formulations of the governing simulation equations. Section III presents the implementation of the proposed methodology used to automatically construct a pipeline system in the Travis County area, given the geographical coordinates of the loads and source. The synthetic electric grid used is the Travis County Transmission and Distribution System made available through [16]. Next, methods like those seen in [6] and [17] are presented to solve for the nodal pressures and pipeline flows. Section IV shows the detailed results and validation of the method along with a case study done on the system. Section V gives the conclusion along with future work that needs to be added to the research.

II. Modeling an Electric-Gas Combined System

A. Natural Gas Pipeline System Formulation

The steady state behavior of a natural gas pipeline system can be described using gas flow equations and nodal balance equations. The gas flow equations consider temperature and pressure conditions of the pipeline in question as well as the chemical composition of the natural gas itself. The core of these equations is expressing the relationship between the pressure drop between two nodes and the pipeline flow rate. For the purposes of this paper, no elevation differences in the pipeline system are accounted for and the transfer process is assumed to be isothermal. According to [4], these assumptions are reasonable for practical purposes. The nodal balance equations are simply a consequence of conservation of mass. One equation given in [4] is used to calculate the gas demand of a generator given the MW output. Other important properties of natural gas systems that are essential in constructing an accurate and realistic simulation are compressor stations and loop equations, which are addressed here in detail.

B. The Weymouth Equation

Reference [6] presents a few equations that can be used to model gas flow in a pipeline, one common equation used

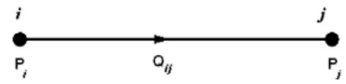

Fig. 1. Pipe joining two consecutive nodes. [18]

for steady-state high pressure conditions is the Weymouth Equation. The equation, given in [18], takes the form

$$P_i^2 - P_j^2 = K_{ij} Q_{ij}^2 \tag{1}$$

where *P* is the pressure and *Q* is the volumetric flow rate. The subscript *i* indicates the upstream node (higher pressure) and *j* indicates the downstream node (lower pressure). K_{ij} , the pipe flow resistance, is a constant that depends on the units used. In this paper, *P* is expressed in kPa, *Q* in m^3/hr, *T* in Kelvin, *L* in km and *D* in mm. Figure 1 shows an example of a pipeline using this notation. From [18], when these are the units used, K_{ij} takes the form

$$4.3599 \times 10^8 \frac{fgZT}{D^5} (\frac{P_n}{T_n})^2 L \tag{2}$$

where *f* is a unit-less friction factor, *g* is the specific gravity of the natural gas, *Z* is the compressibility of the gas, *T* is the temperature of the gas, *D* is the diameter of the pipeline, *L* is the length of the pipeline, P_n is the standard pressure 101.3 kPa and T_n is the standard temperature 288.15 Kelvin. Due to the highly turbulent flow seen in high pressure pipeline transmission systems, the only factor that the friction factor *f* is dependent on is the diameter of the pipeline [4]. Given that *D* is expressed in mm, the friction factor can be found by

$$f = \frac{0.09407}{\sqrt[3]{D}} \tag{3}$$

The specific gravity of natural gas used in pipeline transmission has values of 0.58 to 0.65 [19]. This number depends on the chemical composition of the gas. The chemical composition of the gas also determines what the compressibility of the gas is. The compressibility of the gas must be accounted for since the assumption that the gas is ideal has not been made. Natural gas used in pipeline transmission systems are made up of different chemical components, which makes the calculations to determine the compressibility complicated. There are many different approaches that different authors have presented, such as the one seen in [20]. The value used for the sample test case is the same as what was presented in [18], which is 0.91.

C. Nodal Balance Equation

The nodal balance formulation simply states that the sum of all flows entering a node must equal the sum of flows exiting the node. The mathematical formulation is

$$\sum Q_s + \sum Q_{in} = \sum Q_L + \sum Q_{out} \tag{4}$$

where Q_S is the flow from any source adjacent to the node and Q_L is the flow to any load adjacent to the node.

D. Gas Powered Generators

Gas-powered generators can be modeled as any generic gas load with a specified flow demand. Reference [4] provides an equation that determines the amount of gas demand required in cubic meters per hour for a specified amount of power output at a gas-powered generator

$$Q_d = \frac{3600 P_{GPG}}{\eta_{GPG} LHV} \tag{5}$$

where P_{GPG} is the real power produced at the generator, η_{GPG} is the energy efficiency of the generator, and *LHV* is what is known as the Lower Heating Value. The lower heating value of natural gas is between 35.40 and 39.12 MJ/m^3 . As in [4], the average value of 37.26 MJ/m^3 is used for the *LHV* and 0.8 is used for the energy efficiency of the generator.

E. Compressor Stations

Compressors are machines strategically placed along a pipeline to increase the pressure, which can decrease due to frictional losses. Additionally, compressors can be used to increase or decrease the pressure at a specified node to meet certain demands without obstructing the flow of gas. Compressors play a key role in natural gas pipeline networks. Mathematically, compressors serve as another restriction to the simulation where the inlet pressure, outlet pressure or compressor ratio (the ratio of output pressure to input pressure) are specified and remain fixed throughout the solution process. Compressors require power to increase the pressure given a certain amount of natural gas flow through them. Depending on their design, they are either powered by gas turbines or electrical motors. If they are electrically powered, the compressor can be modeled as an electric load in the power flow. If the compressor is gas powered, the amount of gas needed can be tapped from the pipeline and is accounted for in (4). The formula used to describe the amount of power needed by a compressor, given in kilowatts, is

$$BP = 9.753 \times 10^{-5} Z_a Q_{SC} [\frac{T_s}{E\eta_C}][\frac{k}{k-1}][(CR)^{\frac{k-1}{k}} - 1] \tag{6}$$

where *BP* is the break power, Z_a is the average compressibility, Q_{SC} is the standard volumetric flow rate in m^3/hr, T_s is the suction temperature in Kelvin, *k* is the specific heat ratio, *E* is the parasitic efficiency, η_C is the compression efficiency and *CR* is the compression ratio.

F. Loops

Most pipeline systems are originally constructed with single pipes connecting a point of origin to its destination. As more capacity is needed, these single pipelines are looped, meaning a parallel pipeline is installed along the route of the main pipeline with both ends connected to the original pipe [15]. By splitting the path, more capacity is added and the flow rate can be increased with reduced risk of reaching maximum flow limits. Another property of loops is that the pressure difference between the two common nodes for both the pipelines are the same. Loops play an important role in everyday natural gas pipeline operations and are used in most pipeline systems,

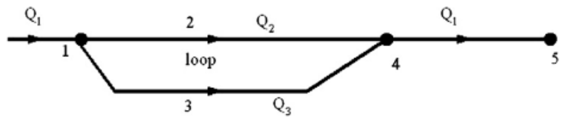

Fig. 2. Part of pipeline network with loops [18]

therefore, the appropriate equations must be included in the simulations to present an accurate model. Mathematically, this adds the restriction that the pressure drop between the two common nodes must be equal. Referring to the Figure 2 and equating both sides of (1) for each segment, the formulation for a loop is

$$K_{14(2)}Q_{14(2)}^2 = K_{14(3)}Q_{14(3)}^2 \tag{7}$$

where the term on the left-hand side refers to pipe number 2 and the right-hand side refers to pipe number 3.

G. Electric System Formulation

The governing equations for power systems are the power balance equations at each bus. The voltage at bus *k* (expressed in polar form) and the admittance (expressed in rectangular form) between two adjacent busses *k* and *n* are given by

$$\tilde{V_k} = |V_k|\angle\theta_k \tag{8}$$

$$\tilde{Y_{kn}} = G_{kn} + jB_{kn} \tag{9}$$

where the voltage angle θ_k is given in radians and the conductance and susceptance, G_{kn} and B_{kn} respectively, are given in Siemens. The tilde indicates a phasor variable. The power balance equations can then be formulated as

$$P_{G,k} - P_{L,k} = |V_k|\sum_{n=1}^{N}|V_n|(G_{kn}cos\theta_{kn} + B_{kn}sin\theta_{kn}) \tag{10}$$

$$Q_{G,k} - Q_{L,k} = |V_k|\sum_{n=1}^{N}|V_n|(G_{kn}sin\theta_{kn} + B_{kn}cos\theta_{kn}) \tag{11}$$

where $P_{G,k}$ and $Q_{G,k}$ are the real and reactive power at bus *k* and $\theta_{kn} = \theta_k - \theta_n$.

H. Combined Electric and Natural Gas System

With the governing equations for each respective system formulated, the interaction between the two must be defined. One point of interaction occurs at the gas-powered generators in the electric system. The generators can be modeled as gas loads in the natural gas system with a fixed demand. Once the power flow has been solved, the real power output of the gas-powered generators is known. Using (5), the required gas demand can be calculated. The other point of interaction is at the compressor stations. With the flows known, the compressor stations are placed where needed and their parameters are set. The power required by the compressor stations can be calculated and they are modeled as loads in the electric system.

III. Methodology for Building and Solving Synthetic Combined Electric-Gas Grids

The integrated natural gas and electric power system is composed of a gas network and an electric network. With geographically accurate, synthetic electric grids already available under prior methods [14], a geographically accurate natural gas system needs to be constructed using a systematic method. The Texas Railroad Commission [12] provides a map of the actual pipeline network in Texas, but available data is not sufficiently detailed to use for the solution process. This section outlines the general procedure used for automatically constructing a natural gas pipeline network and solving it with a dependency on an electric transmission network.

A. Obtaining Natural Gas Production and Demand Data

Data was obtained from the U.S. Energy Information Administration to aid in constructing a geographically accurate natural gas system. The location of all processing plants, monthly gas flow capacity of each processing plant and monthly gas consumption by state, along with other useful data, is publicly available via sources such as [21]. The locations of the gas-powered generators can be obtained from the synthetic electric grid along with the output power from the power flow. The natural gas system for this case requires only one processing plant to serve as the main source of gas and “slack” node for the system. Using the data provided by the EIA [21], a nearby processing facility with enough monthly capacity to fuel this synthetic system can be picked as the source. The units given by the EIA are in MMCFD but should be converted to m^3/hr to be consistent with the units presented in (2).

B. Building the Natural Gas Pipeline Network

The first step in the algorithm is to build a synthetic electric network following a similar procedure outlined in [14] or to obtain one readily available. The gas-powered generators in the synthetic network will serve as some of the gas load where the amount of gas demand can be found using equation (5). Then the general gas loads and source are placed by geographical location with a specified amount of gas consumption for the loads. These loads represent local distribution facilities, large industrial facilities or commercial facilities. The location and output power of the gas-powered generators are already known from the synthetic electric grid. The remaining general gas loads can be stochastically (or manually) placed around the area of interest with a respective demand specified. It is important to check that the total gas consumption of the system does not exceed the maximum capacity of the source.

It is also crucial to number all the nodes and assign them a ‘type’ (generator, load, or junction), as this information is needed to solve the system. The algorithm presented uses a numbering system to organize the nodes by type, where type 0 indicates a generator, type 1 indicates a gas-powered generator, type 2 indicates a general gas load, type 3 indicates a junction point and type 4 indicates a compressor’s inlet and outlet nodes. Once all the loads’ and source’s locations and node numbers have been specified, the next step is to find the geographical center of the cluster of the loads, just as is done in a k-means clustering algorithm [22]. A line is drawn from the processing plant through the center of the cluster, this will represent the main artery of the pipeline system. The end point can be chosen to be as far as the furthest load, where the closest distance from the furthest load and the end of the pipe is a perpendicular line.

The distance of every load to the closest point on the main pipeline is calculated (which is also a line perpendicular to the main pipeline) and sorted. Starting from the furthest load away from the line, a branch is drawn. Then the next furthest load is looked at. The algorithm calculates the closest distance to each line present and draws a line to the closest one. This process is repeated until every load is accounted for. If the closest point to a load happens to be another load, then the next closest point on an existing pipeline is chosen as the endpoint for that branch. This is done to avoid stringing loads together and to achieve a proper tree-branch structure. During the process each node or junction is assigned a number and the geographical location is recorded. The algorithm then incrementally moves along each line to determine which nodes are adjacent to one another and calculates the distance between them by using their geographical coordinates, then calculates the k value, as this is needed for the Weymouth Equation. Data is used from [4] to assign the pipelines with appropriate diameter lengths. Compressor stations and loops are then manually placed where needed.

All necessary data is then exported to an Excel spreadsheet in an organized manner. The dataset includes tables for different properties of the network, such as the nodes, loads, sources, and pipes. Figure 3 shows the natural gas system that was constructed using this algorithm. The triangle represents the processing plant, the squares represent general gas loads, the hollow circles represent the gas-powered substations, and the black circles represent compressor stations. The red line represents the main pipeline while the black lines are the branches. Figure 4 shows the synthetic electric network of the Travis County area.

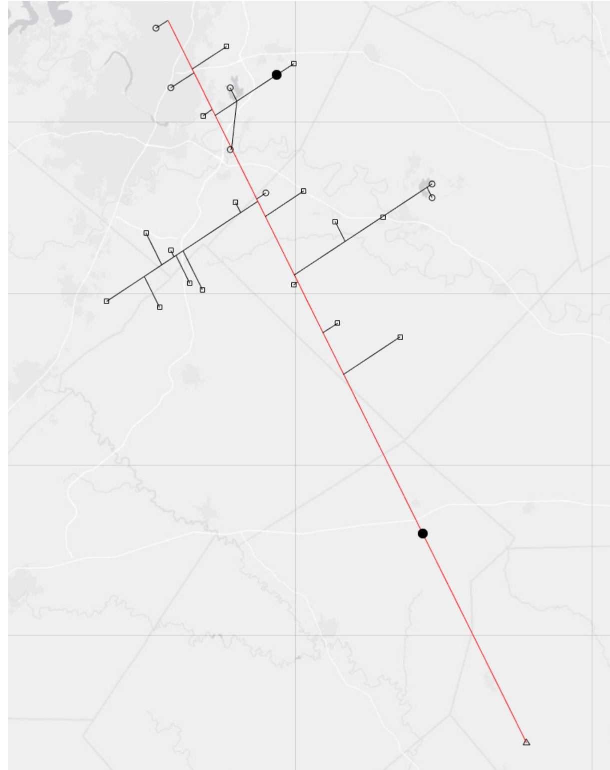

Fig. 3. Natural gas network test case, where the triangle represents the processing plant, the squares represent the general gas loads, the hollow circles represent the gas-powered substations and the black circles represent the compressor stations

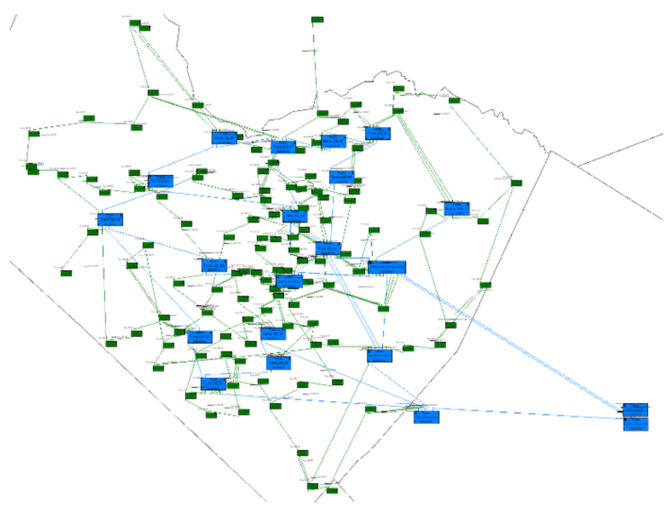

Fig. 4. Synthetic electric network used in test case. Larger boxes represent 230 kV substations and smaller boxes represent 69 kV substations [1]

C. Solving for Nodal Pressures and Pipeline Flow Rates

The core of the solution process are the governing equations presented in Section II. The general methodology is derived from [6] and [17]. The algorithm sets up the pipeline system using a depth-first search tree branch algorithm to define the branches and numbers the nodes using the data obtained from sub-section B. Since no loops are present and all the flows for the loads are known, no iterations are needed, and a backward-forward sweep is used. First, the unknown pipeline flows are solved for using the principle of nodal balance and the information already known from the flows at the loads. With all the pipeline flows known and the pressure at the source defined, the Weymouth equation can then be used to solve for the nodal pressures. A matrix formulation is used for this step. The nodal balance equation is re-written as

$$\mathbf{A_x Q + L = 0} \tag{12}$$

where $\mathbf{Q}$ is a vector of all the pipeline flows, $\mathbf{L}$ is a vector of the defined flows at all nodes, which are 0 for junctions and Q_n for the known values of the loads. The dimensions of $\mathbf{L}$ and $\mathbf{Q}$ are $(m \times 1)$, where m is the number of pipes in the system. A gives the node-branch incidence matrix, with dimensions $(n \times m)$, where n is the number of nodes. In $\mathbf{A}$, each column represents a pipe, and each row represents a node. To construct this matrix, each column has a 1 and a -1 at the two ending nodes of that respective pipeline, and the rest of the indexes are 0. The selection of the "from" and "to" ending points are arbitrary. Once this is done, the row corresponding to the slack node is removed. The resulting matrix is $\mathbf{A_x}$, with dimensions $(n-1 \times m)$. Through this, $\mathbf{A_x}$ is a non-singular, square matrix. In a system with no loops or compressors, $m = n-1$. $\mathbf{Q}$ can be solved and all the branch flows in the system can be found. The matrix formulation for the Weymouth equation is as follows

$$\mathbf{A^T P} - \mathbf{C}\phi(\mathbf{Q}) = \mathbf{0} \tag{13}$$

where $\mathbf{P}$ is a $(n \times 1)$ vector of all the squares of the nodal pressures, $\mathbf{C}$ is a diagonal matrix of the pipe flow resistance and $\phi(Q_i) = Q_i|Q_i|$. The reason for expressing the flows in this form is to keep any negative signs acquired from (12). In this form, the elements in $\mathbf{P}$ cannot directly be solved for because $\mathbf{A^T}$ is not square. Note that $\mathbf{P}$ includes both the unknown pressures and the known pressure of the slack node. Equation (13) can be written as a function of the unknown pressures alone and can then be solved for. This is given by

$$\mathbf{A^T P_0} + \mathbf{A^T D_n P_x} - \mathbf{C}\phi(\mathbf{Q}) = \mathbf{0} \tag{14}$$

where $\mathbf{P_0}$ is a $(n \times 1)$ vector with the pressure at the slack node specified and the remaining nodes being 0, $\mathbf{P_x}$ is a $(n-1 \times 1)$ vector of the unknown pressures and $\mathbf{D_n}$ is a matrix used to describe the pressure dependence between the nodes. The dimension of this matrix is $(n \times n-1)$, where the rows correspond to all the nodes and the columns also correspond to all the nodes but the slack node is not included. The $\mathbf{D_n}$ matrix is constructed by placing a 1 at the indices where the respective row and column represent the same node, and 0 otherwise. With the multiplication of $\mathbf{A^T}$ and $\mathbf{D_n}$ resulting in an $(n-1 \times n-1)$ square matrix, the unknown pressures $\mathbf{P_x}$ can be solved for. To find the actual pressures at a node, the square root of that element must be taken. The ordering of the nodes and pipes are arbitrary but must be kept consistent throughout the solution process.

D. Solving for Nodal Pressures and Pipelines Flow Rates with Loops or Compressors

Including loops and compressors introduces complications with the solution process. Most notably, the inclusion of loops prohibits the use of a backward-forward sweep, and a recursive method is needed. Compressor stations now introduce the dependence of nodes on other nodes (i.e., the outlet pressure as a multiple of the inlet pressure). Given this, distinctions between "tree" pipes, "chord" (loop causing) pipes and "tree" non-pipes (such as compressors), as well as between dependent and non-dependent nodes need to be made. As a result, the matrices defined in sub-section B need to be slightly re-defined. New methods and equations are also introduced for the recursive solution process, but are still derived from the core equations mentioned in sub-section B.

The new notation is as follows: n, just like the previous section, is defined as the number of nodes in the system. The nodes can be defined as slack nodes, dependent nodes,where the pressure is a linear combination of other pressures, and independent nodes, whose pressures are being solved for. These nodes are denoted as n_r, n_d and n_x respectively. Therefore

$$n = n_r + n_d + n_x = n_r + n_{dx}. \tag{15}$$

The total number of pipes, or branches, is still denoted with m. The tree pipes in the system are denoted with m_{tp}, the tree non-pipes (such as compressors) are denoted with m_{tn} and the chord pipes are denoted with m_c. The pipes are also organized into several subsets that will be used in the solution process. m_t are all the tree elements in the system

$$m_t = m_{tp} + m_{tn} \tag{16}$$

while m_p are all the pipe elements in the system

$$m_p = m_{tp} + m_c. \tag{17}$$

The total number of pipes in the system can be expressed as

$$m = m_c + m_t = m_p + m_{tn}. \tag{18}$$

When these subscripts are seen on matrices or vectors, they will indicate the subsets of rows and columns for certain nodes and branches. With the different subsets now defined, the nodal balance equation is written as

$$\mathbf{A_{dx} Q} + \mathbf{L} = \mathbf{0} \tag{19}$$

where $\mathbf{A}$ and $\mathbf{Q}$ retain the same definition from the previous sub-section. $\mathbf{L}$ is the same but has length n_{dx} instead of m. The nodal balance equation can be split into tree and non-tree branches

$$\mathbf{A_{dx,t} Q_t} + \mathbf{A_{dx,c} Q_c} + \mathbf{L} = \mathbf{0}. \tag{20}$$

The objective is to define a function in terms of $\mathbf{Q_c}$, where $\mathbf{Q_c} = f(\mathbf{Q_c})$, since the loops cause non-linearity in the system. Once $\mathbf{Q_c}$ has been solved for recursively, the remaining branch flows and node pressures can be solved for directly. $\mathbf{Q_t}$ is written in terms of $\mathbf{Q_c}$

$$\begin{aligned} \mathbf{Q_t} &= -\mathbf{A_{dx,t}^{-1} L} - \mathbf{A_{dx,t}^{-1} A_{dx,c} Q_c} \\ &= \mathbf{Q_t^0} - \mathbf{B_t^T Q_c} \end{aligned} \tag{21}$$

where $\mathbf{Q_t^0} = -\mathbf{A_{dx,t}^{-1} L}$ and $\mathbf{B_t^T} = \mathbf{A_{dx,t}^{-1} A_{dx,c}}$. Equation (13) is also re-written as a function of only the independent pressures.

$$\begin{aligned} &\mathbf{A_p^T P^0} + \mathbf{A_p^T D_n P_x} - \mathbf{C}_\phi(\mathbf{Q_p}) \\ &= \mathbf{P_p^0} + \mathbf{A_{pD}^T P_x} - \mathbf{C}\phi(\mathbf{Q_p}) = \mathbf{0} \end{aligned} \tag{22}$$

where $\mathbf{P_0}$ retains the same definition from the previous subsection, $\mathbf{P_p^0} = \mathbf{A_p^T P^0}$ and $\mathbf{A_{pD}^T} = \mathbf{A_p^T D_n}$. Equation (22) is split into two parts, separating the tree and chord pipes.

$$\mathbf{P_{p,tp}^0} + \mathbf{A_{pD,tp}^T P_x} - \mathbf{C_{tp}}\phi(\mathbf{Q_{tp}}) = \mathbf{0} \tag{23}$$

$$\mathbf{P_{p,c}^0} + \mathbf{A_{pD,c}^T P_x} - \mathbf{C_c}\phi(\mathbf{Q_c}) = \mathbf{0}. \tag{24}$$

After rearranging (23) to be in terms of $\mathbf{P_x}$ and substituting into (24), the function $\mathbf{Q_c} = f(\mathbf{Q_c})$ is obtained and expressed as

$$\begin{aligned} &\phi^{-1}\mathbf{C_c^{-1}}(\mathbf{P_{p,c}^0} + \mathbf{A_{pD,c}^T}(\mathbf{A_{pD,t}^T})^{-1}(-\mathbf{P_{p,t}^0} \\ &\qquad + \mathbf{C_{tp}}\phi(\mathbf{Q_{tp}^0} - \mathbf{B_{tp}^T Q_c}))) \end{aligned} \tag{25}$$

where ϕ^{-1} is defined as $sign(x)\sqrt{|x|}$ and $\mathbf{Q_{tp}^0} - \mathbf{B_{tp}^T}\mathbf{Q_c} = \mathbf{Q_{tp}}$, which was derived the same way as $\mathbf{Q_t}$ in (21). With (25), a golden section line search is used to find the optimal step size $0 < a_* < 1$ such that $Q_* = Q_0 + a_*(Q_+ - Q_0)$ minimizes $m_* = L_\infty(f(Q_*) - Q_*)$. Where L_∞ is the infinity norm, Q_0 is the chord flows for the current iteration, $Q_+ = f(Q_0)$ is the chord flows for the following iteration and Q_* is the optimal chord flow values. The initial values used in this paper for the chord flows were 0. Q_* as well as m_* are returned after a fixed number of iterations for the golden section line search. Q_* is used for the next set of iterations and the whole process is repeated until m_* is less than the specified convergence tolerance. Note that (19)-(24) reduce to the equations shown in sub-section B when no loops or compressors are present. As a result, an iterative search algorithm will no longer be needed, and the flows can directly be solved for.

IV. Example Results for a 173 Bus, 51-Node Combined Case

A. Results for a Combined Electric-Gas Network

The natural gas pipeline system that was created for the test case was a section of the Texas natural gas pipeline system in Travis county and the synthetic electric grid that was used was the Travis County Transmission and Distribution System [16]. The synthetic grid contains 173 busses, with 7 natural gas-powered generator substations. The location of all the natural gas-powered plants and their power outputs were obtained from this system. The optimal power flow was solved on the commercial software PowerWorld Simulator to obtain the power output of these generators. The optimal power flow is solved rather than the conventional power flow to allow generators to optimally change their dispatch values to account for the change in load, while taking into consideration generator cost curves, system conditions and other factors. For simplicity, each substation was modeled as a single respective load in the gas system, with the summation of the power produced by each generator at the substation used to calculate the required gas flow.

As stated in Section III, EIA data for the natural gas consumption of the state of Texas is publicly available and was used to place the remaining loads around Travis County. The gas consumption was proportioned for the population of Travis County. The loads were then probabilistically placed within this area, each with a gas consumption until the total gas consumption was met. A processing plant nearby the county with enough capacity pressure of 2500 kPa. A loop was then manually placed to provide a direct path between two previously unconnected nodes. Once the test case dataset is built, it is solved to get benchmark results. The parameters for (2) used in this test case were the same presented in [18], where f is 0.00855, g is 0.58, Z is 0.91 and T is 308 K. The pipe flow resistance K_{ij} then becomes $6.4575 \times 10^7 L/D^5$ .The assumptions made here are that the friction factor and compressibility are constant for the entire system. The entire data set for this test case, as well as a figure with the nodes numbered can be viewed at [23].

With the initial results obtained, the compressor stations can be manually placed to meet specific requirements. The requirements for this system were arbitrarily chosen to mimic real-life scenarios in which certain pressure demands need to be met by pipeline operating companies. The first requirement was that the pressure at node 25 must be at 2500 kPa, the second is that the pressure at node 9 must be at 2500 kPa. The first compressor station (between nodes 24 and 25) was set to have a fixed compressor ratio of 1.29 and the second compressor station (between nodes 47 and 9) was set to have a fixed outlet pressure of 3000.1 kPa. With the flows along the respective pipelines, and pressures at the respective nodes already calculated, equation (1) can be used to calculate the resulting pressure at the inlet of the compressor, and the required output pressure (either directly input for a fixed pressure or as a multiple of the inlet pressure for a fixed compressor ratio) to meet the requirements stated. The results of this can also be viewed at [23].

For the dependence on the electric system, the compressors were then modeled as electric loads using equation (6). The optimal power flow of the electric system was run again to account for the slight difference in generation due to the compressor stations now being included. The required flows at the generators were re-calculated and the gas system was solved once more, and the compressor parameters were slightly tuned to meet the requirements. The process was repeated until the differences in the power generated by the gas-fueled generators between the iterations are minimal, which required 3 iterations to a convergence tolerance of 0.01 MW. This process converged successfully and the compressors were modeled as electric loads consuming 5.93 MW and 0.137 MW. The resulting MW outputs of the substations containing gas-powered generators can be found at [23].

Once the consumption of the compressor stations and outputs of the generators were fixed, the solution for the gas network was obtained with 6 iterations to a convergence tolerance of 10^{-6} cubic meters per hour and 10 iterations for the golden section line search. The solutions of the gas system were obtained without violating the fundamental equations. That is, the Weymouth equation was obeyed for every pipe and nodal balance was achieved at every node. Table I shows the decrease of the error in the loop with each iteration. Table II gives the general statistics of the combined network. Tables III and IV show samples of the end results.

B. Time-Series Case Study

To test the robustness and accuracy of the system and solution method, a simple time series study was conducted

TABLE I
Flow Error in Loop

Iteration No.	Error (m^3/hr)
1	0.2593E + 06
2	418.9
3	0.6869
4	9.572E - 04
5	1.333E - 06
6	1.877 - 09

TABLE II
COMBINED ELECTRIC-GAS NETWORK STATISTICS

Network Characteristics	Amount
Gas Nodes	51
Pipelines	51
Electric Gas Loads	7
Non-Electric Gas Loads	16
Electric Load Consumption (m^3/hr)	387,000
Non-Electric Load Consumption (m^3/hr)	292,800
Electric Busses	173
Electric Loads	132
Transformers	5
Substations	140
Gas-Powered Substations	7
Gas-Powered Generators	16

TABLE III
SAMPLE OF NODE DATA

Node No.	Node Name	Lat.	Lon.	Type	P (kPa)
24	Processing Plant #1	29.34	-97.11	0	2500
1	Load #1	30.38	-97.74	1	2261
2	Load #2	30.30	-97.71	1	2261
3	Load #3	30.21	-97.61	1	2262
4	Load #4	30.30	-97.61	1	2258
5	Load #5	30.15	-97.55	1	2273

TABLE IV
SAMPLE OF PIPE DATA

Starting Node	Ending Node	D (mm)	L (km)	k	Q (m^3/hr)
24	48	915	33.48	3.37E+06	679,800
49	25	915	33.48	3.37 E+06	679,800
35	2	650	4.438	2.47 E+06	684.8
32	3	650	0.237	1.32 E+06	79,090
47	4	650	2.418	1.35 E+06	104,100
31	5	650	1.841	1.02 E+06	70,060

to obtain metrics of the relationship between the electric and natural gas transmission networks. A 48-hour time-step simulation was used, where the input to the system was the changing electric system load. Hourly load data for the area, which is publicly available from ERCOT [24], was obtained and the load for the synthetic system was proportioned accordingly. A constant load multiplier is applied to each load at each hour to simulate the fluctuation of the load over a 48-hour time period. The OPF was solved at each time step, which allows for the generators' MW dispatch to optimally change throughout the simulation. At each time step, the gas-powered generator's power output was converted to a flow rate using (5) and the new steady state flows and pressures are solved for. For simplicity, the compressor stations' parameters remain constant throughout the simulation. The general gas loads also remained constant throughout the simulation to conduct a sensitivity analysis of the relationship explicitly between the electric load fluctuation and how much natural gas the system requires for each hour. The electric load fluctuation can be seen in Figure 5 and the natural gas required for each substation with at least one gas-powered generator at each hour is seen in Figure 6. The results depicted in Figure 6 demonstrates the relationship between the two systems. As the load is changing throughout the time series, the generators are optimally changing the required MW output to be able to supply the system's load. For each hour, the required natural gas consumption to be able to generate the MW output is calculated; thus presenting a detailed simulation where the two systems are shown directly interacting with one another.

C. Validation of the Numerical Solutions

The solver was validated by solving two networks presented in [6] in which the author provides sufficient data to be able to re-create the systems and uses the Weymouth equation to solve them. The first system (system 3.19) that was simulated is a 22-node, 36-pipe system with 15 loops and no compressor stations. The error between the flows obtained via the proposed algorithm and the flows shown in the literature were calculated for each pipe and were then averaged. No pressure data was given for this network. The average error between the flows for this system was found to be 1.52%. The second system (system 6.17) that was simulated is a 25-node, 35-pipe system with 14 loops and three compressor stations. The average errors between the flows and pressures for this system were found to be 0.0124% and 0.116% respectively. This validates that the solver was implemented correctly and that the numerical results found in this study are correct.

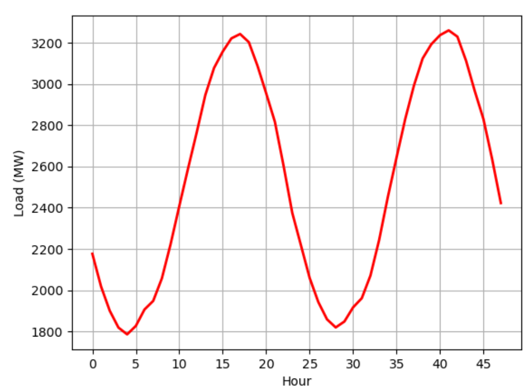

Fig. 5. Total electric system demand

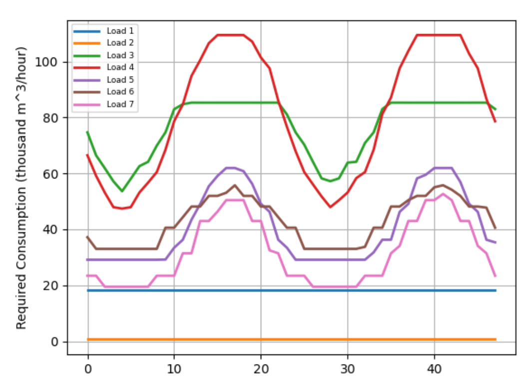

Fig. 6. Required natural gas demand for each substation

TABLE V
OPERATIONAL BEHAVIOR OF PROPOSED NETWORK AND NETWORKS IN PREVIOUS LITERATURE

System	Average Pressure (kPa)	Average Flow (m^3/hr)	No. CS	No. Sources	No. Loads
Proposed System	2327	137,642	2	1	23
System 3.19 [6]	3098	23,929	0	1	21
System 6.17 [6]	3593	83,792	3	1	18
Reference [8]	5983	361,110	2	7	17

TABLE VI
TOPOLOGICAL COMPARISON BETWEEN PROPOSED NETWORK AND NETWORKS IN PREVIOUS LITERATURE

System	No. Nodes	No. Edges	Average Node Degree	EPNR
Proposed System	51	51	2	1
Reference [4]	10	11	1.83	0.9167
Reference [5]	24	24	2	1
Reference [8]	20	24	2.4	1.2
Reference [18]	12	12	2	1

D. Operational and Structural Validation of the Constructed Natural Gas System

In addition to validating the numerical results to ensure that the solver was implemented correctly, the operation and the structure of the system need to be validated to ensure it is consistent with previous literature. The operation of the constructed network is validated by comparing operational parameters, such as the average pressure of the system, the average flow of the system, the number of compressor stations (abbreviated as CS) to networks seen in previous literature. Table V shows these comparisons between the system from this work and other works.

Table V shows that the parameters of the constructed system are consistent with the systems used in other literature and that the values of the pressures and flows are all within the same orders of magnitude. Pressure data was not given for system 3.19, so values from the solution obtained by the authors' solution algorithm were used. Furthermore, the total consumption for the electric and non-electric loads of the area is consistent with the data provided by [12].

With the behavior of the constructed system shown to be consistent with previous literature and available real-world data, it is also important to confirm that the structure of the system itself is consistent with the test systems used by other authors for research purposes in this field. This is done by treating the network as a small-world complex network [25] and comparing quantifiable properties of the system to the ones used in previous literature. The properties being compared are the average node degrees and the edge per node ratios. Both these parameters provide a general sense of the level of connectivity and structure of a network, and they can be used to compare networks of different sizes and structures. The comparisons are shown in Table VI, where compressor stations between two adjacent nodes are considered as edges for these calculations.

The accuracy of the network was validated by comparing its topology to other networks reported in previous literature. The systems are slightly different because some have more loops or no loops at all, which explains the slight differences in the parameters. Regardless, the comparison showed that the topology of the system was consistent with the systems used in previous studies, ensuring that the network was properly modeled.

V. CONCLUSION AND FUTURE WORK

In this paper, a method is presented to build a combined electric and natural gas network. The network is a synthetic test case and does not contain actual, confidential data. It is validated against actual data, so that it is realistic and useful for research and development. The methodology presented in this paper builds on previous work on solving combined systems and introduces a method to build a natural gas pipeline network with a dependence on an electric network. The result was a 51-node 49 pipeline system with 23 loads, seven of which are substations with gas powered generators. This system has an average pressure of 2327 kPa and an average flow of 137,642 cubic meters per hour, which is consistent with the magnitudes seen in previous works. The system also has an average node degree of 2 and an edge per node ratio of 1, which was also shown to be consistent with the systems used in other studies. This paper also implements a methodology to solve the system once the network has been built, thus establishing a quantitative interaction between the two systems. The system was solved within 6 iterations to a convergence tolerance of 10^{-6} cubic meters per hour. Results were obtained that mathematically satisfy the governing equations and have been validated to be consistent and correct. By implementing the two systems seen in [6], the solution algorithm was shown to be accurate, with the highest error being 1.5% for the flows and the pressures. A case study was conducted to provide a demonstration of how the combined system can be used for joint system analyses but also practically implements the method used to obtain the results.

The test case presented in this paper can be used for demonstration, validation, and evaluation purposes in order to enhance system operations planning and assist engineers in making effective decisions. Firstly, the test case provides a detailed understanding of the relationship between the electric and natural gas transmission networks. The results showed how electric system conditions and changes in the electric load manifest themselves in the natural gas system. This information is crucial for power engineers to consider when making decisions on the operations and planning of the transmission systems. As demonstrated by the time series case study, power engineers can have a quantitative measure of how much natural gas is required throughout the day to reliably operate their generators and can coordinate with pipeline operators accordingly. Secondly, the results of this paper demonstrate the importance of considering both electric and natural gas transmission systems when making decisions.

By understanding the interaction between the two systems, power engineers can better anticipate the impacts of changes in one system on the other. The systems used in this work provide a close representation of the actual network, allowing engineers to test different scenarios and act accordingly. By using real data, simulations such as these accurately reflects the conditions and constraints of the actual systems, providing a more accurate representation of their behavior. This information is particularly important in light of events such as Winter Storm Uri, where extreme weather conditions caused significant disruptions to both systems. By having a test case that represents real-world conditions, power engineers can better prepare for such events and take actions to maintain the reliability of the combined systems.

Several future extensions of this work are possible. First, current research is being done to expand the system to capture the entire Texas interstate and intrastate natural gas network. Unlike the real system, the network presented in this paper is an isolated system. The real system is much larger involving many sources, transactions between pipelines either directly or at trading hubs, imports, exports, and connections to underground storage reservoirs, all of which are being accounted for. The current algorithm that constructs the network provides a solid framework to be built upon to handle a higher number of nodes that are being introduced when the network is expanded. Next, a more formal method is needed to adjust compressor stations' parameters to account for changes in the power systems and should be included directly in the solution process. The results in this paper are promising and show that a converging solution is possible. Furthermore, fluctuating gas loads will be included in further case studies on this system to obtain data that more closely represents everyday gas consumption. Principles in complex network theory will be used to provide formally defined metrics to validate the network topology, as demonstrated by [25], which will prove to be useful for larger systems. Finally, future simulations can include dynamic behavior and linepack in the pipelines, to account for gas stored in a pipeline when contingencies occur or when there is a rapid change in demand.

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Yousef A. Abu-Khalifa (S'18) completed his B.S. degree in Electrical Engineering with a focus in Energy and Power at Texas A&M University in 2020. Continuing with a focus in electrical power systems, Yousef is currently pursuing a M.S. degree in Electrical Engineering and is projected to defend his thesis in the Spring 2023 semester. His primary research area is exploring methods for creating and simulating combined electric and natural gas transmission grids.

Adam B. Birchfield (S'13-M'18)is an Assistant Professor in the Department of Electrical and Computer Engineering at Texas A&M University. He received the B.E.E. degree from Auburn University in 2014, M.S. in electrical and computer engineering from the University of Illinois at Urbana-Champaign in 2016, and Ph.D. in electrical engineering from Texas A&M University in 2018. Dr. Birchfield's research is in power system modeling, large system transient dynamics, applications of synthetic power grid datasets, and the resilience of power systems to high-impact, low-frequency events.