UIN: \_\_\_\_\_ Section: \_\_\_\_ Score: \_\_\_\_

Consider a system of two non-linear equations with the variables  $\theta$  and V,

$$f_1(\theta, V) = 10 V \sin \theta + 2 = 0$$
  
$$f_2(\theta, V) = -10 V \cos \theta + 10 V^2 + 1 = 0$$

1. In preparation for the Newton-Raphson method, write the Jacobian matrix in terms of  $\theta$  and V.

2. Suppose our initial guess was  $\theta^{(0)} = 0$  and  $V^{(0)} = 1$ ; what are the values of  $f^{(0)}$  and  $J^{(0)}$ ?

3. Find the first Newton-Raphson iteration values of  $\theta^{(1)}$  and  $V^{(1)}$ . (Hint: To take the inverse of a diagonal matrix, just take the reciprocal of each element.)

1. 
$$J = \begin{bmatrix} 10 \ V \cos \theta & 10 \sin \theta \\ 10 \ V \sin \theta & -10 \cos \theta + 20 \ V \end{bmatrix}$$
2. 
$$f^{(0)} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}; J^{(0)} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$
3. 
$$\begin{bmatrix} \theta^{(1)} \\ V^{(1)} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0.9 \end{bmatrix}$$

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$$f^{(0)} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}; J^{(0)} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

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$$\begin{bmatrix} \theta^{(1)} \\ V^{(1)} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0.9 \end{bmatrix}$$