- A **phasor** is a complex number that represents a cosine-valued AC function
- The Root Mean Square (RMS) for cosine is found by dividing the maximum value by $\sqrt{2}$
- In polar form, $R\angle\theta$, a phasor represents the RMS voltage or current and phase angle
  - $R\angle\theta \rightarrow \sqrt{2}\,R\cos(2\pi f t + \theta)$
- Conversions to rectangular form: a+jb, and back can be done with these identities:
  - $R = \sqrt{a^2 + b^2}$ $\quad \theta = aTan\left(\frac{b}{a}\right)$ $\quad a = R\cos\theta$ $\quad b = R\sin\theta$
- Complex number addition can be done in rectangular form, and complex number multiplication can be done in polar form.
- Phasor diagrams have the real part on the x axis and imaginary part on the y axis.
- The angular frequency is $\omega = 2\pi f$.
- KVL, KCL, and Ohm's law all apply with AC phasor analysis exactly as with DC.
- The effect of resistors, inductors, and capacitors upon phasors is handled with **impedance (Z),** which acts like complex resistance. $V = I \cdot Z$
- The impedance of inductors and capacitors depends on frequency
- Instantaneous power from the time signal $p(t) = v(t)i(t)$

Element	Time Domain	Phasor Domain	Z (impedance)
Resistor	$v(t) = Ri(t)$	$\boldsymbol{V} = \boldsymbol{I}R$	R
Inductor	$v(t) = L\frac{di(t)}{dt}$	$\boldsymbol{V} = j\omega L\boldsymbol{I}$	$j\omega L$
Capacitor	$i(t) = C\frac{dv(t)}{dt}$	$\boldsymbol{V} = \frac{1}{j\omega C}\boldsymbol{I}$	$\frac{1}{j\omega C}$

- **Complex power**: $S = VI^* = |S|\angle\theta_s = P + jQ$ Don't forget the conjugate!! (*)
- Average power or active power or real power: $\mathrm{Re}[S] = P = |S|\cos\theta_s$
  - This is what's normally thought of as "power". Units are W.
  - It's also what you get if you take the average value of the instantaneous power
- Reactive power: $\mathrm{Im}[S] = Q = |S|\sin\theta_s$ Units are "var".
- Apparent power: $|S| = |V| \cdot |I|$ Units are "VA".
- Power factor angle: θ_s is the angle of S or $\theta_v - \theta_i$
- **Power factor**: $\cos(\theta_s) = P/|S|$. It must be indicated as "leading" (negative θ_s) or "lagging" (positive θ_s). A unity (1) power factor indicates zero reactive power and is neither leading nor lagging.
- **At every node in the system, both active (real) and reactive power are conserved**
- Inductors only absorb reactive power, capacitors only produce reactive power
- Capacitor banks are used for power factor correction by supplying reactive power locally
- A three-phase system is **balanced** if (1) all voltages are equal in magnitude and shifted in phase by 120°, (2) loads are equal on each phase, (3) impedances are equal on each phase.
- Line-to-line voltages are related to line-to-neutral (phase) voltages as: $V_{phase} = \frac{V_{line}}{\sqrt{3}\angle 30°}$
- Delta-connected loads can be replaced with wye-connected loads: $Z_Y = \frac{1}{3}Z_\Delta$
- Delta-connected sources can be replaced by wye-connected sources $V_{phase} = \frac{V_{line}}{\sqrt{3}\angle 30°}$
- Per-phase analysis works for balanced systems if there is no mutual inductance between phases. Steps: (1) convert delta sources and loads to equivalent wye (2) solve phase "a" circuit independent of other phases (3) total system power is $3V_aI_a^*$ (4) if needed, go back to original circuit to find "b" or "c" values and internal Δ values.

Homework #1 due Tuesday, Jan 24th Solutions

1. Practice problems for phasor conversion (solutions not given)

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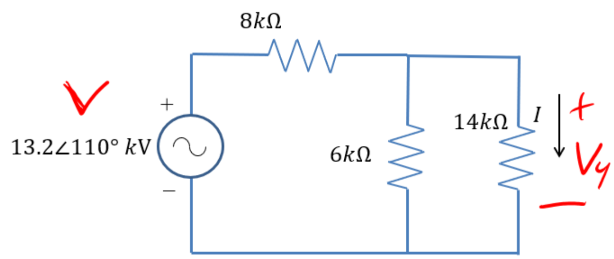
2. Solve for I as a phasor using any method. Assuming the frequency is 60 Hz, write the time signals for V and I.

- Solve for I as a phasor using any method
- Assuming the frequency is 1 kHz, write the time signals for V and I



$I_4 = \frac{V_s}{12.2k\Omega}$

$V_4 = 4k \cdot I_4 = V_s \frac{4}{12.2}$

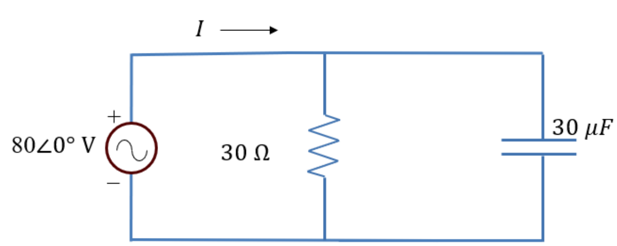
$I = V/R = V_s \frac{4}{12.2 \cdot 14k}$

$I = 0.324\angle 110°\ A$

$$v(t) = 13.2\sqrt{2}\cos(2\pi 60t + 110°)\ \mathrm{kV}$$
$$i(t) = 0.324\sqrt{2}\cos(2\pi 60t + 110°)\ \mathrm{kV}$$

3.

- Find the current phasor I for this 1 kHz circuit
- Impedance of the resistor?
  $$Z = 30\ \Omega$$
- Impedance of the capacitor?
  $$Z = \frac{1}{j\omega C} = \frac{1}{j2\pi 1k \cdot 30\mu} = -j5.3\Omega$$
- You can treat impedances just like complex resistances! The capacitor and resistor are in parallel
  $$Z = \frac{1}{\frac{1}{30} + \frac{1}{-j5.3}} = (0.91 - j5.14)\ \Omega$$
  $$I = \frac{V}{Z} = \frac{80}{0.91 - j5.14} = 15.32\angle 80°\mathrm{A}$$



What would this be as a time function?
What would it be in a lower frequency?

4.

Solve for the phasor V_c for this circuit if it is operated at 400 Hz

$Z_R = 50\,\Omega$

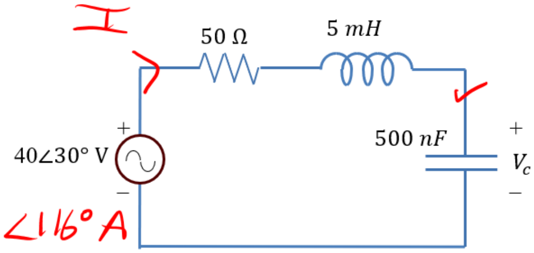
$Z_L = j12.566\,\Omega$

$Z_C = -j795.77\,\Omega$

$Z = 50 - j783.2\,\Omega$

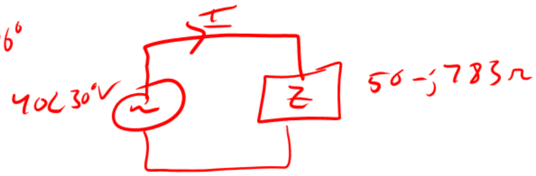
$I = \frac{V}{Z} = \frac{40\angle 30°\,V}{50 - j783\,\Omega} = .051\angle 116°\,A$

$784\angle -86°$



$V_c = I \cdot Z_c = (.051\angle 116°)(-j795)$

$= 40.5\angle 26°\ V$



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5. (Solution not given.)

6.

Calculate for the voltage source

1. Current I
2. Complex power S $= VI^* = 698\angle 14°\ VA = 677 + j169$
3. Active power P $(120\angle 30)(5.82\angle 16)^* =$
4. Reactive power Q
5. Apparent power $|S|$
6. Power factor angle θ_s
7. Power factor

$Z_{Total} = 20 + j6.4 - j1.4 = 20 + 5j$

$I = \frac{V}{Z} = \frac{120\angle 30}{20.6\angle 14} = 5.82\angle 16°\,A$

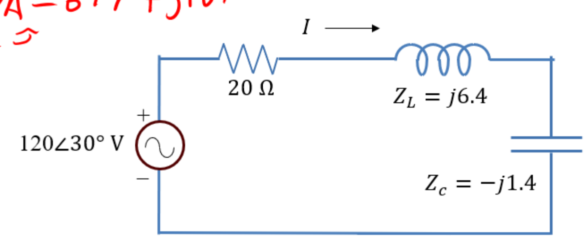
677 W

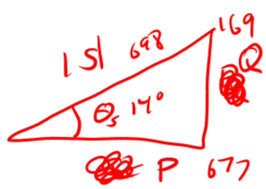
168.9 var

698 VA

14°

0.97 lagging





Bonus: Can you verify conservation of both active and reactive power?

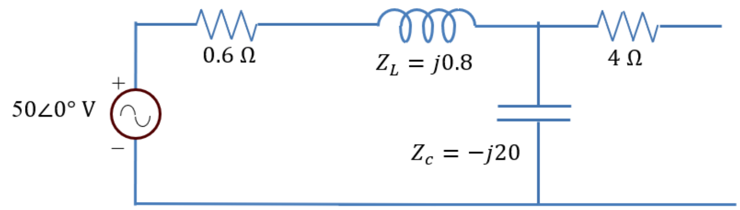
7. (Solution not given – quite similar to online video.)

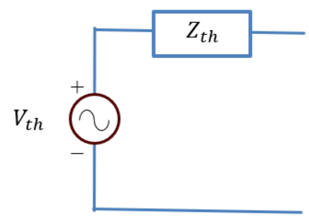
8.

We want to find

(1) The Thevenin equivalent for this circuit, as shown.

(2) The maximum power that could be delivered to a load with impedance Z_{load}





Maximum power transfer occurs when $Z_{load} = Z_{th}^*$

$$V_{th} = 52.06\angle -1.79° \text{ V}$$
$$Z_{th} = 4.65 + j0.81\ \Omega$$
$$S_{\text{Max}} = 145.69 - j25.47 \text{ W}$$