ECEN 460 handout for Jan 1, 2023

Class 1: Three-Phase AC Power Calculations

- A phasor is a complex number that represents a cosine-valued AC function
- The Root Mean Square (RMS) for cosine is found by dividing the maximum value by $\sqrt{2}$
- In polar form, $R \ge \theta$, a phasor represents the RMS voltage or current and phase angle - $R \angle \theta \rightarrow \sqrt{2} R \cos(2\pi f t + \theta)$
- Conversions to rectangular form: a+jb, and back can be done with these identities:

$$R = \sqrt{a^2 + b^2}$$
 $\theta = aTan\left(\frac{b}{a}\right)$ $a = R\cos\theta$ $b = R\sin\theta$

- Complex number addition can be done in rectangular form, and complex number multiplication can be done in polar form.
- Phasor diagrams have the real part on the x axis and imaginary part on the y axis.
- The angular frequency is $\omega = 2\pi f$.
- KVL, KCL, and Ohm's law all apply with AC phasor analysis exactly as with DC.
- The effect of resistors, inductors, and capacitors upon phasors is handled with **impedance** (**Z**), which acts like complex resistance. $V = I \cdot Z$
- The impedance of inductors and capacitors depends on frequency
- Instantaneous power from the time signal p(t) = v(t)i(t)
- Complex power: $S = VI^* = |S| \angle \theta_s = P + jQ$ Don't forget the conjugate!! (*)
- Average power or active power or real power: $\operatorname{Re}[S] = P = |S| \cos \theta_s$
 - This is what's normally thought of as "power". Units are W.
 - It's also what you get if you take the average value of the instantaneous power
- Reactive power: $\text{Im}[S] = Q = |S| \sin \theta_s$
- Apparent power: $|S| = |V| \cdot |I|$
- Power factor angle: θ_s is the angle of S or $\theta_v \theta_i$
- **Power factor**: $\cos(\theta_s) = P/|S|$. It must be indicated as "leading" (negative θ_s) or "lagging" (positive θ_s). A unity (1) power factor indicates zero reactive power and is neither leading nor lagging.
- At every node in the system, both active (real) and reactive power are conserved
- Inductors only absorb reactive power, capacitors only produce reactive power
- Capacitor banks are used for power factor correction by supplying reactive power locally
- A three-phase system is **balanced** if (1) all voltages are equal in magnitude and shifted in phase by 120°, (2) loads are equal on each phase, (3) impedances are equal on each phase.
- Line-to-line voltages are related to line-to-neutral (phase) voltages as: $V_{phase} = \frac{V_{line}}{\sqrt{3} \neq 30^{\circ}}$
- Delta-connected loads can be replaced with wye-connected loads: $Z_Y = \frac{1}{3}Z_{\Delta}$
- Delta-connected sources can be replaced by wye-connected sources $V_{phase} = \frac{V_{line}}{\sqrt{3} \angle 30^{\circ}}$
- · Per-phase analysis works for balanced systems if there is no mutual inductance between phases. Steps: (1) convert delta sources and loads to equivalent wye (2) solve phase "a" circuit independent of other phases (3) total system power is $3V_aI_a^*$ (4) if needed, go back to original circuit to find "b" or "c" values and internal Δ values.

Element	Time Domain	Phasor	Z
		Domain	(impedance)
Resistor	v(t) = Ri(t)	V = IR	R
Inductor	$v(t) = L \frac{di(t)}{dt}$	$\boldsymbol{V} = j\omega L \boldsymbol{I}$	jωL
Capacitor	$i(t) = C \frac{dv(t)}{dt}$	$\boldsymbol{V} = \frac{1}{j\omega C} \boldsymbol{I}$	$\frac{1}{j\omega C}$

Units are "var". Units are "VA". ECEN 460 handout for Jan 1, 2023 Class 1: Three-Phase AC Power Calculations

Homework #1 due Tuesday, Jan 24th

For more help, read Chapters 1 and 2 in the textbook, view the videos and slides on the website, and take advantage of office hours of TAs and the instructor.

- 1. Practice problems for phasor conversion
 - a. Convert $5 \angle 12^\circ$ A to rectangular form.
 - b. Convert $14 \angle 20^{\circ}$ V to the cosine time function, assuming a frequency of 14 kHz.
 - c. Find the polar form phasor for $20\cos(377t 40^\circ)$ kV.
 - d. Convert the phasor 12-j3 A to polar form.
 - e. Sketch a time plot of the phasor 18∠12° mA, assuming a frequency of 100 MHz.
 - f. Draw a phasor diagram for the phasor $35 \angle -110^{\circ}$ V.
 - g. Convert $24 \angle -60^\circ$ A to rectangular form.
 - h. Convert $30 \ge 0^\circ$ V to the cosine time function, assuming a frequency of 50 Hz.
 - i. Find the rectangular form phasor for $20 \cos((6.28 \times 10^6)t + 18^\circ)$ kV.
 - j. Convert the phasor 30+j30 kA to polar form.
 - k. Sketch a time plot of the phasor $1.32 \angle 10^{\circ}$ MV, assuming a frequency of 60 Hz.
 - 1. Draw a phasor diagram for the two phasors $3.5 \ge 10^{\circ} A$ and $2.7 \ge 40^{\circ} A$.
 - m. Convert $16 \ge -90^\circ$ A to rectangular form.
 - n. Convert $100.5 \ge 0^{\circ}$ V to the cosine time function, assuming a frequency of 400 Hz.
 - o. Find the polar form phasor for $200 \sin(377t)$ kV.
 - p. Convert the phasor j5 V to polar form.
 - q. Sketch a time plot of the phasor $300 \ge -90^{\circ}$ V, assuming a frequency of 10 Hz.
 - r. Draw a phasor diagram for the phasor $3.25 \angle 0^{\circ}$ V.
 - s. Convert $2.0 \angle 90^{\circ}$ MV to rectangular form.
 - t. Convert $74.5 \angle 14^{\circ}$ V to the cosine time function, assuming a frequency of 2500 Hz.
 - u. Find the rectangular form phasor for $55 \cos(10^9 t 108^\circ)$ V. What is the frequency?
 - v. Convert the phasor 10-j30000 kA to polar form.
 - w. Sketch a time plot of the phasor $100 \ge 90^{\circ}$ A, assuming a frequency of 6000 Hz.
 - x. Draw a phasor diagram for the two phasors $90 \ge 90^\circ A$ and $90 \ge -90^\circ A$.

2. Solve for *I* as a phasor using any method. Assuming the frequency is 60 Hz, write the time signals for *V* and *I*.



3. Find the current phasor I for this 1 kHz circuit



4. Solve for the phasor V_c for this circuit if it is operated at 400 Hz



5. Solve for the phasor I_1 . The impedance for Z_c is given so you don't need the frequency.



6.

Calculate for the voltage source

- 1. Current I
- 2. Complex power S
- 3. Active power P
- 4. Reactive power Q
- 5. Apparent power |S|
- 6. Power factor angle θ_s
- 7. Power factor



ECEN 460 handout for Jan 1, 2023

Class 1: Three-Phase AC Power Calculations

7. A factory is acting like a 250 Ω resistor in parallel with a 500 mH inductor. A single-phase power distribution line supplying the factory from the substation can be modeled as a 65 mH inductor. At the substation, the voltage is $12 \ge 6.5^{\circ}$ kV.

- a. Draw this circuit and find the impedance of the circuit elements. The substation can be modeled as a voltage source. The frequency is 60 Hz.
- b. What is the voltage at the factory?
- c. How much active and reactive power is the factory absorbing?
- d. What is the factory's power factor?
- e. If a 50 μ F capacitor is installed in parallel with the factory, how do the voltage and power factor change?
- f. What should the value of the capacitor be so that the power factor is 0.95 lagging?



Homework #2 due Tuesday, Jan 31st

Textbook problems: 2.9, 2.22, 2.28, 2.43, 2.48