Homework 1 Solutions

2.9

$$KVL: \ 120\angle 0° = (60\angle 0°)(0.1 + j0.5) + \overline{V}_{LOAD}$$

$$\therefore \ \overline{V}_{LOAD} = 120\angle 0° - (60\angle 0°)(0.1 + j0.5)$$

$$= 114.1 - j30.0 = 117.9\angle -14.7° \text{V}$$

2.22

$$\text{(a)} \ \overline{Y}_1 = \frac{1}{\overline{Z}_1} = \frac{1}{(4 + j5)} = \frac{1}{6.4\angle 51.34°} = 0.16\angle -51.34°$$

$$= (0.1 - j0.12)\text{S}$$

$$\overline{Y}_2 = \frac{1}{\overline{Z}_2} = \frac{1}{10} = 0.1\text{S}$$

$$P = V^2(G_1 + G_2) \Rightarrow V = \sqrt{\frac{P}{G_1 + G_2}} = \sqrt{\frac{1000}{(0.1 + 0.1)}} = 70.71 \text{ V}$$

$$P_1 = V^2 G_1 = (70.71)^2 \, 0.1 = 500 \text{ W}$$

$$P_2 = V^2 G_2 = (70.71)^2 \, 0.1 = 500 \text{ W}$$

$$\text{(b)} \ \overline{Y}_{eq} = \overline{Y}_1 + \overline{Y}_2 = (0.1 - j0.12) + 0.1 = 0.2 - j0.12$$

$$= 0.233\angle -30.96° \text{ S}$$

$$I_S = V \, Y_{eq} = 70.71(0.233) = 16.48 \text{ A}$$

2.28

$$\bar{S}_1 = 15 + j6.667$$
$$\bar{S}_2 = 3(0.96) - j3\left[\sin\left(\cos^{-1} 0.96\right)\right] = 2.88 - j0.84$$
$$\bar{S}_3 = 15 + j0$$
$$\bar{S}_{TOTAL} = \bar{S}_1 + \bar{S}_2 + \bar{S}_3 = (32.88 + j5.827)\,\text{kVA}$$

(i) Let $\bar{Z}$ be the impedance of a series combination of R and X

Since $\bar{S} = \bar{V}\bar{I}^* = \bar{V}\left(\dfrac{\bar{V}}{\bar{Z}}\right)^* = \dfrac{V^2}{\bar{Z}^*}$, it follows that

$$\bar{Z}^* = \frac{V^2}{\bar{S}} = \frac{(240)^2}{(32.88 + j5.827)10^3} = (1.698 - j0.301)\,\Omega$$
$$\therefore \bar{Z} = (1.698 + j0.301)\,\Omega \quad \leftarrow$$

(ii) Let $\bar{Z}$ be the impedance of a parallel combination of R and X

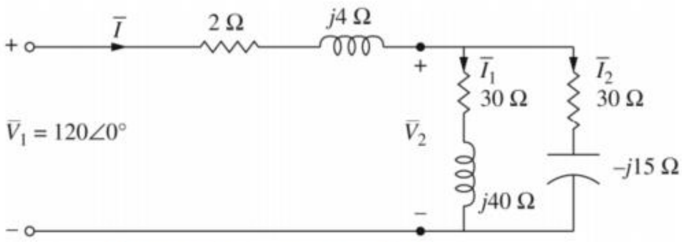
Then $$R = \frac{(240)^2}{(32.88)10^3} = 1.7518\,\Omega$$
$$X = \frac{(240)^2}{(5.827)10^3} = 9.885\,\Omega$$
$$\therefore \bar{Z} = (1.7518 \| j9.885)\,\Omega \quad \leftarrow$$

2.43

(a) Transforming the Δ-connected load into an equivalent Y, the impedance per phase of the equivalent Y is

$$\bar{Z}_2 = \frac{60 - j45}{3} = (20 - j15)\,\Omega$$

With the phase voltage $V_1 = \frac{120\sqrt{3}}{\sqrt{3}} = 120\,\text{V}$ taken as a reference, the per-phase equivalent circuit is shown below:



Total impedance viewed from the input terminals is

$$\bar{Z} = 2 + j4 + \frac{(30 + j40)(20 - j15)}{(30 + j40) + (20 - j15)} = 2 + j4 + 22 - j4 = 24\,\Omega$$

$$\bar{I} = \frac{\bar{V}_1}{\bar{Z}} = \frac{120\angle 0°}{24} = 5\angle 0°\,\text{A}$$

The three-phase complex power supplied = $\bar{S} = 3\bar{V}_1\,\bar{I}^* = 1800\,\text{W}$

$P = 1800\,\text{W}$ and $Q = 0\,\text{VAR}$ delivered by the sending-end source

(b) Phase voltage at load terminals $\bar{V}_2 = 120\angle 0° - (2 + j4)(5\angle 0°)$

$$= 110 - j20 = 111.8\angle -10.3°\,\text{V}$$

The line voltage magnitude at the load terminal is

$$(V_{\text{LOAD}})_{L\text{-}L} = \sqrt{3}\,111.8 = 193.64\,\text{V}$$

(c) The current per phase in the Y-connected load and in the equiv. Y of the Δ-load:

$$\bar{I}_1 = \frac{\bar{V}_2}{\bar{Z}_1} = 1 - j2 = 2.236\angle -63.4°\,\text{A}$$

$$\bar{I}_2 = \frac{\bar{V}_2}{\bar{Z}_2} = 4 + j2 = 4.472\angle 26.56°\,\text{A}$$

The phase current magnitude in the original Δ-connected load

$$(I_{ph})_\Delta = \frac{I_2}{\sqrt{3}} = \frac{4.472}{\sqrt{3}} = 2.582\,\text{A}$$

(d) The three-phase complex power absorbed by each load is

$$\overline{S}_1 = 3\overline{V}_2\overline{I}_1^* = 430\,\text{W} + j600\,\text{VAR}$$
$$\overline{S}_2 = 3\overline{V}_2\overline{I}_2^* = 1200\,\text{W} - j900\,\text{VAR}$$

The three-phase complex power absorbed by the line is

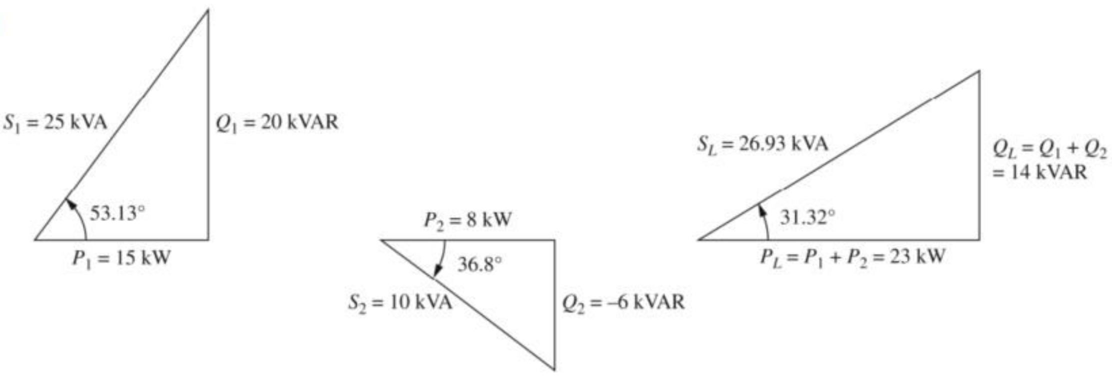
$$\overline{S}_L = 3\left(R_L + jX_L\right)I^2 = 3\left(2 + j4\right)(5)^2 = 150\,\text{W} + j300\,\text{VAR}$$

The sum of load powers and line losses is equal to the power delivered from the supply:

$$\overline{S}_1 + \overline{S}_2 + \overline{S}_L = \left(450 + j600\right) + \left(1200 - j900\right) + \left(150 + j300\right)$$
$$= 1800\,\text{W} + j0\,\text{VAR}$$

2.48

(a)



(b) $pf = \cos 31.32° = 0.854$ Lagging

(c) $I_L = \dfrac{S_L}{\sqrt{3}V_{LL}} = \dfrac{26.93\times10^3}{\sqrt{3}\left(480\right)} = 32.39\,\text{A}$

(d) $Q_C = Q_L = 14\times10^3\ \text{VAR} = 3\left(V_{LL}\right)^2 / X_\Delta$

$$X_\Delta = \frac{3\left(480\right)^2}{14\times10^3} = 49.37\,\Omega$$

(e) $I_C = V_{LL} / X_\Delta = 480 / 49.37 = 9.72\,\text{A}$

$$I_{LINE} = \frac{P_L}{\sqrt{3}\ V_{LL}} = \frac{23\times10^3}{\sqrt{3}\ 480} = 27.66\,\text{A}$$