

Class 16: Phasor analysis for AC circuits

- A **phasor** is a complex number that represents a cosine-valued AC function
 - The Root Mean Square (RMS) for cosine is found by dividing the maximum value by $\sqrt{2}$
 - In polar form, $R\angle\theta$, a phasor represents the RMS voltage or current and phase angle

$$R\angle\theta \rightarrow \sqrt{2} R \cos(2\pi f t - \theta)$$
 - Conversions to rectangular form: $a+jb$, and back can be done with these identities:

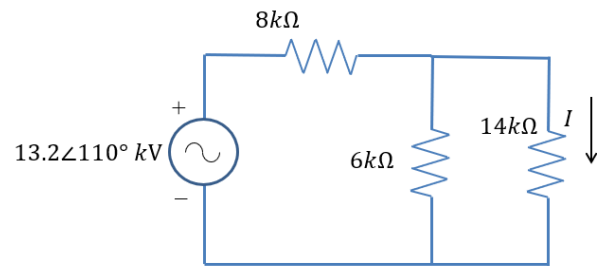
$$R = \sqrt{a^2 + b^2} \quad \theta = a \text{Tan} \left(\frac{b}{a} \right) \quad a = R \cos \theta \quad b = R \sin \theta$$
 - Complex number addition can be done in rectangular form, and complex number multiplication can be done in polar form.
 - Phasor diagrams have the real part on the x axis and imaginary part on the y axis.
 - The angular frequency is $\omega = 2\pi f$.
 - **KVL, KCL, and Ohm's law all apply with AC phasor analysis exactly as with DC.**
 - This means you can use Node-Voltage and Mesh-Current analysis methods too.
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Practice problems for phasor conversion

1. Convert $5\angle 12^\circ$ A to rectangular form.
2. Convert $14\angle 20^\circ$ V to the cosine time function, assuming a frequency of 14 kHz.
3. Find the polar form phasor for $20 \cos(377t - 40^\circ)$ kV.
4. Convert the phasor $12-j3$ A to polar form.
5. Sketch a time plot of the phasor $18\angle 12^\circ$ mA, assuming a frequency of 100 MHz.
6. Draw a phasor diagram for the phasor $35\angle -110^\circ$ V.
7. Convert $24\angle -60^\circ$ A to rectangular form.
8. Convert $30\angle 0^\circ$ V to the cosine time function, assuming a frequency of 50 Hz.
9. Find the rectangular form phasor for $20 \cos((6.28 \times 10^6)t + 18^\circ)$ kV.
10. Convert the phasor $30+j30$ kA to polar form.
11. Sketch a time plot of the phasor $1.32\angle 10^\circ$ MV, assuming a frequency of 60 Hz.
12. Draw a phasor diagram for the two phasors $3.5\angle 10^\circ$ A and $2.7\angle 40^\circ$ A.
13. Convert $16\angle -90^\circ$ A to rectangular form.
14. Convert $100.5\angle 0^\circ$ V to the cosine time function, assuming a frequency of 400 Hz.
15. Find the polar form phasor for $200 \sin(377t)$ kV.
16. Convert the phasor $j5$ V to polar form.
17. Sketch a time plot of the phasor $300\angle -90^\circ$ V, assuming a frequency of 10 Hz.
18. Draw a phasor diagram for the phasor $3.25\angle 0^\circ$ V.
19. Convert $2.0\angle 90^\circ$ MV to rectangular form.
20. Convert $74.5\angle 14^\circ$ V to the cosine time function, assuming a frequency of 2500 Hz.
21. Find the rectangular form phasor for $55 \cos(10^9 t - 108^\circ)$ V. What is the frequency?
22. Convert the phasor $10-j30000$ kA to polar form.
23. Sketch a time plot of the phasor $100\angle 90^\circ$ A, assuming a frequency of 6000 Hz.
24. Draw a phasor diagram for the two phasors $90\angle 90^\circ$ A and $90\angle -90^\circ$ A.

Example 1

- Solve for I as a phasor using any method
- Assuming the frequency is 1 kHz, write the time signals for V and I



Example 2

- $i_s(t) = 7.1 \cos(1256t - 10^\circ) \text{ A}$
- $v_s(t) = 113.1 \cos(1256t + 45^\circ) \text{ V}$
- Find $v_1(t)$ using phasor analysis
- Steps:
 - Convert time signals into phasors (complex numbers)
 - Solve the circuit using regular circuit techniques (just with complex numbers)
 - Convert the answer back into a time signal with the same frequency

