Solving differential equations in this class of the form $a\frac{dy}{dx} + by(t) = x(t), y(0) = y_0$

1. Find the **homogenous** solution of the form $Ae^{-\frac{bt}{a}}$ by setting $x(t) = 0$, with an arbitrary constant A.
2. Guess the **particular** solution based on the form of $x(t)$, using the table at right.
3. Combine the two solutions to find the **general** solution $y(t) = y_h(t) + y_p(t)$
4. Apply **initial condition** y_0 to find the unknown constant.

$x(t)$	$y_p(t)$
Constant - b_0	c_1
Linear - $b_0 + b_1 t$	$c_0 + c_1 t$
Quadratic - $b_0 + b_1 t + b_2 t^2$	$c_0 + c_1 t + c_2 t^2$
Exponential - e^{rt}	$c_1 e^{rt}$
Sine - $\sin(\omega t)$	$c_1 \cos(\omega t) + c_2 \sin(\omega t)$
Cosine - $\cos(\omega t)$	$c_1 \cos(\omega t) + c_2 \sin(\omega t)$

Example 1: Solve this differential equation for $v(t)$ for $t \geq 0$, given initial condition $v(0) = 2$ V

$$5\frac{dv(t)}{dt} + 100{,}000\, v(t) = 0$$

Example 2: Solve this differential equation for $i(t)$ for $t \geq 0$, in terms of R and L, given initial condition $i(0) = 0$.

$$L\frac{di(t)}{dt} = 5 - i(t) \cdot R$$

Example 3:

$$\frac{dy}{dt} + 3y(t) = 2t + 5, \quad y(0) = -4$$

Example 4:

$$2\frac{dy}{dt} + 5y(t) = 1, \quad y(0) = 3$$

Example 5:
Solve for both of the following voltages for $t > 0$, given that they both begin at 150 V at t=0. The values of the capacitances are $C_1 = 1\mu F$ and $C_2 = 100\mu F$.

$$C_1\frac{dv_1}{dt} = \frac{v_1}{0.2} \qquad\qquad C_2\frac{dv_2}{dt} = \frac{v_2}{0.5}$$

Example 6:

$$\frac{dy}{dt} + 3y(t) = e^{-3t}, y(0) = -1$$